# The Dynamics of 3-dimentional micro-mechanic sensor of angle motions of a robot-hexapod 

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#### Abstract

MEMS sensor of angular motions and spectral density of noises measurements were estimated with the help of the method of structural identification for first time. This provides the possibility to control and operate the angle position of the platform of robot-hexapod.


Keywords - micro-mechanic gyroscope, angular motion, hexapod, identification, factorization, separation, noise

## I. INTRODUCTION

Realization of potential possibilities of hexapods in the sphere of high-precision modelling of the motions of dynamic objects, stabilization of measuring appliances on the rolling foundation, the control of working organs of a robot are impossible without using micro-electromechanic sensors (MEMS) [1]. These kinds of appliances include 3d MEMS gyroscopes.

They are designed to measure projections of the vector of angular velocity of the foundation of the appliance on the axis of sensitivity of the gyroscope. The advantage of the sensors is their small dimensions and relatively low price. The main disadvantage is the presence of considerable errors and noises [2] with the characteristics dependable on the character of the motions of the appliance. One of the methods of increasing the accuracy of MEMS gyroscopes is the optimal stochastic filtration of signals. The advantage of the method is the low price of the resulting appliance under high accuracy of measurements.

There are many well-known methods of optimal filtration of signals at the output of the appliance, for example [3,4]. All of them involve the knowledge of mathematic models of dynamics of the measuring appliance and noises which act at its outputs in the real conditions of functioning.

The analysis of the models of dynamics of these appliances according to the sources $[5,6,7]$ shows the necessity to specify the models, especially in the conditions of using 3d sensors under many-dimensional swinging of the foundation. As it is shown in the article [8], the precision of the models is possible to achieve as a result of dynamic attestation of the measuring appliance [9], held in the conditions close to natural.

The article presents the search for the model of dynamics of micro-mechanical 3d measurer of angle velocities according to experimental data.

## II. Purpose and task of research

Let us consider 3d sensor of angular velocities as an automation element (Fig. 1). The vector of programme signals $r$ acts at its input,

$$
\mathrm{r}=\left[\begin{array}{lll}
\dot{\gamma}_{\mathrm{r}}, & \dot{\vartheta}_{\mathrm{r}}, & \dot{\psi}_{\mathrm{r}} \tag{1}
\end{array}\right],
$$

where $\dot{\gamma}_{r}$ is the angular velocity of the roll of the sensor foundation, $\dot{\vartheta}_{\mathrm{r}}$ is the angular velocity of the pitch, $\dot{\psi}_{\mathrm{r}}$ is the angular velocity of the yaw. As it is shown on Fig. 1 the vector of measured signals $x$ acts at the output

$$
\begin{equation*}
\mathrm{x}=\left[\dot{\gamma}_{\mathrm{x}}, \quad \dot{乌}_{\mathrm{x}}, \quad \dot{\psi}_{\mathrm{x}}\right] \tag{2}
\end{equation*}
$$

where $\dot{\gamma}_{x}$ is the measured value of the velocity of the change of the roll angle, $\dot{\vartheta}_{x}$ is the measured value of the velocity of the change of the pitch angle, $\dot{\psi}_{x}$ is the measured value of the velocity of the yaw angle. Generally, the vectors (1) and (2) do not coincide because of the presence of the noise measure vector $\varphi$ and the dynamics of the sensor which is characterized by the matrix of transfer functions W .


Figure 1. Structural scheme of the multidimensional sensor
Let us consider the vector $r$ to be a centered stationary random process and the regular composition of the errors of measurements is fully compensated.

The objective of the research is to determine the model of dynamics of 3d MEMS gyroscope and to assess the degree of mutual influence of the one channel of measurement on the other in the conditions of stationary very-low-frequency random rolling of the foundation of the appliance.
In order to reach the goal it is supposed to use experimental and analytical method [9]. Its realization is based on the placement of the sensor under research on the platform of the three-stage dynamic stand (Fig. 2).


Figure 2. The three-stage dynamic stand
The input of the control system of the stand received the realization of the stationary random process of rolling of the "Yuriy Gagarin" ship. The matrix of spectral densities is given in [10]. Whereupon, the foundation of the sensor performed random spatial motions according to the roll and the pitch.
The result of the experiment presented the records of measurement of the two components of the vector $r$ : $\dot{\gamma}_{r}$ and $\dot{\vartheta}_{\mathrm{r}}$, if $\dot{\psi}_{\mathrm{r}}=0$ (Fig. 3).


Figure 3. The records of change of the two components of the vector

The reaction on these input signals is the vector of angle velocities (2), which was received at the output of
the 3d MEMS gyroscope L3G4200D of STMicroelectronics Company. The realizations of the components of the vector $\dot{\gamma}_{\mathrm{x}}$ and $\dot{\vartheta}_{\mathrm{x}}$ are shown on Fig. 4.


So, in order to reach the goal of the research it is advisable to solve the following task. Using the known records of the vectors $r$ and $x$, which were received as a result of the experiment, we should find the transfer functions of the 3d MEMS gyroscope W and the matrix of spectral densities of the noises of measurements $\mathrm{S} \varphi \varphi$.

## III. METHODOLOGY OF SOLVING THE TASK

The calculation of the estimates of the average of distribution of the realizations of the components $r$ and $x$ show that the random processes may be considered as centered, two-dimensional and stationary. So, in order to solve the above-mentioned task we may use the algorithm which is well-grounded in the book [11].

If to denote as $S_{r r}^{\prime}$ the transposed matrix of spectral densities of the vector (1), $S_{x x}^{\prime}$ - the transposed matrix of spectral densities of the vector (2), $\mathrm{S}_{\mathrm{rx}}^{\prime}$ - the transposed matrix of mutual spectral densities between the vectors $r$ and $x$, then the matrix of transfer functions of the sensor W (Fig. 1), is determined by the relation:

$$
\begin{equation*}
\mathrm{W}=\mathrm{S}_{\mathrm{rx}}^{\prime}\left(\mathrm{S}_{\mathrm{rr}}^{\prime}\right)^{-1} \tag{3}
\end{equation*}
$$

If there is no correlation between the signals of the rolling of the foundation (vector $r$ ), which acts at the input of the sensor and the noise of the measurement (vector $\varphi$ ), which acts at the output of the sensor, then the transposed matrix of spectral densities of the noises may be calculated as:

$$
\begin{equation*}
S_{\varphi \varphi}^{\prime}=S_{x x}^{\prime}-S_{r x}^{\prime}\left(S_{r r}^{\prime}\right)^{-1} S_{x r}^{\prime} \tag{4}
\end{equation*}
$$

where $S_{\mathrm{xr}}^{\prime}$ is the transposed matrix of mutual spectral densities between vectors $x$ and $r$.

Taking into account the results (3), (4), general methodology of solving the task will be to carry out the following actions:

- to determine the estimates of the spectral and mutual spectral densities of the signals "input-output" of the sensor which were received as a result of the experiment;
- to make approximation of the received estimates on the class of fractional rational functions of the complex variable $\mathrm{s}=\mathrm{j} \omega, j=\sqrt{-1}$;
- to calculate the matrix of transfer functions of the measurer W;
- to find the transposed matrix of spectral densities of the noises of measurement $S_{\varphi \varphi}^{\prime}$;
- to determine the structure and parameters of the matrix of transfer functions of the filter G, which forms the noise of measurement;
- to make a model of the sensor and to input the record of the vector $r$;
- to assess the accuracy of the identification.


## IV. THE RESULTS OF THE RESEARCH

Processing of the records of the components of the vectors $r$ and $x$, with the help of Blackman-Tukey method and approximation of the received curves let us formulate the following transposed matrices of the spectral and mutual spectral densities.

$$
\begin{gather*}
\mathrm{S}_{\mathrm{r}}^{\prime}=\left[\begin{array}{cc}
\frac{3.52|\mathrm{~s}+0.38|^{2}}{\left|\mathrm{z}_{1}\right|^{2}} & \frac{-3.14(\mathrm{~s}+0.71)|\mathrm{s}+0.38|^{2}}{\mathrm{z}_{1} \cdot \overline{\mathrm{z}}_{2} \cdot \overline{\mathrm{z}}_{3}} \\
\frac{3.14(\mathrm{~s}-0.71)|\mathrm{s}+0.38|^{2}}{\overline{\mathrm{z}}_{1} \cdot \mathrm{z}_{2} \cdot \mathrm{z}_{3}} & \frac{5.73(\mathrm{~s}+0.71)(\mathrm{s}+0.38) \mid}{\left|\mathrm{z}_{2}\right|^{2} \cdot\left|\mathrm{z}_{3}\right|^{2}}
\end{array}\right],  \tag{5}\\
\mathrm{S}_{\mathrm{rx}}^{\prime}=\left[\begin{array}{cc}
1.52 \mathrm{~s}+\left.0.38\right|^{2}\left(\mathrm{~s}^{2}+0.34 \mathrm{~s}+0.28\right) \times & 1.39|\mathrm{~s}+0.38|^{2}(\mathrm{~s}+0.26)(\mathrm{s}+0.71)(\mathrm{s}-2.27) \times \\
\frac{\times(\mathrm{s}+0.13)\left(\mathrm{s}^{2}+0.21 \mathrm{~s}+1.67\right)}{(\mathrm{s}+0.06) \mathrm{z}_{2} \cdot\left|\mathrm{z}_{1}\right|^{2} \cdot \mathrm{z}_{3}} & \frac{\times\left(\mathrm{s}^{2}+0.19 \mathrm{~s}+0.24\right)\left(\mathrm{s}^{2}-0.03 \mathrm{~s}+0.98\right)}{(\mathrm{s}+0.06) \cdot\left|\mathrm{z}_{2}\right|^{2} \cdot \mathrm{z}_{1} \cdot\left|\mathrm{z}_{3}\right|^{2}} \\
-0.74|\mathrm{~s}+0.38|^{2}\left(\mathrm{~s}^{2}+0.39 \mathrm{~s}+0.33\right) \times & 0.32|\mathrm{~s}+0.38|^{2}(\mathrm{~s}+0.71)(\mathrm{s}-3.85) \times \\
\frac{\times\left(\mathrm{s}^{2}+0.53 \mathrm{~s}+1.57\right)}{\mathrm{z}_{2} \cdot\left|\mathrm{z}_{1}\right|^{2} \cdot \mathrm{z}_{3}} & \frac{\times\left(\mathrm{s}^{2}+0.32 \mathrm{~s}+0.29\right)\left(\mathrm{s}^{2}+0.18 \mathrm{~s}+1.13\right)}{\left|\mathrm{z}_{2}\right|^{2} \cdot \mathrm{z}_{1} \cdot\left|\mathrm{z}_{3}\right|^{2}}
\end{array}\right],  \tag{6}\\
\mathrm{S}_{\mathrm{xx}}^{\prime}=\left[\begin{array}{cc}
\frac{\mathrm{A}}{\frac{\mathrm{~s}^{6}+0.98 \mathrm{~s}^{4}+0.38 \mathrm{~s}^{4}+0.13 \mathrm{~s}^{2}-0.0005}{2}} \\
\mathrm{~A} & \frac{1.45 \mathrm{~s}^{2}+0.14}{\mathrm{~s}^{4}+0.69 \mathrm{~s}^{2}+0.13}
\end{array}\right] . \tag{7}
\end{gather*}
$$

where $\mathrm{z}_{1}=\left(\mathrm{s}^{2}+0.12 \mathrm{~s}+0.35\right), \mathrm{z}_{2}=\left(\mathrm{s}^{2}+0.48 \mathrm{~s}+0.35\right), \mathrm{z}_{3}=\left(\mathrm{s}^{2}+0.69 \mathrm{~s}+1.29\right)$, $\mathrm{A}=\frac{0.28 \mathrm{~s}^{11}+0.54 \mathrm{~s}^{10}+0.6 \mathrm{~s}^{9}+1.47 \mathrm{~s}^{8}+0.8 \mathrm{~s}^{7}+1.78 \mathrm{~s}^{6}+0.63 \mathrm{~s}^{5}+0.69 \mathrm{~s}^{4}+0.13 \mathrm{~s}^{2}}{\mathrm{~s}^{13}-0.06 \mathrm{~s}^{12}+3.29 \mathrm{~s}^{11}-0.2 \mathrm{~s}^{10}+4.72 \mathrm{~s}^{9}-0.28 \mathrm{~s}^{8}+3.34 \mathrm{~s}^{7}-0.2 \mathrm{~s}^{6}+1.3 \mathrm{~s}^{5}-0.08 \mathrm{~s}^{4}+0.28 \mathrm{~s}^{3}-0.02 \mathrm{~s}^{2}}$.

The degree of compliance of the results of approximation and estimates of the spectral densities are shown on the graphs of Fig. 5.

The substitution of the matrices (5), (6), (7) into the formulas (3), (4) let us solve the task in the following

$$
\mathrm{W}=\left[\begin{array}{cc}
\frac{0.43(\mathrm{~s}+0.02)}{(\mathrm{s}+0.06)} & \frac{0.24(\mathrm{~s}+0.4)}{(\mathrm{s}+0.06)}  \tag{8}\\
0.2 & 0.055
\end{array}\right],
$$



Figure 5. Results of approximation and estimates of the spectral densities
$S_{\varphi \varphi}^{\prime}=\left[\begin{array}{lc}\frac{-6.32|s+0.17|^{2}}{\left|s^{2}+0.18 s+0.36\right|^{2}} & \frac{0.6(s-0.8)(s-0.3)}{\left|s^{2}+0.18 s+0.36\right|^{2}} \\ \frac{0.6(s+0.3)(s+0.8)}{\left|s^{2}+0.18 s+0.36\right|^{2}} & \frac{-1.3|s+0.3|^{2}}{\left|s^{2}+0.18 s+0.36\right|^{2}}\end{array}\right]$

The received in this way matrix of the spectral densities of the noise (9) enabled us find the matrix of transfer functions of the forming filter $G$ as a result of Wiener's factorization
$\mathrm{G}=\left[\begin{array}{cc}\frac{2.5(\mathrm{~s}-0.008)}{\left(\mathrm{s}^{2}+0.18 \mathrm{~s}+0.36\right)} & \frac{0.43}{\left(\mathrm{~s}^{2}+0.18 \mathrm{~s}+0.36\right)} \\ \frac{-0.24(\mathrm{~s}+0.3)}{\left(\mathrm{s}^{2}+0.18 \mathrm{~s}+0.36\right)} & \frac{1.11(\mathrm{~s}+0.3)}{\left(\mathrm{s}^{2}+0.18 \mathrm{~s}+0.36\right)}\end{array}\right]$,
The assessment of the accuracy of the identification of the dynamics of the sensor which was carried out with the help of the method of imitation modeling showed that the mean square deviation of the error of identification is not higher than $0.5 \%$ in the angle velocity of the roll change as well as in the angle velocity of the pitch change.

## CONCLUSIONS

While operation the 3d MEMS sensor of angle velocities in the conditions of stationary multidimensional rolling of the foundation, it is advisable to consider the sensor as a multidimensional dynamic object with considerable cross links.

If the power of rolling of the foundation of the appliance is in the very-low-frequency domain then the sensor of the class under consideration may be related to the multidimensional combined filters.

The random constituent of the noises of measurement of the appliance may be multidimensional stationary narrow-band random process. And its basic power is focused on the constant frequency in both channels of measurement.

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