MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

Central Ukrainian National Technical University

V. V. Pirogov, G. B. Filimonikhin,
L. S. Olijnichenko, Ju. A. Nevdaha

THEORY OF MECHANISMS AND MACHINES.

STRUCTURE AND CLASSIFICATION OF MECHANISMS

Methodical instructions for lectures

Kropivnitskiy - 2026

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

Central Ukrainian National Technical University

Chair of Machine Parts and Applied Mechanics

V. V. Pirogov, G. B. Filimonikhin,
L. S. Olijnichenko, Ju. A. Nevdaha

THEORY OF MECHANISMS AND MACHINES. Structure and classification of mechanisms

Methodical instructions for lectures

Recommended by the Chair of Machine Parts and Applied Mechanics for students of engineering, transport and electrical engineering specialties.
Protocol № 10, 26.05.2026

Kropivnitskiy – 2026

Methodical instructions for lectures. Theory of mechanisms and machines. Structure and classification of mechanisms / V. V. Pirogov, G. B. Filimonikhin, L. S. Olijnichenko, Ju. A. Nevdaha. – Kropivnitskiy: CNTU, 2026. – 46 p.

Compiler: candidate of physical and mathematical sciences,
associate professor Pirogov Vladimir Vasilyevich

Reviewer: doctor of technical sciences, professor,
Matsui Anatoly Mykolayovych, professor of the department
of automation of production processes
Central Ukrainian National Technical University.

CONTENT

Preface

Mechanical engineering - the main branch of a modern industrially developed country - determines the level of development of the productive forces of society, is the foundation of technical progress in all branches of the national economy. In turn, the progress of mechanical engineering is determined by the perfection of the machines that are created. Therefore, deep theoretical knowledge and experience are required from the engineer, the ability not only to manage complex equipment, to use it successfully, but also to ensure its rapid progress. A modern engineer must perfectly master the methods of calculating and designing new high-speed, automated and high-performance machines.

The creation of new machines is based on the achievements of many fundamental and applied sciences, among which the theory of mechanisms and machines occupies an important place.

TMM is one of the main general engineering disciplines that provides the necessary theoretical training for mechanical engineers.

Knowledge of TMM is necessary not only for design engineers who design machines, but also for engineers engaged in their production and operation.

The basis of TMM is courses in physics, higher and applied mathematics, theoretical mechanics, engineering graphics, computing and programming.

The task of the TMM course is to prepare students for listening to courses on machine detailing, mechanical engineering technology, automated design systems, the basics of scientific research, and courses on the calculation and design of various special machines.

The study guide can be used both in the educational process and in engineering practice.

1. Structure and classification of mechanisms

1.1. Kinematic pairs and their classification

In general, any absolutely rigid body *ABC* (Fig. 1.1.1) moving freely in space has six degrees of freedom. That is, such a body has six types of independent possible motions: three rotational ones around the x, y, and z axes and three translational ones along these axes.

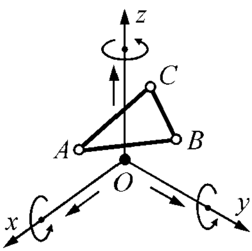


Fig. 1.1.1

The entry of a link into a kinematic pair with another link imposes certain restrictions on the relative motions of these links. These restrictions are called the connection conditions in kinematic pairs. It is obvious that the number of these connection conditions can only be an integer and less than six, since when the number of connection conditions is equal to six, the body loses relative mobility. Similarly, the number of connection conditions cannot be less than one, because in this case the links do not touch, that is, a kinematic pair does not exist. We have two bodies that move freely in space.

Thus, the number of coupling conditions imposed on the relative motion of each link of the kinematic pair varies within 1–5. Then the number of degrees of freedom H of the link of the kinematic pair in relative motion can be expressed by the equation

$$H = 6 - S, \qquad (1.1.1)$$

where S is the number of coupling conditions imposed by the kinematic pair on the relative motion of the links.

The classification of kinematic pairs is carried out according to the following criteria:

a) the number of connection conditions imposed by the kinematic pair on the relative motion of the links;

b) the shape of the elements of the links that form the kinematic pair;

c) the method of closing the links.

Depending on the number of connection conditions (Artobolevsky's classification) imposed by a kinematic pair on the relative motion of the links, pairs are divided into five classes: I, II, III, IV, V.

The class of the kinematic pair is determined by the dependence

$$S = 6 - H, \tag{1.1.2}$$

which follows from the dependence (1.1.1).

Less commonly used is the Dobrovolsky classification, according to which kinematic pairs are divided by the number of degrees of freedom H into one-, two-, three-, four-, and five-moving.

Table 1.1 gives examples of kinematic pairs and their symbols, taking into account the elements of kinematic pairs.

A class I pair is a ball-plane pair, the scheme of which is shown in Fig. 1.1.2. Here, the ball relative to the plane has the ability to rotate around three axes (x, y, z) and move translationally along the x, y axes (H=5, S=1) (the x, y and z axes are not shown in the figures). The movement of the ball along the z axis is impossible, since in one direction it is limited by the plane, and when moving in the opposite direction.

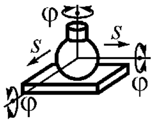
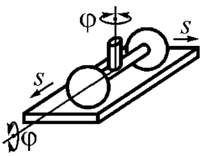
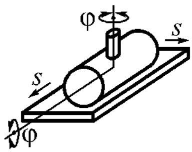
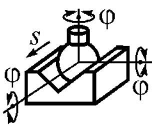
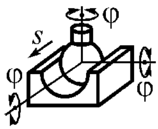
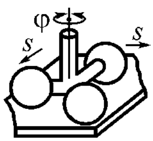
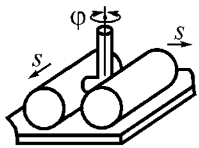
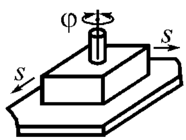
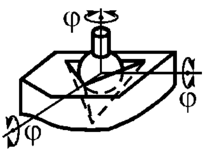
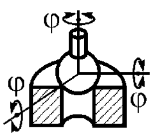
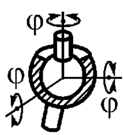
The kinematic pair cylinder-plane (Fig. 1.1.3 and 1.1.4) allows rotational movements of one link relative to the other around the x and z axes and translational movements along the x, y axes (the x, y and z axes are not shown in the figures). Therefore, this pair belongs to class II (H=4, S=2). The pair ball-cylinder (Fig. 1.1.5 and 1.1.6) is also a class II pair, which allows, in addition to three rotational movements, also translational movement along the cylinder axis.

In a spherical pair (Fig. 1.1.10 – 1.1.12), one link relative to the other can rotate around one of three mutually perpendicular axes passing through the center of the sphere. Therefore, the spherical pair belongs to class III (H=3, S=3). In a planar kinematic pair (Fig. 1.1.7 – 1.1.9), one link relative to the other can move along the x and y axes and rotate around the z axis (the x, y and z axes are not shown in the figures). This pair is of class III.

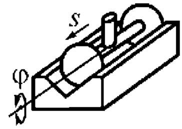
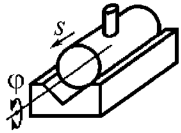
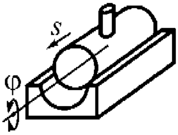
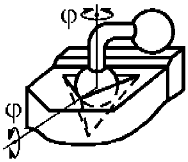
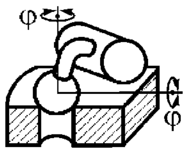
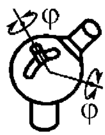
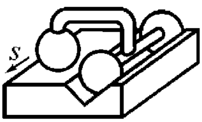
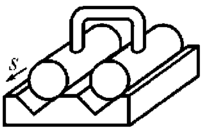
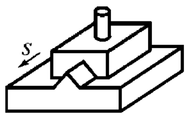
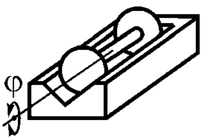
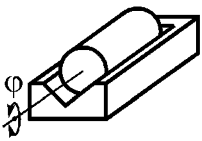
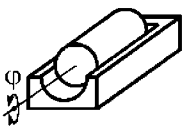
In a cylindrical pair (Fig. 1.1.13 – 1.1.15) the cylinder can rotate around its own longitudinal axis and move along it. These movements are independent, so this pair belongs to the IV class pair (H=2, S=4). This also applies to the spherical pair with a finger (Fig. 1.1.16 – 1.1.18).

In a rotating pair (Fig. 1.1.22 – 1.1.24), one of the links can rotate around its own longitudinal axis, and in a translational pair (Fig. 1.1.19 – 1.1.21) – move along the groove. The number of degrees of freedom of one link in its movement relative to the other is H=1, therefore the number of connection conditions S=5, therefore, these kinematic pairs are of class V. A screw pair can also be attributed to a class V kinematic pair, for example, with a fixed nut, the screw can rotate around the axis of the longitudinal axis and simultaneously move along this same axis.

Table 1.1

Kinematic pair class	Possible moves	Elements of kinematic pairs: points	Elements of kinematic pairs: lines	Elements of kinematic pairs: surfaces	Symbols (name)
I (S=1)	2s 3φ	 Fig. 1.1.2	—	—	 sphere-plane
II (S=2)	2s 2φ	 Fig. 1.1.3	 Fig. 1.1.4	—	 cylinder-plane
	1s 3φ	 Fig. 1.1.5	 Fig. 1.1.6	—	 ball-cylinder
III (S=3)	2s 1φ	 Fig. 1.1.7	 Fig. 1.1.8	 Fig. 1.1.9	 planar
	3φ	 Fig. 1.1.10	 Fig. 1.1.11	 Fig. 1.1.12	 spherical

Continuation of table 1.1

IV (S=4)	1s 1φ	 Fig. 1.1.13	 Fig. 1.1.14	 Fig. 1.1.15	 cylindric al
	2φ	 Fig. 1.1.16	 Fig. 1.1.17	 Fig. 1.1.18	 spherical with finger
V (S=5)	1s	 Fig. 1.1.19	 Fig. 1.1.20	 Fig. 1.1.21	 translatio nal
	1φ	 Fig. 1.1.22	 Fig. 1.1.23	 Fig. 1.1.24	 rotating

Depending on the shape of the elements, kinematic pairs are divided into lower and higher. Lower kinematic pairs are those pairs in which the elements of kinematic pairs touch surfaces (Fig. 1.1.9, 1.1.12, 1.1.15, 1.1.18, 1.1.21, 1.1.24). Higher kinematic pairs are those pairs in which the elements of kinematic pairs touch along a line or at a point (see Fig. 1.1.2–1.1.8, 1.1.10, 1.1.11, 1.1.13, 1.1.14, 1.1.16, 1.1.17, 1.1.19, 1.1.20, 1.1.22, 1.1.23).

Lower kinematic pairs are characterized by the fact that they can transmit more force than higher ones, due to the larger contact area between the links. However, friction costs in such pairs are higher compared to higher ones (for example, in rolling bearings). In modern mechanisms, lower kinematic pairs of classes IV and V are most common.

The lower pairs have the property of inversion (reversibility of motion), that is, the nature of the relative motion of the links does not change depending on which link moves (A relative to B, or B relative to A, see Fig. 1.1.4–1.1.10). The higher pairs do not have this property. Thus, when a cylinder rolls without slipping along a fixed plane (Fig. 1.1.11), the trajectory of some point M, which lies on the surface of the cylinder, will be a cycloid, and vice versa, when a plane rolls without slipping around a fixed cylinder, some point M of the plane will describe an involute.

In order for the elements of kinematic pairs to be in constant contact, the pairs must be closed. The closure can be geometric or force. Geometric closure is carried out by the appropriate geometric shape of the elements of the links of the kinematic pair or by the design of the kinematic pair.

For example, all the pairs shown in Fig. 1.1.12, 1.1.18 are closed geometrically, since the contact of the elements of these pairs is ensured by their geometric shapes. In order for the pairs shown, for example, in Fig. 1.1.12, 1.1.4 to be closed, it is necessary to press the body A to the plane B with any force. Force closure is ensured by the force of gravity, the force of spring elasticity, etc.

1.1.1. Flat mechanisms with lower kinematic pairs

Mechanisms whose links form only lower (rotational, translational, cylindrical, spherical) kinematic pairs are called lever mechanisms. These mechanisms have found wide use in mechanical and instrument-making due to the fact that when the links interact, the forces in the kinematic pairs are distributed over the surfaces. Due to this, the load, and therefore, the wear of these elements is lower than that of the elements in the higher pairs. The advantages of lever mechanisms include the simple geometric shape of the links, which simplifies the technology of their manufacture.

The basis for many flat mechanisms is a hinged four-link lever mechanism (Fig. 1.1.25, a). The axes of the kinematic pairs of the 5th A, C, D and 4th B classes in this mechanism are made perpendicular to the planes in which the trajectories of the points of the links are located. A similar mechanism is formed by the same links when replacing the pair C of the 5th class with a pair of the 3rd class (Fig. 1.1.25, b).

Based on the four-link lever mechanism, a flat crank-rod or crank-slider mechanism (Fig. 1.1.26) is built, which is used to convert the rotational motion of the input link 1 into the translational motion of the output link – slider 3. The axes of the rotational and cylindrical kinematic pairs in this mechanism are located perpendicular to the direction of the slider movement.

Flat crank-rocker mechanisms (Fig. 1.1.27) consist of an input link - crank 1, which forms a rotational pair A with slider 2. Slider 2 enters into a translational pair B with rocker 3, which is the output link. In these mechanisms, with uniform rotation of the input link, it is possible to obtain uneven oscillatory (Fig. 1.1.27, a), rotational (Fig. 1.1.27, b), and translational (Fig. 1.1.27, c) motion of the output link.

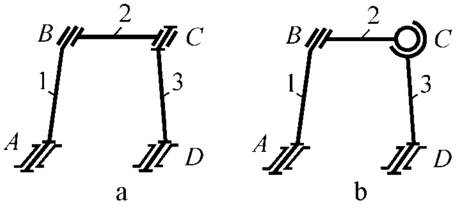


Fig. 1.1.25

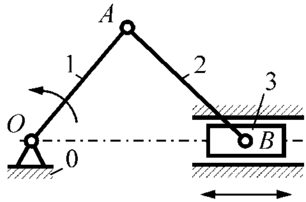
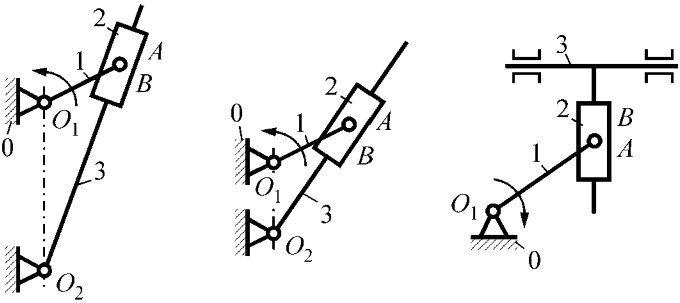


Fig. 1.1.26



a b c

Fig. 1.1.27

1.1.2. Spatial mechanisms with lower kinematic pairs

Spatial lever mechanisms are used as transmission or guide mechanisms to reproduce spatial curves if the axes of kinematic pairs intersect or cross in space.

Similar to the flat one, a spatial four-link mechanism is widely used, which consists only of pairs of the 5th class (Fig. 1.1.28, a) or which also contains pairs of the 3rd class (Fig. 1.1.28, b). It is used to transmit motion between two cross axes.

Structural transformations of the spatial four-link mechanism allow obtaining various modifications of the kinematic connections of universal joint mechanisms (Fig. 1.1.29). They are used in metal-cutting machines, automobiles and other machines to transmit motion between shafts located at a certain angle, as well as in cases where the position of the shafts changes during operation.

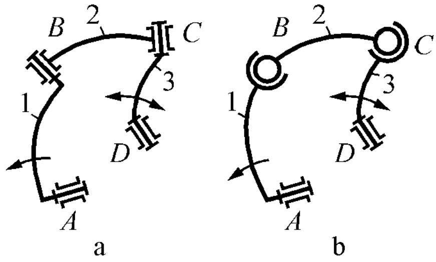


Fig. 1.1.28

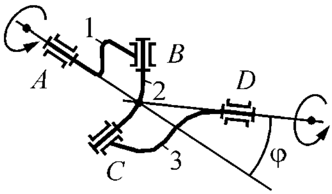


Fig. 1.1.29

The spatial crank-slider mechanism (Fig. 1.1.30) is used in the case when the axis of rotation of the input link 1 is not perpendicular to the plane in which the slider 3 moves. Fig. 1.1.31 shows an example of the use of the spatial crank-slider mechanism in aircraft landing gear control devices, in which the input links have a different nature of movement.

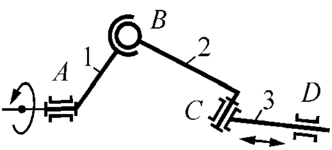


Fig. 1.1.30

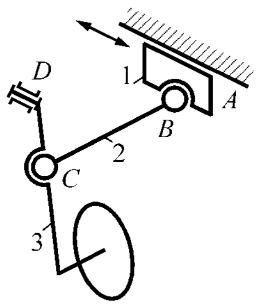


Fig. 1.1.31

Spatial lever mechanisms include manipulator mechanisms and screw mechanisms.

1.1.3. Flat mechanisms with higher kinematic pairs

With the help of lever mechanisms, most of the laws of motion and the trajectory of the points of the output links are reproduced only approximately, and sometimes they are completely impossible to implement (for example, the movement of the output link with a constantly increased or decreased speed relative to the speed of the input link). This is due to the fact that the lower kinematic pairs have limited possibilities when choosing mobility. Much greater possibilities for reproducing the given laws of motion and trajectories of the points of the output links are mechanisms whose links form higher kinematic pairs.

The simplest mechanism for ensuring a constant speed of the output link is a friction transmission (Fig. 1.1.32), in which the transmission from the input link 1 to the output link 2 is carried out due to the friction forces that arise on the elements of the higher kinematic pair B. The element of the kinematic pair is a point or a line. The friction force is formed due to the force closure of the higher kinematic pair. With significant distances between the axes of rotation of the input and output links, friction transmissions with a flexible link are used - chain or belt transmissions (Fig. 1.1.33).

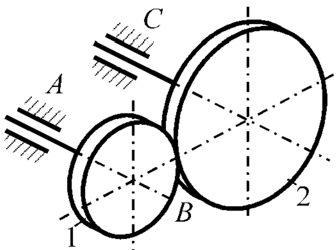


Fig. 1.1.32

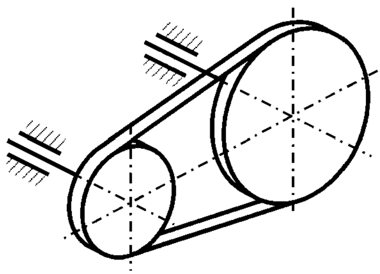


Fig. 1.1.33

Cam mechanisms are used as a transmission mechanism to reproduce the required law of motion of the output link for a given motion of the input link (Fig. 1.1.34). The required law of motion is achieved by giving the input link – cam 1 – the appropriate geometric shape. The cam performs rotational (Fig. 1.1.34, a, b) or translational (Fig. 1.1.34, c, d) motion, and the output link 2 – translational (Fig. 1.1.34, a, c) or rotational-oscillating (Fig. 1.1.34, b, d) motion. To reduce friction losses in the higher kinematic pair A, an additional link 3 is used – a roller, which forms a rotational pair C with link 2. Cam mechanisms are used in various control devices (electrical, hydraulic, pneumatic, valve systems, etc.).

If, with a given rotation of the input link (Fig. 1.1.35), it is necessary to obtain continuous movement of the output link in one direction, gear mechanisms or transmissions are used. The contacting teeth of the gears form a higher kinematic pair, which is called a gear mesh. Flat gear mechanisms, which include cylindrical gears with teeth located on cylindrical surfaces, are designed to transmit motion between parallel axes.

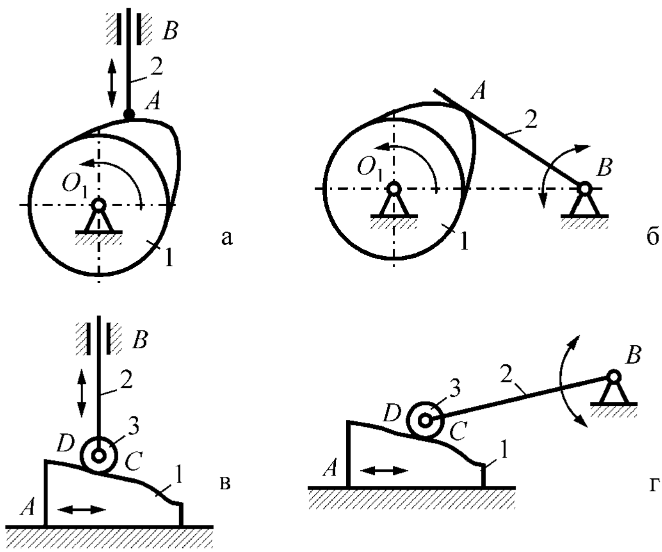


Fig. 1.1.34

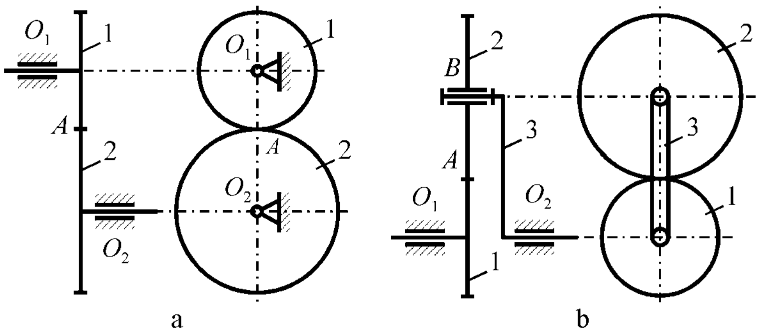


Fig. 1.1.35

Gear mechanisms are divided into ordinary (Fig. 1.1.35, a), in which the axes of all wheels are stationary, satellite (Fig. 1.1.35, b), in which some wheels perform two rotational movements - around their own axis and around the axis of another link, and gear-lever with round (Fig. 1.1.36, a) and non-round wheels (Fig. 1.1.36, b).

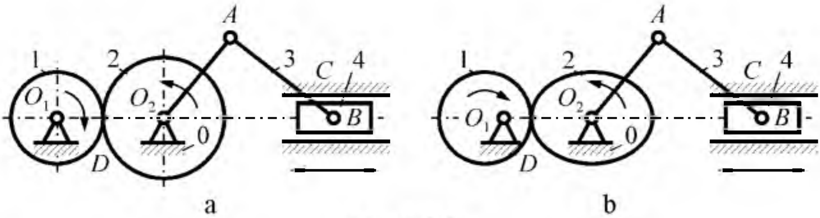


Fig. 2.1.36

1.1.4. Spatial mechanisms with higher kinematic pairs

Friction mechanisms or gears with intersecting axes (Fig. 1.1.37) transmit motion not only with a constant speed ratio (Fig. 1.1.37, a) of the input 1 and output 2 links, but also with a smooth change in this ratio using link 3 (Fig. 1.1.37, b).

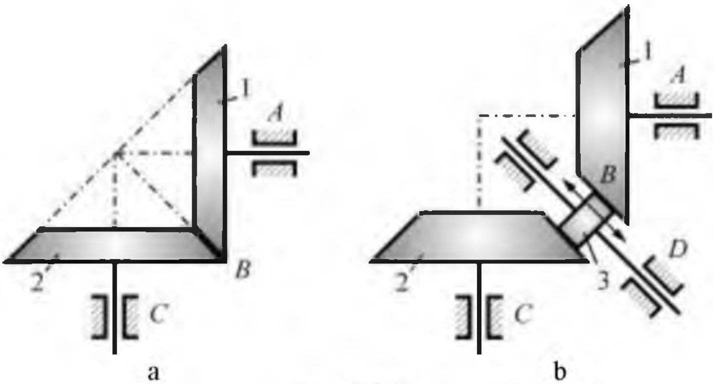


Fig. 2.1.37

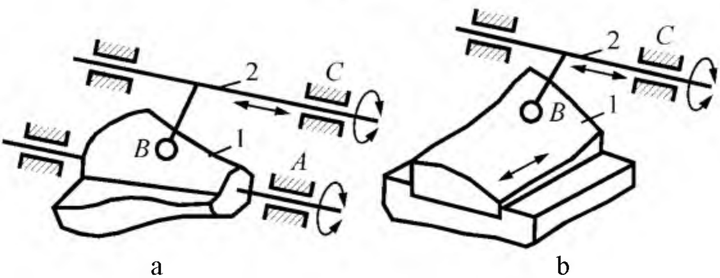


Fig. 2.1.38

Spatial cam mechanisms are widely used (Fig. 1.1.38). The input link 1 performs rotational (Fig. 1.1.38, a) or translational (Fig. 1.1.38, b) motion, and the output link 2 performs complex motion in space.

Geared spatial mechanisms (conical, helical, hypoid, worm, globoid) are used to transmit motion between axes that intersect or cross.

1.2. Kinematic chains and their classification

A kinematic chain is a system of links that are interconnected by kinematic pairs. Fig. 1.2.1 shows a diagram of a kinematic chain consisting of four links that form three kinematic pairs. Links 1 and 2 belong to the rotating pair A (V class), links 2, 3 - to the translational pair B (V class), links C, 4 - to the rotating pair C (V class).

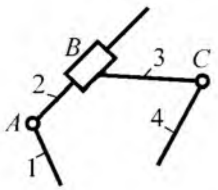


Fig. 1.2.1

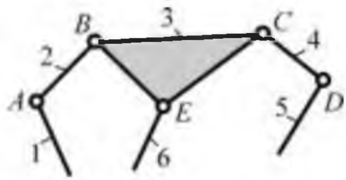


Fig. 1.2.2

Kinematic chains are divided into simple and complex. A simple kinematic chain is one in which each link is included in no more than two kinematic pairs (Fig. 1.2.1). A complex kinematic chain is one in which there is at least one link included in more than two kinematic pairs (Fig. 1.2.2 – link 3 is included in three kinematic pairs B, C, E).

In turn, simple and complex kinematic chains are divided into closed and open. In an open kinematic chain there are links that are included in only one kinematic pair (Fig. 1.2.1, 1.2.2), in a closed kinematic chain (Fig. 1.2.3, 1.2.4) each link is included in at least two kinematic pairs.

So, the figures show: simple open-ended (Fig. 1.2.1), complex open-ended (Fig. 1.2.2), simple closed (Fig. 1.2.3), complex closed (Fig. 1.2.4) kinematic chains.

Depending on the form of movement of the links, kinematic chains are divided into planar and spatial. A planar chain is a chain in which all points of the links describe trajectories that lie in the same or parallel planes.

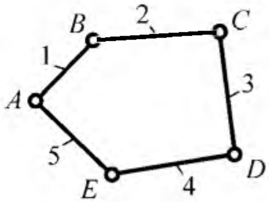


Fig. 1.2.3

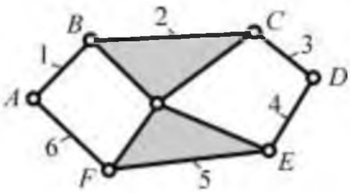


Fig. 1.2.4

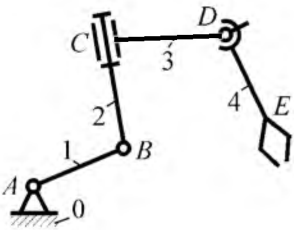


Fig. 1.2.5

A chain in which the points of the links move in different non-parallel planes is called spatial. If the points of the links describe trajectories on concentric spheres, then the chain is called spherical. Spatial kinematic chains are used in the design of manipulator and robot mechanisms. An example of such a mechanism is shown in Fig. 1.2.5.

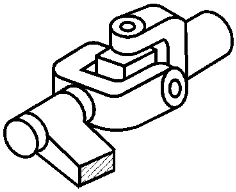
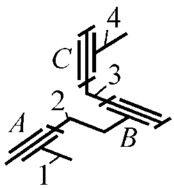
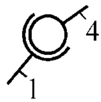
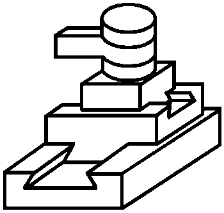
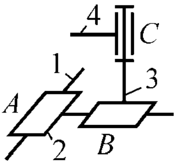
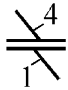
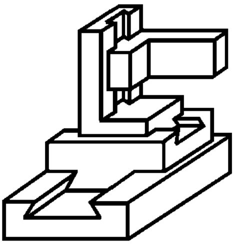
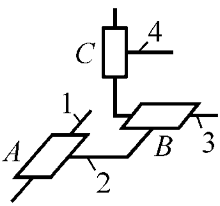
1.3. Kinematic joints

In some cases, it is convenient to replace kinematic pairs of classes I–IV with equivalent kinematic chains formed only by pairs of class V. Such a chain is called a kinematic connection, which is understood as an open kinematic chain that can replace a kinematic pair by the nature of the relative movements of the links.

Table 1.2 shows examples of kinematic joints and their equivalent kinematic pairs.

The use of kinematic joints instead of kinematic pairs allows you to increase the load-bearing capacity of the machine structure, reduce friction costs, and simplify manufacturing technology.

Table 1.2

Possible moves	**Kinematic connection**	**Connection block diagram**	**Symbol of the kinematic pair being replaced**
3φ	 Fig. 1.3.1		
2*s* 1φ	 Fig. 1.3.2		
3*s*	 Fig. 1.3.3		no analogues

1.4. Formulas for the mobility of kinematic chains. The determinism of the movement of the links of the mechanism

It is known that when no coupling conditions are imposed on the motion of a link in space, then it has six degrees of freedom. If the number of links in a kinematic chain is k, then the total number of degrees of freedom will be $6k$. Each kinematic pair imposes a different number of couplings on the relative motion of the links, which depends on the class of the pair. Let us denote the number of pairs of class I that are part of the chain by p_1, II – p_2, III – p_3, IV – p_4, V – p_5. To determine the total number of degrees of freedom of the links of a kinematic chain, it is necessary to subtract from $6k$ degrees of freedom those degrees of freedom that are subtracted by the kinematic pairs. From Table 2.1 it is seen that one pair of class I imposes one coupling condition (S=1) on the relative motion of the links, of class II – two (S=2), etc. Then the number of degrees of freedom H that the kinematic chain has is

$$H = 6k - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1 . \quad (1.4.1)$$

Since one of the links of the kinematic chain is necessarily fixed, the total number of degrees of freedom (mobility) of the links of the chain relative to the fixed link will be

$$W = H - 6$$

or after transformations

$$W = 6n - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1 , \quad (1.4.2)$$

where $n = k - 1$ is the number of moving links of the kinematic chain.

Formula (1.4.2) is called the Somov-Malyshev mobility formula or the structural formula of a spatial kinematic chain of general form.

For planar kinematic chains, the number of degrees of freedom is equal to $(6–3)n=3n$, since three general restrictions are imposed on the movement of all links of the mechanism as a whole. Therefore, the following formula holds

$$W = (6-3)n - (5-3)p_5 - (4-3)p_4 - (3-3)p_3 =$$
$$= 3n - 2p_5 - p_4 . \quad (1.4.3)$$

Formula (1.4.3) is called the Chebyshev mobility formula or the structural formula of a planar kinematic chain of general form.

Only pairs of classes IV and V can be included in flat mechanisms, with pairs of class IV being higher and V being lower.

The general formula for the mobility of kinematic chains, taking into account the connections imposed on the movement of the links of the mechanism, has the form

$$W = (6-m)n - \sum_{k=5}^{m+1} (k-m) p_k \quad , \qquad (1.4.4)$$

where m – is the number of general connections imposed on the movement of the mechanism links (m=0...4); k – kinematic pair class (k=1...5). Formula (1.4.4) is also called Dobrovolsky's formula.

To clarify the relationship between the degrees of freedom W and the certainty of the movement of the links of the mechanism, consider two examples. Fig. 1.4.1 shows a diagram of a flat four-link kinematic chain, which includes three moving links (n=3), four rotating kinematic pairs of class V (p_5=4). The degrees of freedom of such a kinematic chain can be determined by the Chebyshev formula (p_4=0)

$$W = 3\cdot 3 - 2\cdot 4 - 0 = 1 .$$

If any link, for example AB, is given a law of motion, in this case rotational, then all other links BC and CD) will also have a completely defined motion.

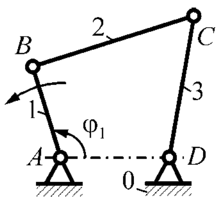


Fig. 1.4.1

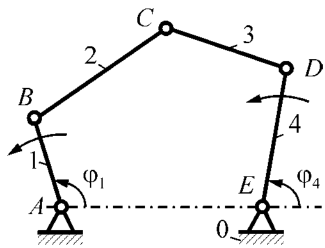


Fig. 1.4.2

As is known, the position of a rigid body that moves freely in space is determined by six independent coordinates. They are usually called generalized, since they determine the position of the entire rigid body. Similarly, generalized coordinates of the mechanism are independent linear or angular coordinates that determine the position of all links of the mechanism relative to the riser. In this case (Fig. 1.4.1), the angle of rotation of the crank φ1 can be taken as the generalized coordinate, since the position of link 1 determines the position of all other moving links of the articulated four-link mechanism.

A link to which one or more generalized coordinates are assigned is called the initial link. This term is related to the fact that the positions of all links of the mechanism begin with finding the positions of the initial links.

For the kinematic chain, the diagram of which is shown in Fig. 1.4.2, the degree of freedom (n=4, p_5=5, p_4=0)

$$W = 3 \cdot 4 - 2 \cdot 5 - 0 = 2.$$

If in this chain only the position of the link AB is given, then it is obvious that the position of the remaining moving links will be indefinite. If we also specify the position of another link, for example link 4, by the angle φ4, then all the links of the mechanism will have a completely definite movement. Therefore, in the mechanism shown in Fig. 1.4.2, there must be two initial links.

Thus, the degrees of freedom of the kinematic chain relative to the riser determine the number of initial links of the mechanism. The latter may coincide with the input links of the mechanism, or may not coincide. The selection of the initial link is determined by the convenience of determining the positions of the links of the mechanism and the convenience of its analysis.

1.5. The basic principle of mechanism formation

The basic principle of mechanism formation reveals not only the method of mechanism formation by sequentially connecting kinematic chains, but also forms the basis of the most rational classification of mechanisms (formulated in 1914 by the Russian scientist L. V. Assur). This principle is that any mechanism can be obtained if kinematic chains with zero degrees of freedom are sequentially connected to the initial link (or initial links) and the riser.

Indeed, as shown above, each mechanism consists of a fixed link (strut), initial links, i.e. links, the laws of motion of which are given and on which the laws of motion of all other links depend. Therefore, starting to create a mechanism of the required degree of freedom, we fix one of the links (form a strut) and introduce into kinematic pairs with this link the initial links according to the number of degrees of freedom that the mechanism should have. In this case, each initial link should have only one degree of freedom.

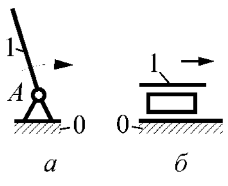


Fig. 1.5.1

Let us conventionally call the initial link and the riser, which form a kinematic pair of class V, a class I mechanism. Fig. 1.5.1 shows class I mechanisms, the initial links of which form a rotating (Fig. 1.5.1, a) or translational (Fig. 1.5.1, b) pair with the riser.

To obtain a mechanism with the desired degree of freedom, it is necessary to attach to the mechanism(s) of class I a system of links that constitutes one or more kinematic chains with zero degrees of freedom. The latter condition follows from the fact that the entire mechanism must have a degree of freedom equal to the sum of the degrees of freedom of the mechanisms of class I.

As an example, consider the planar mechanism shown in Fig. 1.5.2, a. The degree of freedom of this mechanism is:

$$W = 3 \cdot 5 - 2 \cdot 7 - 0 = 1.$$

If we take riser 0 and link 1 as a class I mechanism (Fig. 1.5.2, b), then links 2–5 form a system of links with zero degrees of freedom (n=4, p_5=6).

It is easy to see that the kinematic chain of links 2–5 can be divided into two kinematic chains: one composed of links 2–3 (Fig. 1.5.2, c), and the second composed of links 4–5 (Fig. 1.5.2, d). Each of these kinematic chains, composed of two links and three kinematic pairs of class V, has a degree of freedom W, which is equal to zero. It is impossible to divide these chains into simpler kinematic chains that would have zero degrees of freedom.

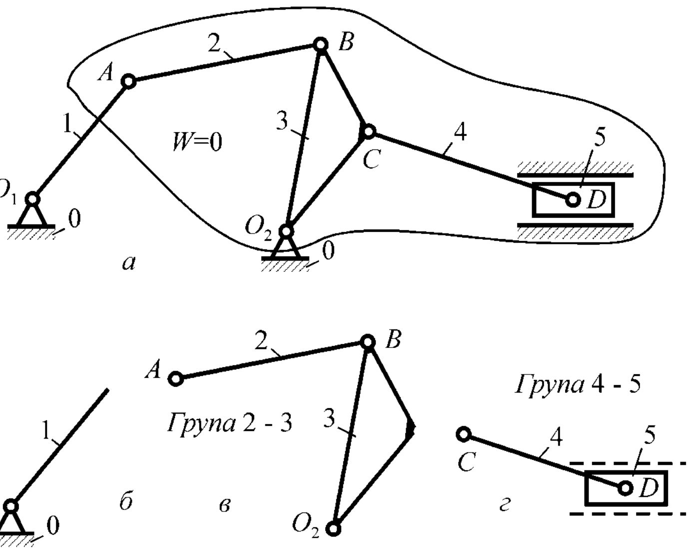


Fig. 1.5.2

A kinematic chain that, after connecting its free elements in pairs to other links of the mechanism, does not change its degree of freedom and which cannot be separated into simpler kinematic chains of zero degrees of freedom is called a structural group, or an Assur group.

Thus, a planar mechanism (see Fig. 1.5.2, a) with one degree of freedom can be considered as one formed by the sequential addition of two groups to a class I mechanism: groups 2–3 and groups 4–5. Now we can give the following definition of the basic principle of mechanism formation.

Any mechanism can be obtained by sequentially adding structural groups to the mechanism(s) of class I.

When sequentially connecting groups, it is necessary to follow certain rules. When forming a mechanism with one degree of freedom, the first group is connected by free elements of the links to the initial link and riser. Subsequent groups can be connected to any links of the resulting mechanism only in such a way that the links of the group can move relative to each other. It is impossible to connect a group by free elements to one link, since in this case we will get a fixed contour.

Structural groups of planar mechanisms satisfy the condition

$$W = 3n - 2p_5 - p_4; \quad (1.5.1)$$

structural groups of spatial mechanisms –

$$W = 6n - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1. \quad (1.5.2)$$

Both planar and spatial structural groups are used not only in structural synthesis, but also in mechanism analysis.

1.6. Structural classification of planar mechanisms

In modern mechanical engineering, flat mechanisms are especially widespread, the links of which are included in pairs of classes IV and V. Below we will consider the principles of their structural classification.

The structural classification of mechanisms is one of the most rational classifications of flat mechanisms (the foundations were laid by L. V. Assur and further developed by I. I. Artobolevskiy, V. V. Dobrovolskiy and other scientists). The advantage of this classification is that it is associated with the methods of kinematic, force and dynamic research of mechanisms.

The structural classification of mechanisms is based on the basic principle of mechanism formation. Any mechanism can be obtained by attaching to the mechanism of class I structural groups, the condition for the existence of which is the equality (1.5.1) or (1.5.2).

All class IV kinematic pairs that are part of the flat mechanism can be replaced by class V pairs, so dependence (1.5.1) can be rewritten as follows:

$$3n - 2p_5 = 0,$$

where

$$p_5 = \frac{3}{2}n. \quad (1.6.1)$$

Since the numbers of links and pairs can only be integers, condition (1.6.1) will be satisfied only by the following combinations of the numbers of links and kinematic pairs that are included in the group (Table 1.3):

Таблиця 1.3

n	2	4	6	8	…
p_5	3	6	9	12	…

It is characteristic that only an even number of links can be included in the structural group.

By choosing different combinations of these numbers that satisfy condition (1.6.1), we can obtain groups of various types. All groups obtained in this way can be divided into classes. As will be shown below, the division of groups into classes is determined by the methods of kinematic and force analysis inherent in the groups of each class.

1.6.1. Structural groups and mechanisms of class II

As can be seen from the above, the simplest group will be the group consisting of two links and three kinematic pairs of class V (Fig. 1.6.1, a) (n=2, p_5=3). Such a group was called the structural group (Assur group) of class II of order II, or a two-link group. The order of the group is determined by the number of element pairs by which the group is connected to the main mechanism. In the group shown in Fig. 1.6.1, a, the free elements have two pairs (*B* and *D*), by which the group can be connected to other links.

Class II groups are of five types depending on the number of rotating and translational pairs and their relative arrangement (Fig. 1.6.1). Let us call a group that has two links and three rotating pairs the first type of class II group.

All other types of class II groups can be obtained by replacing individual rotating pairs with translational ones. If one of the extreme rotating pairs is replaced by a translational one, we obtain a group of type II (Fig. 1.6.1, *b*, *c*). The group depicted in Fig. 1.6.1, *c* is a special case of the group depicted in Fig. 1.6.1, *b*, in which the length of the segment *CD*=0. The third type of class II group is depicted in Fig. 1.6.1, *d*, *e* (in Fig. 1.6.1, *e* *CD*=0). Here, the middle rotating pair is replaced by a translational one. If the two extreme rotating pairs are replaced by translational ones, we obtain a group of class II of type IV (Fig. 1.6.1, *e*, *g*). In Fig. 1.6.1, there are segments *BC*=*CD*=0. And finally, in group V of the type (Fig. 1.6.1, *g*, *h*) the extreme and middle rotating pairs are replaced by translational ones (in

Fig. 1.6.1, with *BC*=0). When replacing all rotating pairs with translational ones, we obtain a wedge mechanism (*W*=1).

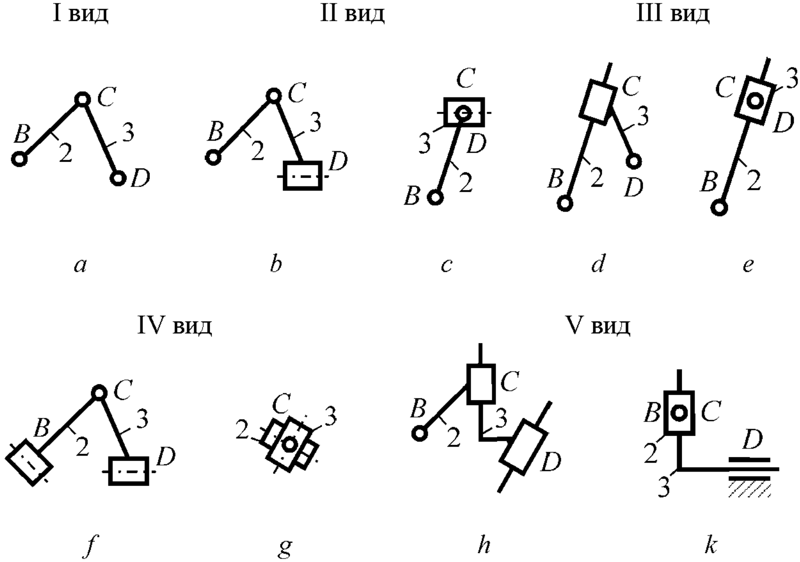


Fig. 1.6.1

Fig. 1.6.2 and 1.6.3 show examples of the simplest mechanisms that use class II groups of all five types.

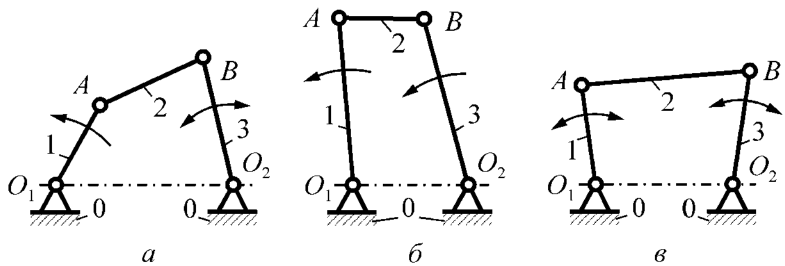


Fig. 1.6.2

Fig. 1.6.2, a shows a hinged four-link mechanism *ABCD* with links: 0 – riser; 1 – crank; 2 – connecting rod; 3 – rocker arm. Depending on the size of the links, the hinged four-link mechanism can be of three types: crank-rocker arm (Fig. 1.6.2, a); double-crank (links 1, 3 make a full turn, Fig. 1.6.2, b); double-rocker arm (links 1, 3 oscillate, Fig. 1.6.2, c).

An example of mechanisms where group II of class II type is used is a crank-slider (or rocker-slider) mechanism (Fig. 1.6.3, a); type III - a crank-rocker mechanism (Fig. 1.6.3, b); type IV - a tangent mechanism (Fig. 1.6.3, c); type V - a sine mechanism (Fig. 1.6.3, d).

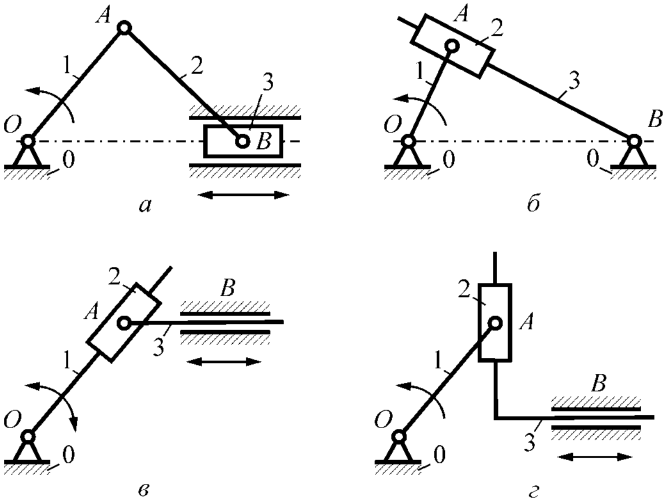


Fig. 1.6.3

Mechanisms that include only class II groups are called class II mechanisms. Most mechanisms used in modern technology belong to this class of mechanisms.

1.6.2. Structural groups and mechanisms of class III

Let us consider the second possible combination of the numbers of links and kinematic pairs forming a structural group (n=4, p_5=6). For this combination, three types of kinematic chains can be obtained, the structural principles of their formation being different.

The first kinematic chain (Fig. 1.6.4, a) consists of link 3 (basic), which is included in three kinematic pairs with links 2, 4, 5 (leashes). Such a kinematic chain is a group III of class III order, or a three-leash group. This group is connected to the main mechanism by means of three leashes (the elements of kinematic pairs *A*, *D*, *F* are free).

A feature of this group is the presence of link 3, which is included in three kinematic pairs and forms a certain rigid triangular contour, as if composed of three links *ED*, *CD*, *EC*, which are included in three kinematic pairs (Fig. 1.6.4, *b*). The relative degree of freedom of such a contour is W=0.

Class III groups can be of various types, which are obtained by replacing rotating pairs with translational ones. Examples of such groups are shown in Fig. 1.6.4, *c*, *d*.

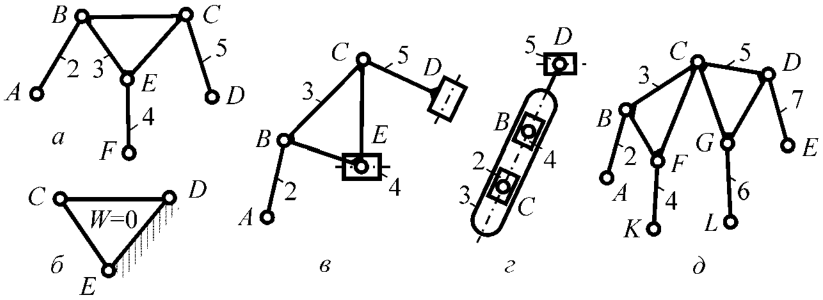


Fig. 1.6.4

Class III groups may have more than four links and more than six pairs (Fig. 1.6.4, e).

Mechanisms that include groups no higher than class III are called class III mechanisms. Examples of such mechanisms are shown in Fig. 1.6.5.

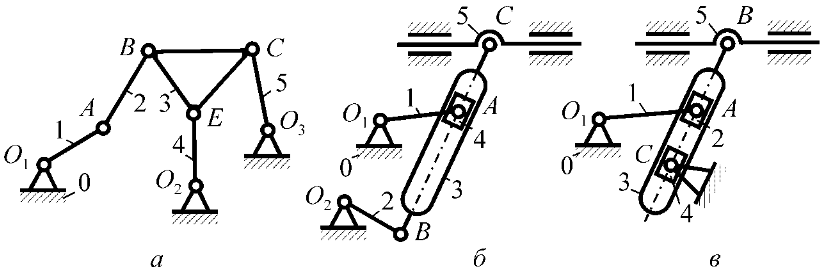


Fig. 1.6.5

1.6.3. Structural groups and mechanisms of classes IV and higher

A kinematic chain of four links and six lower pairs is shown in Fig. 1.6.6, *a*. A characteristic feature of this group is that it includes a quadrilateral moving contour *DEFG* (Fig. 1.6.6, *b*), the relative degree of freedom of which W=1. The group, which includes a quadrilateral closed moving contour, belongs to the group IV of class. Therefore, the group shown in Fig. 1.6.6, *a*, will be a group IV of class II of order, since it is connected to the main mechanism by free elements of kinematic pairs B, C. Fig. 1.6.6, *c* shows an example of a mechanism, which includes this group.

Mechanisms that include groups no higher than class IV are called class IV mechanisms.

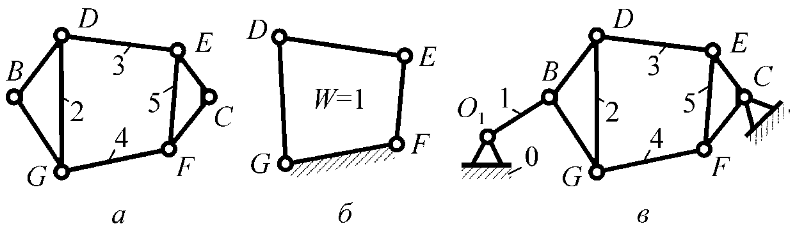


Fig. 1.6.6

The third possible type of kinematic chain with four links and six kinematic pairs is shown in Fig. 1.6.7.

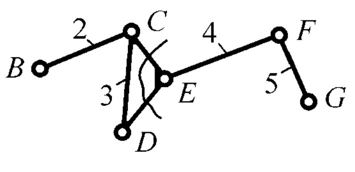


Fig. 1.6.7

This chain splits into two class II groups (group 2–3 and group 4–5), i.e. this chain does not give rise to a new group.

If we proceed to further combinations of links and pairs that satisfy the structural group condition, then the pentagonal contour (*W*=2) will be included in the V class groups, the hexagonal contour (*W*=3) will be included in the VI class groups, etc.

1.6.4. Structural formula of the mechanism structure

Based on the above, the following conclusions can be drawn: the so-called class III contour is included in the class III group (see Fig. 1.6.4), the class IV group – the class IV contour (see Fig. 1.6.6), etc. The class of the contour is determined by the number of kinematic pairs that make up the links that form it (Fig. 1.6.8). The class of the group is determined by the highest class of the contour that is part of it. The class of the mechanism is determined by the highest class of the groups that are part of it. For example, if the mechanism is formed by two groups – a class III group and a class IV group – then it belongs to class IV mechanisms.

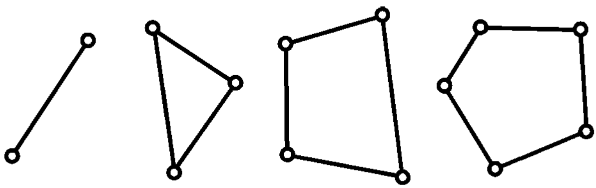


Fig. 1.6.8

The composition and sequence of attachment of the structural groups of a mechanism can be expressed by the structural formula of the mechanism, for example, the structural formulas for the mechanisms depicted:

$$\text{fig. 1.6.3, a} - \text{I}(1) \rightarrow \text{II}\,(2, 3);$$

$$\text{fig. 1.6.5, a} - \text{I}(1) \rightarrow \text{III}\left(\frac{3}{2{,}4{,}5}\right),$$

where I is a class I mechanism; II, III are class groups. The numbers of the links that make up the class I mechanism or structural groups are indicated in brackets. In the class III group, the basic link is separately highlighted.

1.7. Structural synthesis of mechanisms

Structural synthesis of mechanisms allows to develop kinematic chains with a minimum number of links to transform the motion of a given number of input links into the required motion of the output links. As a result of structural synthesis of a mechanism, its structural diagram is obtained, which indicates the links and the nature of their interconnection.

The tasks of structural synthesis are multivariate. The same transformation of motion is obtained by mechanisms of different structures. In addition, when choosing the optimal structural scheme, the technology of manufacturing links and kinematic pairs, the conditions of assembly and installation, as well as the operating conditions of the mechanisms are taken into account. Therefore, as a rule, after the synthesis of a planar structural scheme, one proceeds to a spatial scheme.

1.7.1. Synthesis of mechanisms by layering structural groups

The synthesis of structural diagrams of mechanisms with a given number of input links is carried out by the method of layering structural groups. For example, by attaching a structural group consisting of two links 2 and 3 to the mechanism of the first class, we obtain a crank-rocker mechanism (Fig. 1.7.1, a). A more complex mechanism can be obtained by attaching a structural group consisting of links 4 and 5 to link 3 of the mechanism and the riser (Fig. 1.7.1, b).

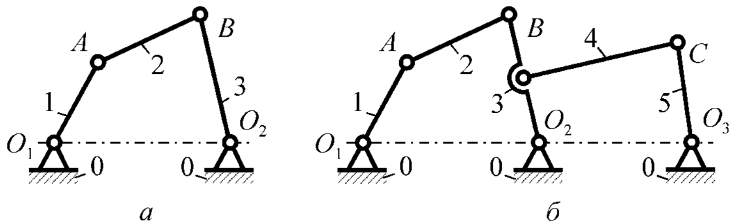


Fig. 1.7.1

Thus, by sequentially adding various structural groups to the first class mechanism, any mechanisms can be constructed.

1.7.2. Synthesis of mechanisms by inversion method

Depending on the choice of the input link and the riser in the kinematic chain, other mechanisms with a different nature of the relative motion of some links can be obtained. For example, if in the flat structural diagram of a four-link mechanism the riser is link 4 (Fig. 1.7.2, *a*), then we will obtain a crank-rocker mechanism. This method of obtaining variants of the mechanism and its component solutions by replacing the functions of one link with the functions of another is called inversion.

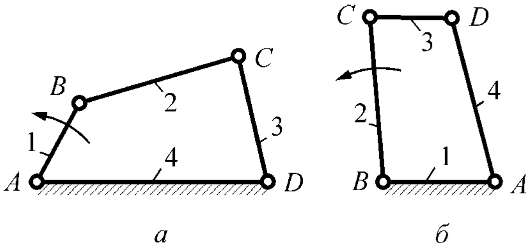


Fig. 1.7.2

If the smallest link 1 is rigidly fixed and link 2 is made the input link, a two-crank mechanism is formed (Fig. 1.7.2, b).

By inverting the crank-slider mechanism (Fig. 1.7.3, a), when the slider 3 is transformed into a riser, and link 2 into an input link, we obtain a mechanism with link 4, which performs translational motion (Fig. 1.7.3, b). This mechanism will be transformed into a crank-rocker mechanism (Fig. 1.7.3, c), if link 1 is the riser, and link 2 is the input (link 4 will be the rocker).

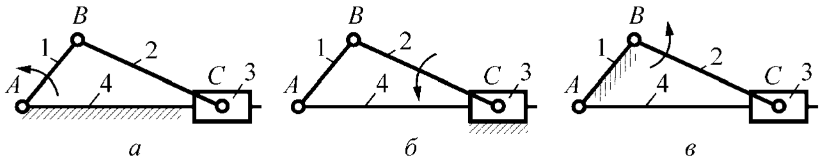


Fig. 1.7.3

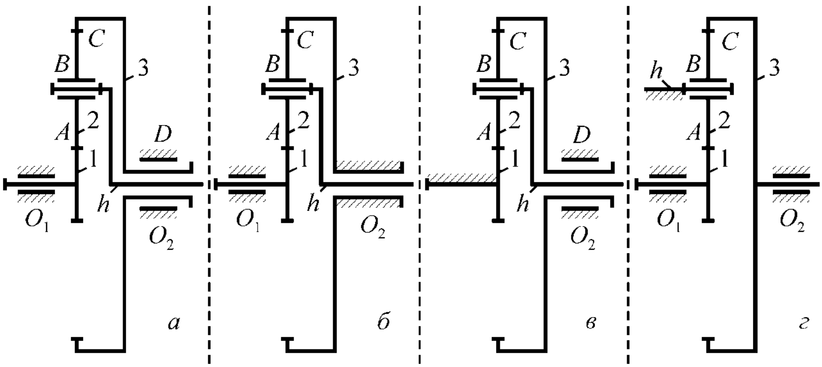


Fig. 1.7.4

By the inversion method from the differential gear mechanism (Fig. 1.7.4, a) it is possible to obtain three different mechanisms. For example, by stopping link 3 (Fig. 1.7.4, b) or 1 (Fig. 1.7.4, c) we obtain two types of planetary gear mechanisms with input link 1 or h and 3 or h; by stopping link h – the carrier – we obtain a regular gear mechanism (Fig. 1.7.4, d).

1.7.3. Transition from a flowchart to a real mechanism

The design of planar mechanisms begins with the synthesis of planar structural diagrams, on which the number of links and the nature of their relative movements are determined, and all kinematic pairs of the 4th and 5th classes are also determined. Since the links in real mechanisms are most often located in different planes, the actual operating conditions of the kinematic pairs will differ from the calculated ones. Therefore, to ensure the normal operation of the mechanism, a spatial structural diagram is additionally constructed.

Let us consider the sequential transition from a flat to a spatial structural diagram of the gas distribution mechanism of an internal combustion engine (Fig. 1.7.5, *a, b*).

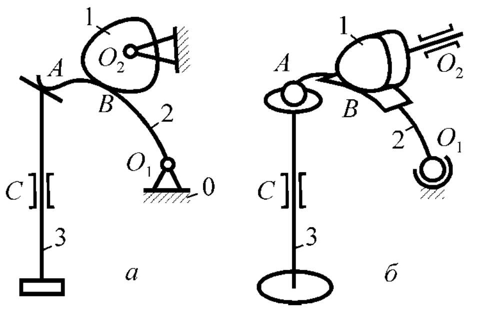


Fig. 1.7.5

The input link of the mechanism is the cam 1 of the camshaft - it interacts with the lever 2, which is in contact with the valve 3. The synthesis of a flat structural diagram to ensure the functional purpose of the gas distribution mechanism can be carried out, for example,

by successive connection to the mechanism of the first class with the rotating pair O2 of two structural groups. The first structural group is composed of link 2 and kinematic pair O1 of the 5th class and kinematic pair B of the 4th class. The second structural group is composed of link 3 and kinematic pair C of the 5th class and kinematic pair A of the 4th class. On the spatial diagram of the mechanism (Fig. 1.7.5, *b*) kinematic pairs of the 4th class A and B will be pairs of the 2nd class, since they allow additional movements. The displacements provided by these kinematic pairs on the flat structural diagram are given in Table 1.4.

Under the conditions adopted in the flat structural diagram, it is necessary to ensure high accuracy of execution of the axes of the kinematic pairs O1 and O2 in the riser, as well as the exact location of the elements of the kinematic pair B relative to these axes. Otherwise, the load distribution along the contact line in the kinematic pair B will be uneven, which in turn will lead to rapid wear of the elements of this kinematic pair. To create favorable contact conditions in the kinematic pair B, it is necessary to provide angular mobility to link 2 by replacing the rotational kinematic pair O1 of the 5th class with the kinematic pair O1 of the 4th class (spherical joint with a finger) (Fig. 1.7.5, *b*). Thus, the constraint imposed by the kinematic pair O1 of the 5th class is eliminated. Eliminating this constraint allows reducing the requirements for the accuracy of manufacturing the elements of the kinematic pairs.

Table 1.4

Kinematic couple	Flat structural diagram		Spatial structural diagram		Real mechanism	
	Personality	Class	Personality	Class	Personality	Class
A	*s*, φ	4	2*s*, 3φ	1	2*s*, 3φ	1
B	*s*, φ	4	2*s*, 2φ	2	2*s*, 2φ	2
O_1	φ	5	2φ	4	3φ	3
C	*s*	5	*s*	5	*s*, φ	4
O_2	φ	5	φ	5	φ	5

The elements of the kinematic pairs O_1, O_2 and *B* determine the position of link 2 relative to the riser. Therefore, if the kinematic pair *A*, as well as the kinematic pair *B* of the 2nd class, are made with contact along the line, then it is necessary to accurately manufacture the elements of this kinematic pair relative to the elements of the kinematic pairs *C* and *B*. This requirement can be prevented if the ligature is eliminated by replacing the kinematic pair *A* of the 2nd class with a pair of the 1st class (Fig. 1.7.5, *b*). In this case, the point contact of links 2 and 3 ensures the direction of the force of their interaction along the axis of the kinematic pair C, which contributes to the creation of favorable operating conditions in this pair and an increase in the durability of its elements.

In real operating conditions, additional relative movements of the links are provided. Thus, for uniform wear of the valve head chamfer under operating conditions (when in contact with the seat), it should be given additional rotation around its own longitudinal axis. Therefore, in a real mechanism, the kinematic pair *C* is made cylindrical of the 4th class. To simplify the manufacturing and assembly technology, it is advisable to replace the kinematic pair O_1 (spherical joint with a finger) with a kinematic pair of the 3rd class (spherical joint). But in this case, the rotation of link 2 relative to its own axis passing through the center of the pair O_1 will appear, which will disrupt the normal operation of the mechanism. In this case, this movement is harmful and must be eliminated (for example, by introducing special springs).

1.7.4. Extra degrees of freedom and coupling conditions

Degrees of freedom and connections that do not affect the mobility of the mechanism as a whole and do not change the kinematics of its links, but affect the degree of mobility, are called redundant. Mechanisms that have redundant connections are called statically indeterminate.

Identifying redundant connections is important, as it allows you to correctly select the class of kinematic pairs that connect the links of the mechanism and make the structure more workable.

Fig. 1.7.6, a shows the kinematic diagram of a flat four-link hinge mechanism with the initial link 1. Its degree of freedom according to the formula (1.4.3)

$$W = 3\cdot 3 - 2\cdot 4 = 1.$$

If, due to inaccuracy in manufacturing and installation, the axes of the hinges are non-parallel, then the links of the mechanism will move in parallel planes only under the condition of their deformation. In this case, deformations can lead either to jamming of the mechanism or to premature failure of one of the links. Since formulas (1.4.2 – 1.4.4) do not reflect the geometric relationships between the links, then when preventing deformation of the links, formula (1.4.2) more fully reflects the possibility of movement of the links in non-parallel planes. Degree of freedom of the mechanism under consideration

$$W = 6\cdot 3 - 5\cdot 4 = -2,$$

which indicates the possibility of loss of mobility due to the presence of redundant connections.

To identify redundant relationships, formulas (1.4.2) and (1.4.3) are transformed to the form

$$W = 6n - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1 + q; \quad (1.7.1)$$

$$W = 3n - 2p_5 - p_4 + q, \quad (1.7.2)$$

q – the number of redundant connections and degrees of freedom.

From (1.7.1) for the spatial mechanism we obtain

$$q = W + 5p_5 + 4p_4 + 3p_3 + 2p_2 + p_1 - 6n; \quad (1.7.3)$$

From (1.7.2) for a flat

$$q = W + 2p_5 + p_4 - 3n \quad (1.7.4)$$

If $q > 0$, then the mechanism has redundant connections, if $q < 0$, then there are redundant degrees of freedom.

For the mechanism in Fig. 1.7.6, and according to formula (1.7.3), we obtain

$$q = 1 - 6 \cdot 3 + 5 \cdot 4 = 3,$$

which indicates the presence of three redundant connections.

Considering that the axes of the hinges are non-parallel, we replace the pairs of the 5th class *B* and *C* with pairs of the 3rd class (spherical hinges) (Fig. 1.7.6, *b*). In this case, we obtain

$$q = 1 - 6 \cdot 3 + 5 \cdot 2 + 3 \cdot 2 = -1.$$

The result indicates the appearance of an extra degree of freedom, which is manifested in the possibility of free rotation of link 2 around its axis. If for some reason the rotation of link 2 is undesirable, then it can be prevented by using a kinematic pair of class 4 (Fig. 1.7.6, *c*) or a spherical pair with a finger (Fig. 1.7.6, *d*) instead of a pair of class 3 *B* or *C*.

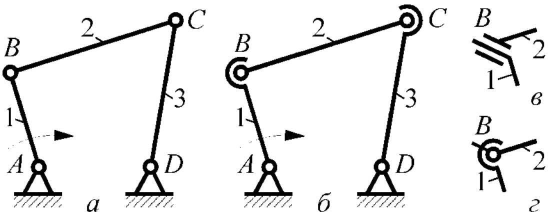


Fig. 1.7.6

The following conclusions can be drawn from the example under consideration. To eliminate unnecessary connections, it is necessary to reduce the class of the corresponding kinematic pairs adopted in the flat scheme (to move from a kinematic pair of class V to a kinematic pair of class III, II or I). Taking into account the spatial structural scheme, a real mechanism is designed in which small displacements relative to the position of the links and elements of the kinematic pairs, caused by manufacturing inaccuracies or deformations of the links from loads, do not affect its normal operation. Mechanisms in which unnecessary connections are eliminated are called rational. In some cases, on the contrary, it is advisable to introduce unnecessary connections, for example, to increase rigidity or distribute the load over several flows.

Excess degrees of freedom that appear when replacing the corresponding kinematic pair with a pair of a lower class, depending on the specific conditions, can be both harmful and useful. Excess degrees of freedom appear in real mechanisms as a result of the synthesis of structural schemes when introducing additional links into them that do not affect the relative motion of the original link, but, in particular, reduce the wear of higher kinematic pairs and improve the operational characteristics of the mechanism or contribute to better load distribution.

1.8. Structural analysis of mechanisms

Structural analysis of mechanisms allows us to determine the number of links, the number and class of kinematic pairs that connect them into kinematic links, the functional purpose of kinematic connections, and to give a comparative characteristic of mechanisms that perform the same functions, even at the stage of choosing their structure. Using the structural diagram of the mechanism, the presence of unnecessary connections and degrees of freedom is determined. Using the methods of structural analysis, it is possible to transform the structural diagram by removing links, changing the class of kinematic pairs that form unnecessary connections and degrees of freedom, and replacing higher kinematic pairs with kinematic connections with lower kinematic pairs. These transformations allow us to present the mechanism as a set of statically defined structural groups with lower kinematic pairs attached to the input links of the mechanism, and to reduce the task of kinematic and dynamic analysis to the use of a number of corresponding operator functions developed for these structural groups.

1.8.1. Determining the mechanism class

Structural analysis is performed in the reverse order of synthesis. Since the structural diagram of the mechanism is formed by the sequential addition of structural groups to the input links, their separation from the structural diagram begins with those groups that include the output links. In this case, the degree of mobility of the rest of the mechanism is determined, which should be equal to the degree of mobility of the output mechanism, and it is checked whether the kinematic chain has not broken up into unrelated parts.

Let us consider the process of structural analysis of the lever mechanism depicted in Fig. 1.8.1, *a*.

1st case (input link 1). The output link of the mechanism is link 5, connected to the link 3 by the kinematic pair *D*, and to the riser by the kinematic pair *E*. It is impossible to distinguish these links into a two-link structural group, since in this case the kinematic chain is broken: link 4 is not connected to the input link, links 4 and 2 receive mobility. For the same reason, links 4 and 3 are not distinguished into a two-link structural group. Therefore, only group 2–3–4–5 is distinguished (Fig. 1.8.1, *b*). The degree of its mobility when connected to the riser

$$W = 3 \cdot 4 - 2 \cdot 6 = 0.$$

Link 3 forms a circuit with kinematic pairs *C*, *B*, *D*, therefore, it is a structural group of class 3. The entire mechanism will also be of class 3.

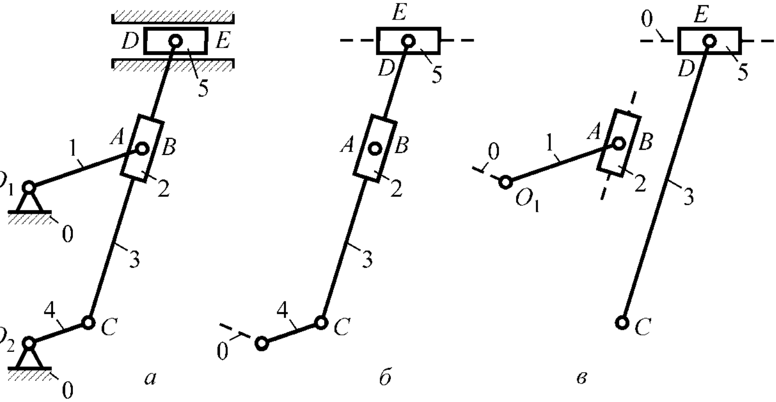


Fig. 1.8.1

2nd case (input link 4). Link 1 and link 2 form a two-link group of the 2nd class of the 2nd type (Fig. 1.8.1, *c*). The degree of mobility of the rest of the mechanism, which consists of links 4–3–5

$$W = 3 \cdot 3 - 2 \cdot 4 = 1.$$

The next two-link group (links 5 and 3) is also a group of the 2nd class of the 2nd kind. Therefore, the entire mechanism in this case is a mechanism of the 2nd class.

From the analysis it follows that the class of structural groups in the mechanism depends on the choice of the input link. Since the methods of kinematic and dynamic research are simpler for mechanisms of lower classes, in structural analysis it is advisable to take as the input such a link, at which the class of the mechanism decreases. This allows to apply simpler algorithms for the calculation.

1.8.2. Replacing higher kinematic pairs with lower ones

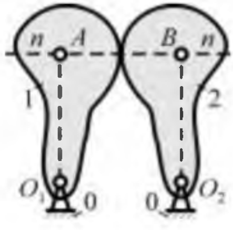
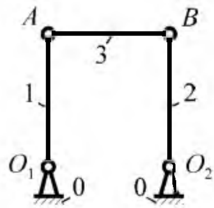
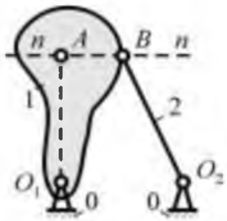
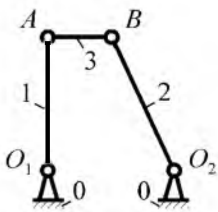
To use simpler algorithms for calculating mechanisms with higher kinematic pairs, structural transformations are performed in groups with higher pairs by replacing them with structurally and kinematically equivalent kinematic chains with lower kinematic pairs. In this case, the condition of structural equivalence must be satisfied, i.e. the nature of the instantaneous relative motion and the number of degrees of freedom of the equivalent mechanism must not change.

Replacing higher class pairs with lower class pairs is done as follows:

1. through the point of contact of links 1 and 2, draw the normal n-n to the contour of the profiles that form the upper pair (Fig. 1.8.2, a; Fig. 1.8.3, a);

2. on the normal in the centers of curvature O_1 and O_2 of the profiles of the output links, rotational kinematic pairs of the 5th class are placed, which are connected by a conditional weightless link 3 (Fig. 1.8.2, b; Fig. 1.8.3, b).

Table 1.5

Radius of curvature	Scheme	
	valid	equivalent
$r_1 > 0$, $r_2 > 0$	 Fig. 1.8.2, a	 Fig. 1.8.2, b
$r_1 > 0$, $r_2 = 0$	 Fig. 2.8.3, a	 Fig. 2.8.3, b

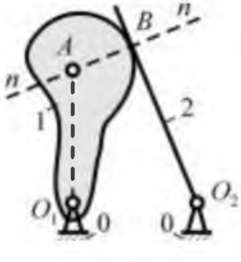
<table>
<tr>
<td>$r_1 > 0$,
$r_2 = \infty$</td>
<td>

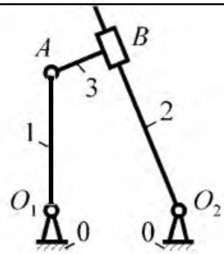
Fig. 2.8.4, a</td>
<td>

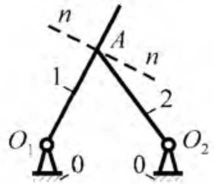
Fig. 2.8.4, b</td>
</tr>
<tr>
<td>$r_1 = \infty$,
$r_2 = 0$</td>
<td>

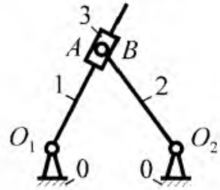
Fig. 2.8.5, a</td>
<td>

Fig. 2.8.5, b</td>
</tr>
</table>

Fig. 1.8.6 shows examples of constructing equivalent mechanisms (a, c – real mechanisms; b, d – their corresponding equivalent mechanisms).

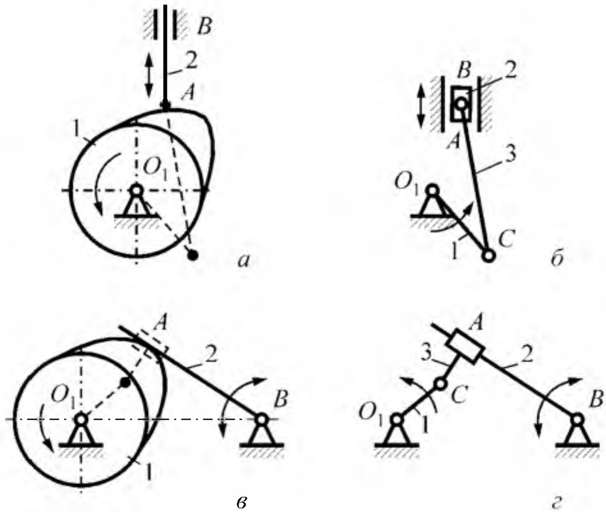


Fig. 1.8.6

It should be noted that when one of the profiles is a straight line, the center of its curvature is infinitely distant, and the corresponding rotational couple is converted into a translational couple (Fig. 1.8.4, a, b; Fig. 1.8.5, a, b).

Comparing the mechanisms depicted in Fig. 1.8.2 – 1.8.5, we obtain that the highest pair in flat mechanisms is equivalent to one conditional link and two kinematic pairs of class V. Note that the obtained equivalent mechanisms in the general case, when the profiles of the highest pairs are curves of variable curvature, are kinematically equivalent to the real mechanism only in the considered position of the input link.

1.8.3. Analysis of the relationships imposed by kinematic pairs

Analysis of flat structural diagrams allows us to determine the number of links, the number of kinematic pairs, the nature of the relative motion of the input and output links and their number, which is equal to the number of degrees of freedom of the mechanism, as well as links that impose redundant connections or links with redundant degrees of freedom.

Links that impose redundant connections are intermediate links that perform the same functions. When one of them is removed, the movement of the output links does not change. An example is a hinged four-link mechanism (Fig. 1.8.7, a). Here, link 2 (or link 4) imposes redundant connections. But for the normal functioning of such a mechanism, it is necessary that the conditions determined by the presence of redundant connections $L_{AD} = L_{EF} = L_{BC}$ are met. If these conditions are violated, the movement of the mechanism is impossible

$$W = 3n - 2p_5 = 3 \cdot 4 - 2 \cdot 6 = 0.$$

The force analysis problem of such a mechanism is statically indeterminate, therefore, to enable the use of static equations when solving force analysis problems, link 2 (or link 4) is eliminated and the kinematically equivalent scheme is used (Fig. 1.8.7, b).

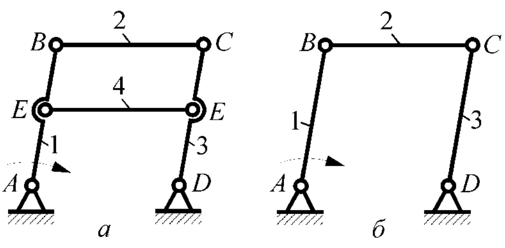


Fig. 1.8.7

Links whose motion does not affect the motion of the output link add extra degrees of freedom. An example is link 3 – a roller in a cam mechanism (Fig. 1.8.8, *a*):

$$W = 3 \cdot 3 - 2 \cdot 3 - 1 = 2 .$$

It is obvious that in this mechanism it is enough to know the position of one cam to uniquely determine the position of the pusher, that is, it is enough to have one initial link, not two, as follows from the Chebyshev formula.

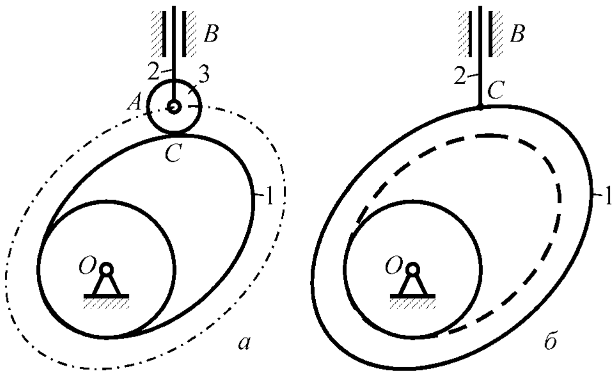


Fig. 1.8.8

In the cam mechanism, the roller 3 creates an extra degree of freedom, it can roll and slide relative to the cam. The roller is a structural element that is introduced to replace sliding friction with rolling friction, that is, to reduce the resistance of friction forces and wear of the links. The kinematics of the mechanism will not change if the roller is removed and the rocker 3 is directly connected to the cam 1 in a kinematic pair of class IV (Fig. 1.8.8,*b*).

To determine a method that allows us to detect redundant connections, we will analyze the mobility in a closed loop formed by structural groups 2 and 3 and connected to the riser by pairs B and D (Fig. 1.8.9). This closed loop is a kinematic chain with a degree of freedom

$$W = 6 \cdot 2 - 4 \cdot 3 = 0 .$$

Analysis of possible movements shows that kinematic pairs C, B and D provide six degrees of freedom relative to a fixed coordinate system. In the considered closed loop, these degrees of freedom in kinematic pairs compensate for possible manufacturing inaccuracies and deformations of the links. When attaching the structural group to the input link 1 and to the riser (Fig. 1.8.10) we obtain a mechanism with the number of degrees of freedom in kinematic pairs 6+1.

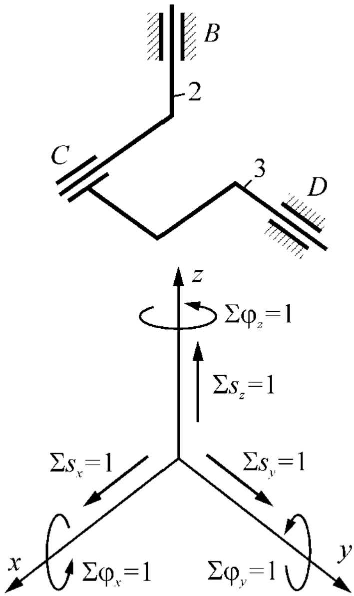


Fig. 1.8.9

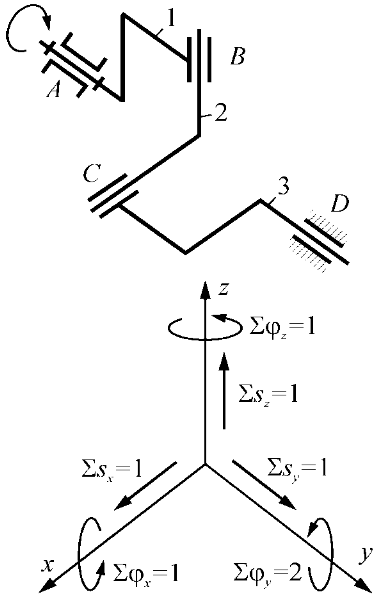


Fig. 1.8.10

The number of degrees of freedom and redundant links in this mechanism

$$W = 6\cdot 3 - 5\cdot 1 - 4\cdot 3 = 1,$$

$$q = 1 + 5\cdot 1 + 4\cdot 3 - 6\cdot 3 = 0.$$

Therefore, if in the connected kinematic link when forming the mechanism six movements are possible relative to the coordinate axes, then taking into account the degrees of freedom of the kinematic pairs in the mechanism there are no unnecessary connections. The absence of any of the six degrees of freedom indicates the presence of an unnecessary connection, except in cases where the absence of a degree of freedom relative to a certain axis is compensated by angular mobility relative to the perpendicular axis.

1.8.4. Procedure for conducting structural analysis of mechanisms

The procedure for structural analysis of mechanisms is as follows.

1. Determine the number of degrees of freedom of the mechanism (or kinematic chain). Links that create unnecessary connections and unnecessary degrees of freedom are discarded during structural analysis. If there are kinematic pairs of class IV, then they must be replaced by pairs of

class V and the structural diagram of the replaced mechanism must be drawn separately.

2. Select the initial links, the number of which is determined by the number of degrees of freedom of the mechanism (kinematic chain). Recall that the initial link and the riser form a class I mechanism.

3. Divide the mechanism into structural groups. The separation of a structural group is most often started with the links and pairs that are farthest from the initial link. They begin with an attempt to disconnect the class II groups from the mechanism. When disconnecting structural groups, it is necessary to check the number of degrees of freedom W of the part of the mechanism that remains, while W should not change. The groups are separated until one initial link and a riser (class I mechanism) remain, if W=1, or several initial links, the number of which is equal to the obtained number of degrees of freedom. If attempts to separate the groups of class II do not give such a result, we must proceed to attempts to separate the groups of class III, then IV, etc.

4. We determine the class and order of the structural groups and the class of the mechanism.

5. We write down the structural formula of the mechanism.

1.9. Questions for self-control

1. What are the connection conditions in kinematic pairs called? What is the difference between the classification of kinematic pairs according to Artobolevsky and Dobrovolsky?

2. How do lower kinematic pairs differ from higher kinematic pairs? Give examples of higher and lower kinematic pairs.

3. What is the geometric and force closure of kinematic pairs called?

4. What is a kinematic chain called? What kinematic chains are distinguished?

5. What are simple and complex, closed and open, flat and spatial kinematic chains called?

6. What is a kinematic connection called? What are kinematic connections used for?

7. What structural formulas are used for flat and spatial kinematic chains?

8. What is an initial link called? How many initial links should there be in a mechanism?

9. What is the difference between structural analysis and structural synthesis of mechanisms?

10. What are called redundant degrees of freedom and connection conditions? How are redundant connections and degrees of freedom determined for a spatial and planar mechanism?

11. What mechanisms are called rational?

12. What are redundant connections used for, and what are redundant degrees of freedom for?

13. What is the content of the structural equivalence condition?

14. What is the basic principle of mechanism formation?

15. What mechanisms are called mechanisms of class I, class II, class III, class IV?

16. What is called the structural group of Assura?

17. What is the basis of structural classification?

18. What structural groups are called structural groups of class II, class III, class IV?

19. What is the class of a contour, group, mechanism determined by?

20. What are the main stages of conducting a structural analysis of a mechanism?

Bibliography

1. Kinytskyi Y.T. Theory of mechanisms and machines: a textbook. - K.: Naukova dumka, 2002. - 660 p.

2. Zablonskyi K.I., Belokonev I.M., Shchekin B.M. Theory of mechanisms and machines. - K.: Vyshcha Shk., 1989. - 335 p.

3. Calculation of lever and cam mechanisms using a personal computer. Study guide / F.Y. Zlatopolskyi, G.B. Filimonikhin, V.V. Kovalenko, O.B. Tchaikovsky. - 2nd ed., trans. and additional - Kirovohrad: PP "Code", 2003. - 149 p.

Theory of mechanisms and machines. Structure and classification of mechanisms

Methodical instructions for lectures

Compiler: V. V. Pirogov

Підп. до друку Формат 60х84 1/16 (А5). Папір друк №3. Друк офсетний.
Умов. друк. арк. Ум. фарбо-відб. Облік.-вид.арк. . Тираж прим.
Зам. №

Центральноукраїнський національний технічний університет
25030, м. Кіровоград. пр. Університетський, 8