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Mechanics

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The tutorial is written in accordance with the Physics course syllabus for engineering and technical specialities of higher education institutions. The first volume contains the following sections: physical foundations of mechanics, electrostatics and direct current, electromagnetism. This textbook is designed to make complex concepts accessible and understandable to a wide range of readers. We start with the basic concepts and gradually move on to more complex topics, making every effort to ensure that everyone can understand the material.The material is presented without cumbersome mathematical transformations, with the main emphasis on the physical essence of the phenomena and the laws that describe them.

Electronic network educational edition

Contents

1. Introduction

Throughout life, each of us has been conditioned to accept, without thought, regular patterns as we conduct our daily business. The Sun rises in the morning and sets at night, dropped objects fall to the ground, water flows downhill, and an ice cube melts when taken out of the freezer. We know that a window will shatter when stuck by a baseball hit by a line drive, and if we live in a glass house it is best to throw nerf balls. The universe constantly reveals its behavior as we go about our daily lives. Conversely, we accept other facts based not on direct experience, but on the authority of others. At times, these facts even contradict our experience and direct observation. For example, it is difficult to comprehend that as you read this book you are sitting on a roughly spherical object rotating at a speed of approximately 800 miles per hour. (This assumes you are reading in the continental United Sates. In Alaska, the speed is only about 500 miles per hour, while at the equator it is just over 1,000 miles per hour.) In addition to its rotation, the Earth is also revolving around the Sun at a speed close to 67,000 miles per hour, and the entire solar system is moving around the center of our Milky Way galaxy at more than 500.000 miles per hour.

The several examples cited above illustrate the science of physics. Physics involves the study of matter and energy in its different forms, and the transformation of matter and energy. This same definition might also apply to chemistry, and the two disciplines are closely related. Chemists tend to focus more on the specific characteristics of matter and how different forms of matter are transformed into other forms of matter. Chemists tend to treat matter and energy as separate entities. Physicists are concerned with the general properties that govern all of matter and energy, and in this sense a clear distinction between the two is unnecessary.

As the quote at the beginning of this chapter states, physics involves trying to explain everyday experience in the simplest terms. The science of physics is a continual quest to explain the behavior of the universe using relatively few basic principles. These basic principles should be applicable to all scales of matter ranging from fundamental particles (quarks) and atoms to galaxies. In a sense, physics can be thought of as the search for a general explanation for everything. During physics' history there have been periods when physicists have claimed they were on the verge of knowing everything needed to explain the universe. For instance, the confidence in Newtonian mechanics at the end of the nineteenth century caused some physicists to claim that little remained to be discovered and this heralded the end of physics. Shortly thereafter, relativity and quantum mechanics gave rise to modern physics. Today the quest for simplicity continues in the form of string theory, but a **general unified theory** also termed a **Theory of Everything** doesn't exist. [1]

1.1 Scalars and Vectors

Before translational motion can be examined, it is important to distinguish between scalar and vector quantities. A **scalar quantity,** or just scalar, is defined by a magnitude and appropriate units. Ten dollars, 3 meters, and 32 °F are examples of scalar quantities. A quantity that is defined by a magnitude and direction with appropriate units is a **vector quantity,** or vector. Wind velocity is a good example of a vector. When wind velocity is reported, both the magnitude and direction are given, for example, 10 miles per hour out of the north. Vectors require a specified or assumed direction. A vector quantity is incomplete without a direction. The importance of including the direction for a vector can be illustrated by considering a person standing at the end of a narrow dock. If the person were told to move three steps forward versus three steps directly backward, it would probably make the difference between falling in the water and staying dry. Three steps is a scalar, whereas three steps backward is a vector.

2. Kinematics

2.1. Rectilinear motion

2.1.1.Speed and Velocity

Speed and **velocity** are terms that are often used interchangeably but are not equivalent. Speed is a scalar and velocity is a vector. In order to understand the difference between speed and velocity, a distinction has to be made between distance and displacement. Before making this distinction, it is important to assume a frame of reference to describe motion. Our most common frame of reference is the Earth. Even though the Earth itself is in motion, it is commonly treated as stationary, and movement is measured with respect to this "stationary" surface. For example, when we are told to measure the distance from a point, it is assumed that the point is fixed at a specific location and the distance is measured from this fixed point. In our treatment of motion, a fixed Cartesian coordinate system attached to the Earth will be assumed, with horizontal directions referenced to compass directions. Vertical motion will simply be referred to as up or down.

Distance is scalar measure of the amount of linear space covered. Displacement is a vector that points from an object's initial position to its final position. The magnitude of the displacement is simply the straight-line distance between the initial and final positions. If a person jogs around a 400 meter track, then the distance covered is 400 meters, but the displacement is zero since the jogger ends up right back at the starting line. Using the Cartesian coordinate system, the distinction between distance and displacement can be demonstrated by considering a bicyclist traveling 4 miles to the east and then 3 miles north. The distance covered by the bicyclist is 7 miles. The displacement calculated using the Pythagorean theorem is 5 miles in a direction 30° north of east.

Distance and displacement can be used to define speed and velocity. Speed is a scalar quantity defined by the distance an object travels divided by the time of travel, whereas velocity is a vector defined by displacement divided by the time of travel. To determine time, the final time is subtracted from the initial time. For example, if the bicyclist started a trip at 1:00 P.M. and ended an hour later at 2:00 P.M., the time of travel would be 1 hour. Since time is always determine by subtracting the start time from the end time, it can be represented mathematically as t. The Δ is called the delta sign and means “change in”; therefore, Δt means the change in time. The average speed of the bicyclist during her trip is found by dividing the distance covered of 7 miles by Δt, which is 1 hour, to give 7 miles per hour (mph). The units for speed will be length divided by time, for example, feet per second (ft/s). or kilometers per hour (km/h). The most familiar unit for speed in the United States is miles per hour. The magnitude of the average velocity is found by dividing the displacement of 5 miles by 1 hour to give 5 mph. This is only the magnitude of the velocity. Since velocity is a vector. a direction must also be given. The average velocity should be reported as 5 mph in a direction 37° to the north of east.

The speed and velocity calculated were specified as the average speed and average velocity. The formulas for the average speed and velocity are

$$\text{average speed} = \frac{\text{distance}}{\Delta t}$$

$$\text{average velocity} = \frac{\text{displacement}}{\Delta t}$$

The equations and examples in this section illustrate the importance of using the correct terminology in physics. When reporting a velocity, a direction should always be given along with the magnitude, but at times the direction can be assumed. For instance, for a jet taking off down a runway with an average velocity reported as 200 mph, the direction is assumed down the runway.

The average speed and velocity were calculated for the bicyclist in the previous example. Average speed and velocity are useful for reporting the general progress of a trip but don’t represent what may be happening at any particular time during the trip. The bicyclist could have pedaled quickly and covered the first 4 miles in 20 minutes, rested for 20 minutes, and then completed the last 3 miles in the remaining 20 minutes. The speed and velocity can be characterized for each of these intervals. In order to describe what is happening at any point in time, the instantaneous speed and velocity can be used. The instantaneous speed is the speed at any instant, while the instantaneous velocity is the velocity at any instant. The latter includes both the magnitude and direction of motion at any instant. Instantaneous motion can be found by calculating the speeds and velocities over smaller and smaller time intervals. A 20 minute interval is much too large, but the question arises as to how small a time interval must be to determine instantaneous motion. Just how long is an instant? In the bicycle example, instantaneous motion can be approximated over short time intervals such as a few seconds. For example, the 1 hour bicycle trip could be broken up into 1,200 3 second intervals and a speed and velocity calculated for each of these 3 second intervals. The 3 second intervals could be broken down further. To determine the true instantaneous velocity requires the use of calculus and taking the limit as Δr approaches zero in the equations for average speed and velocity. A formal derivation using calculus for instantaneous motion will not be presented here; the approximate definition for instantaneous speed and velocity as speed and velocity measured over a short time interval will be adequate

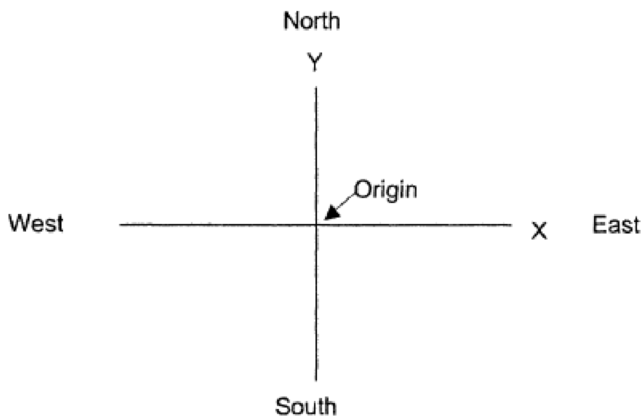


Figure. Motion is referred to a Cartesian coordinate system, with the compass directions indicating horizontal motion

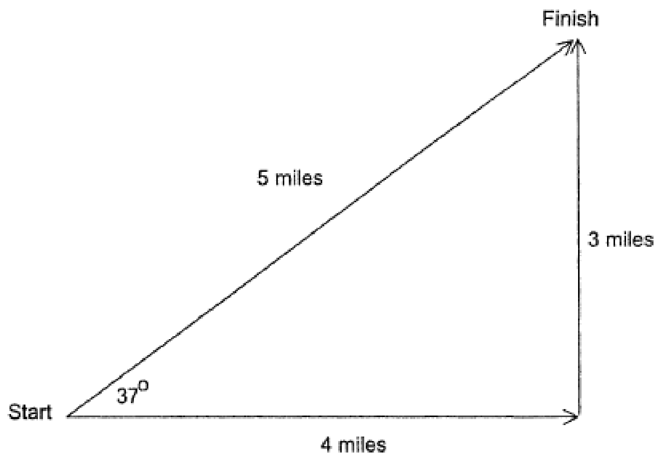


Figure . Moving a distance 4 miles east and then 3 miles north results in a displacement of 5 miles at an angle of approximately 37° north of east

Incidentally, to a good approximation we have another law, which says that the change in distance of a moving point is the velocity times the time interval, or $\Delta s = v\,\Delta t.$. This statement is true only if the velocity is not changing during that time interval, and this condition is true only in the limit as Δt goes to 0. Physicists like to write it $ds = v\,dt,$ because by *dt* they mean Δt in circumstances in which it is very small; with this understanding, the expression is valid to a close approximation. If Δt is too long, the velocity might change during the interval, and the approximation would become less accurate. For a time *dt*, approaching zero, *ds* = *v dt* precisely. In this notation we can write as

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}.$$

2.1.2.Distance as an integral

Now we have to discuss the inverse problem. Suppose that instead of a table of distances, we have a table of speeds at different times, starting from zero. A similar table could be constructed for the velocity of the car, by recording the speedometer reading every minute or half-minute. If we know how fast the car is going at any time, can we determine how far it goes? This problem is just the inverse of the one solved above; we are given the velocity and asked to find the distance. How can we find the distance if we know the speed? If the speed of the car is not constant, and the lady goes sixty miles an hour for a moment, then slows down, speeds up, and so on, how can we determine how far she has gone? That is easy. We use the same idea, and express the distance in terms of infinitesimals. Let us say, "In the first second her speed was such and such, and from the formula $\Delta s = v\, \Delta t$ we can calculate how far the car went the first second at that speed." Now in the next second her speed is nearly the same, but slightly different; we can calculate how far she went in the next second by taking the new speed times the time. We proceed similarly for each second, to the end of the run. We now have a number of little distances, and the total distance will be the sum of all these little pieces. That is, the distance will be the sum of the velocities times the times, or s = 52 $v\, \Delta t$, where the Greek letter Σ (sigma) is used to denote addition. To be more precise, it is the sum of the velocity at a certain time, multiplied by Δt.

$$s = \sum_i v(t_i)\, \Delta t.$$

The rule for the times is that. $t_{i+1} = t_i + \Delta t.$ However, the distance we obtain by this method will not be correct, because the velocity changes during the time interval Δt. If we take the times short enough, the sum is precise, so we take them smaller and smaller until we obtain the desired accuracy. The true s is

$$s = \lim_{\Delta t \to 0} \sum_i v(t_i)\, \Delta t.$$

The mathematicians have invented a symbol for this limit, analogous to the symbol for the differential. The Δ turns into a t to remind us that the time is as small as it can be; the velocity is then called v at the time t, and the addition is written as a sum with a great *"s," ∫* (from the Latin *summa)*, which has become distorted and is now unfortunately just called an integral sign. Thus, we write

$$s = \int v(t)\, dt.$$

This process of adding all these terms together is called integration, and it is the opposite process to differentiation. The derivative of this integral is v, so one operator (d) undoes the other ($\int$). One can get formulas for integrals by taking the formulas for derivatives and running them backwards, because they are related to each other inversely. Thus one can work out his own table of integrals by differentiating all sorts of functions. For every formula with a differential, we get an integral formula if we turn it around. Every function can be differentiated analytically, i.e., the process can be carried out algebraically, and leads to a definite function. But it is not possible in a simple manner to write an analytical value for any integral at will. You can calculate it, for instance, by doing the above sum, and then doing it again with a finer interval Δt and again with a finer interval until you have it nearly right. In general, given some particular function, it is not possible to find, analytically, what the integral is. One may always try to find a function which, when differentiated, gives some desired function; but one may not find it, and it may not exist, in the sense of being expressible in terms of functions that have already been given names.[1]

2.1.3.Acceleration

Acceleration is another important variable used to describe motion and refers to the change in velocity over a period of time. Since acceleration results from a change in velocity, which is a vector, it also is a vector. Acceleration may result from a change in magnitude, direction, or both magnitude and direction of the velocity vector. In this chapter, only a change in speed is considered; in the next chapter it will be seen that circular motion results from a change in direction of the acceleration vector. The equation for average acceleration can be written as

$$\text{average acceleration} = \frac{\Delta\text{velolcity}}{\Delta\text{time}}$$

The units of acceleration will be a length per time per time, for example, kilometers per hour per second. Acceleration will typically be expressed as m/s/s in metric units. In this case, where time units are the same, the acceleration would be expressed as m/s^2.

One of the most familiar examples of acceleration is during acceleration or braking in traffic. When accelerating from a red light, the initial speed of a vehicle is 0 mph. Several seconds later the vehicle has obtained some speed, and using the time it takes to obtain this speed, the magnitude of the acceleration vector can be calculated. For example, if it takes 10 s to reach 40 mph, the acceleration is 4 mph/s. In this case the direction is assumed to be in the direction of the road. Acceleration is a measure of performance for cars and is often reported indirectly by stating the time it takes for a vehicle to reach 60 mph (or 100 km/h in countries using the metric system). Reported values can be used to calculate the average acceleration for a car. A 2003 Ford Mustang has a reported time of 5.6 s, so its acceleration is 10.7 mph/s. This compares to a Honda Civic that takes 10.2 s to reach 60 mph. giving an acceleration of 5.9 mph/s. A dragster reaches 300 mph in approximately 4.5 s down a quarter mile straightaway, giving an acceleration of almost 67 mph/s. This is about the same as the maximum acceleration of the Space

Shuttle during take-off. Greater yet is the acceleration of bullets fired from a gun, which is several hundred miles per hour per second. It must be remembered that these acceleration values are only the magnitude of the acceleration, and that it is assumed that the acceleration vector is in the direction of the motion.

A deceleration is the same as a negative acceleration. This can be seen when a car that is moving 40 mph comes to a stop in 10 s. In this case the change in velocity is -40 mph. so the acceleration is —4 mph/s. Each second the car moves 4 mph slower. The acceleration vector points in the opposite direction of the motion. Just as with velocity, a distinction is made between average acceleration and instantaneous acceleration. If an object accelerates at a uniform rate, then the acceleration is constant over the course of motion. On the other hand, if acceleration is not constant, short time intervals can be used to approximate the instantaneous acceleration. The instantaneous acceleration at any point would equal the limit of the change in velocity divided by the change in time as the time interval approaches zero. The instantaneous acceleration can be approximated by dividing the change in velocity by a short time interval.

2.1.4. General Formulas for Kinematics

Displacement, velocity, and acceleration can be used to describe motion. The description of motion is known as kinematics. When the acceleration is constant, several equations can be used to describe the kinematics of motion in one dimension. These are summarized in Table.1.

1	$v_t = v_0 + at$
2	$\Delta x = \left(\frac{v_t - v_0}{2}\right) t$
3	$\Delta x = v_0 t + \frac{at^2}{2}$
4	$\Delta x = \frac{v_t^2 - v_0^2}{2a}$

v_t – final velocity, v_0 - initial velocity, x – position, Δx – displacement, a-acceleration, t – time.

2.2. Rotational Motion

Introduction

In translational motion, an object moves from one point to another. Another type of motion occurs when an object moves in a curved path. This type of motion is called rotational motion. Examples of rotational motion are all around us. A spinning compact disc, the wheels of a car. and a blowing fan are a few common examples of rotational motion. As you sit reading this book, you are in constant rotational motion as the Earth spins on its axis. Rotational motion may accompany translational motion, as when a ball rolls down a ramp. It may also occur in the absence of translational motion, as when a top spins on its axis. Rotation refers to the circular movement of an object about an axis that passes through the object. Another type of curved motion occurs when an object moves about an axis outside of it. Both rotational and revolutionary motion will be considered in this chapter, and the term rotational motion will be used as a generic description for curved motion. Many of the concepts for rotational motion parallel those for translational motion. The simplest type of rotational motion is uniform circular motion.

2.2.1.Circular Motion: Velocity and Angular Velocity

We begin our description of circular motion by choosing polar coordinates. In Figure we sketch the position vector **r** *(t*) of the object moving in a circular orbit of radius *r* .

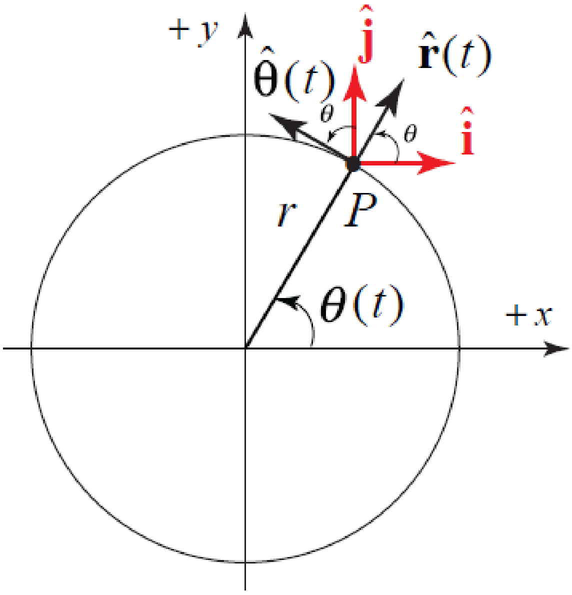


Figure A circular orbit with unit vectors

At time *t* , the particle is located at the point *P* with coordinates *(r, θ(t*)) and position vector given by

$$\vec{\mathbf{r}}(t) = r\,\hat{\mathbf{r}}(t)\,.$$

At the point *P* , consider two sets of unit vectors ($\hat{r}(t)$, $\hat{\theta}(t)$) and ($\hat{\imath},\hat{\jmath}$), as shown in Figure. The vector decomposition expression for $\hat{r}(t)$ and $\hat{\theta}(t)$ in terms of $\hat{\imath}$ and $\hat{\jmath}$ is given by

$$\hat{\mathbf{r}}(t) = \cos\theta(t)\,\hat{\mathbf{i}} + \sin\theta(t)\,\hat{\mathbf{j}}\,,$$
$$\hat{\boldsymbol{\theta}}(t) = -\sin\theta(t)\,\hat{\mathbf{i}} + \cos\theta(t)\,\hat{\mathbf{j}}\,.$$

Before we calculate the velocity, we shall calculate the time derivatives of Eqs.. Let’s first begin with $d\hat{r}\,(t)/\,dt$

$$\frac{d\hat{\mathbf{r}}(t)}{dt} = \frac{d}{dt}(\cos\theta(t)\,\hat{\mathbf{i}} + \sin\theta(t)\,\hat{\mathbf{j}}) = (-\sin\theta(t)\frac{d\theta(t)}{dt}\,\hat{\mathbf{i}} + \cos\theta(t)\frac{d\theta(t)}{dt}\,\hat{\mathbf{j}}) \; ;$$
$$= \frac{d\theta(t)}{dt}(-\sin\theta(t)\,\hat{\mathbf{i}} + \cos\theta(t)\,\hat{\mathbf{j}}) = \frac{d\theta(t)}{dt}\hat{\boldsymbol{\theta}}(t)$$

where we used the chain rule to calculate that

$$\frac{d}{dt}\cos\theta(t) = -\sin\theta(t)\frac{d\theta(t)}{dt} \; ,$$
$$\frac{d}{dt}\sin\theta(t) = \cos\theta(t)\frac{d\theta(t)}{dt} \; .$$

The calculation for $\frac{d\hat{\theta}(t)}{dt}$ is similar:

$$\frac{d\hat{\boldsymbol{\theta}}(t)}{dt} = \frac{d}{dt}(-\sin\theta(t)\,\hat{\mathbf{i}} + \cos\theta(t\,\hat{\mathbf{j}}) = (-\cos\theta(t)\frac{d\theta(t)}{dt}\,\hat{\mathbf{i}} - \sin(t)\frac{d\theta(t)}{dt}\,\hat{\mathbf{j}}) \; .$$
$$= \frac{d\theta(t)}{dt}(-\cos\theta(t)\,\hat{\mathbf{i}} - \sin\theta(t)\,\hat{\mathbf{j}}) = -\frac{d\theta(t)}{dt}\hat{\mathbf{r}}(t)$$

The velocity vector is then

$$\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}(t)}{dt} = r\frac{d\hat{\mathbf{r}}}{dt} = r\frac{d\theta}{dt}\hat{\boldsymbol{\theta}}(t) = v_\theta\,\hat{\boldsymbol{\theta}}(t),$$

where the $\boldsymbol{\theta}$ -component of the velocity is given by

$$v_\theta = r\frac{d\theta}{dt},$$

a quantity we shall refer to as the *tangential component of the velocity*. Denote the magnitude of the velocity by $v \equiv |\vec{\mathbf{v}}|$. The angular speed is the magnitude of the rate of change of angle with respect to time, which we denote by the Greek letter

$$\omega \equiv \left|\frac{d\theta}{dt}\right| .$$

2.2.2. Geometric Derivation of the Velocity for Circular Motion

Consider a particle undergoing circular motion. At time *t*, the position of the particle is ***r****(t)* . During the time interval Δ*t* , the particle moves to the position ***r(****t* + Δ*t*) with a displacement Δ**r** .

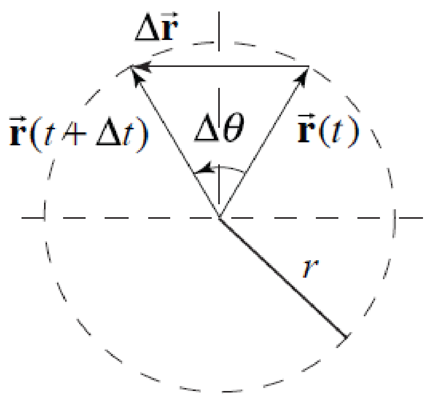


Figure. Displacement vecnor for circular motion

The magnitude of the displacement, |Δ**r** |, is represented by the length of the horizontal vector Δ**r** joining the heads of the displacement vectors in Figure 6.2 and is given by

$$|\Delta\vec{\mathbf{r}}| = 2r\sin(\Delta\theta/2)\,.$$

When the angle Δθ is small, we can approximate

$$\sin(\Delta\theta/2) \cong \Delta\theta/2\,.$$

This is called the *small angle approximation,* where the angle Δθ (and hence Δθ/2) is measured in radians. This fact follows from an infinite power series expansion for the sine function given by

$$\sin\left(\frac{\Delta\theta}{2}\right) = \frac{\Delta\theta}{2} - \frac{1}{3!}\left(\frac{\Delta\theta}{2}\right)^3 + \frac{1}{5!}\left(\frac{\Delta\theta}{2}\right)^5 - \cdots\,.$$

When the angle Δθ/ 2 is small, only the first term in the infinite series contributes, as successive terms in the expansion become much smaller. For example, when $\frac{\Delta\theta}{2} = \frac{\pi}{30} \cong 0{,}1$, corresponding to 6°, $\frac{\left(\frac{\Delta\theta}{2}\right)^3}{3!} \cong 1{,}9 * 10^{-4}$; this term in the power series is three orders of magnitude smaller than the first and can be safely ignored for small angles

Using the small angle approximation, the magnitude of the displacement is

$$\left|\Delta\vec{\mathbf{r}}\right| \cong r\ \Delta\theta\ .$$

This result should not be too surprising since in the limit as $\Delta\theta$ approaches zero, the length of the chord approaches the arc length $r\Delta\theta$.

The magnitude of the velocity, v , is proportional to the rate of change of the magnitude of the angle with respect to time

$$v \equiv \left|\vec{\mathbf{v}}(t)\right| = \lim_{\Delta t\to 0}\frac{\left|\Delta\vec{\mathbf{r}}\right|}{\Delta t} = \lim_{\Delta t\to 0}\frac{r\left|\Delta\theta\right|}{\Delta t} = r\lim_{\Delta t\to 0}\frac{\left|\Delta\theta\right|}{\Delta t} = r\left|\frac{d\theta}{dt}\right| = r\omega\ .$$

The direction of the velocity can be determined by considering that in the limit as $\Delta t \to 0$ (note that $\Delta\theta \to 0$), the direction of the displacement $\overrightarrow{\Delta r}$ approaches the direction of the tangent to the circle at the position of the particle at time t (Figure 6.3).

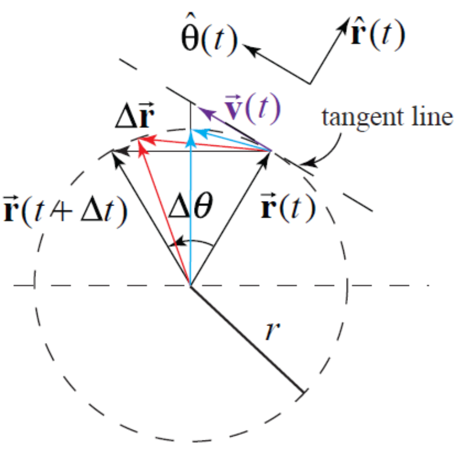


Figure. Direction of the displacement approaches the direction of the tangent line

Thus, in the limit$\Delta t \to 0$, $\Delta\vec{r} \perp \vec{r}$, and so the direction of the velocity $\vec{v}(t)$ at time t is perpendicular to the position vector $\vec{r}(t)$ and tangent to the circular orbit in the $+\hat{\theta}$ −- direction for the case shown in Figure.

2.2.3. Circular Motion: Tangential and Radial Acceleration

When the motion of an object is described in polar coordinates, the acceleration has two components, the tangential component a_θ, and the radial component, a_r We can write the acceleration vector as

$$\vec{\mathbf{a}} = a_r\ \hat{\mathbf{r}}(t) + a_\theta\ \hat{\boldsymbol{\theta}}(t)\ .$$

Keep in mind that as the object moves in a circle, the unit vectors ***r***(*t*) and ***θ***(*t*) change direction and hence are not constant in time.

We will begin by calculating the tangential component of the acceleration for circular motion. Suppose that the tangential velocity $v_\theta = \frac{rd\theta}{dt}$ is changing in magnitude due to the presence of some tangential force; we shall now consider that $d\theta/dt$ is changing in time, (the magnitude of the velocity is changing in time). Recall that in polar coordinates the velocity vector Eq. (6.2.8) can be written as

$$\vec{\mathbf{v}}(t) = r\frac{d\theta}{dt}\hat{\boldsymbol{\theta}}(t)\ .$$

We now use the product rule to determine the acceleration

$$\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}(t)}{dt} = r\frac{d^2\theta(t)}{dt^2}\hat{\boldsymbol{\theta}}(t) + r\frac{d\theta(t)}{dt}\frac{d\hat{\boldsymbol{\theta}}(t)}{dt}\ .$$

Recall from Eq. (6.2.3) that $\hat{\theta}(t) = -\sin\theta(t)\,\hat{\imath} + \cos\theta(t)\,\hat{\jmath}$. So we can rewrite Eq. as

$$\vec{\mathbf{a}}(t) = r\frac{d^2\theta(t)}{dt^2}\hat{\boldsymbol{\theta}}(t) + r\frac{d\theta(t)}{dt}\frac{d}{dt}(-\sin\theta(t)\hat{\mathbf{i}} + \cos\theta(t)\ \hat{\mathbf{j}})\ .$$

We again use the chain rule and find that

$$\vec{\mathbf{a}}(t) = r\frac{d^2\theta(t)}{dt^2}\hat{\boldsymbol{\theta}}(t) + r\frac{d\theta(t)}{dt}\left(-\cos\theta(t)\frac{d\theta(t)}{dt}\hat{\mathbf{i}} - \sin\theta(t)\frac{d\theta(t)}{dt}\ \hat{\mathbf{j}}\right).$$

Recall that $= \frac{d\theta}{dt}$, and from Eq., $\hat{r}(t) = \cos\theta(t)\hat{\imath} + \sin\theta(t)\hat{\jmath}$, therefore the acceleration becomes

$$\vec{\mathbf{a}}(t) = r\frac{d^2\theta(t)}{dt^2}\hat{\boldsymbol{\theta}}(t) - r\left(\frac{d\theta(t)}{dt}\right)^2\hat{\mathbf{r}}(t) .$$

The *tangential component of the acceleration* is then

$$a_\theta = r\frac{d^2\theta(t)}{dt^2} .$$

The *radial component of the acceleration* is given by

$$a_r = -r\left(\frac{d\theta(t)}{dt}\right)^2 = -r\omega^2 < 0 .$$

Because $a_r < 0$, that radial vector component $\vec{a_r}(t) = -r\omega^2\hat{r}(t)$ is always directed towards the center of the circular orbit.

2.2.4.Period and Frequency for Uniform Circular Motion

If the object is constrained to move in a circle and the total tangential force acting on the object is zero, $F_\theta^{total} = 0$ then (Newton's Second Law), the tangential acceleration is zero,

$$a_\theta = 0 .$$

This means that the magnitude of the velocity (the speed) remains constant. This motion is known as *uniform circular motion.* The acceleration is then given by only the acceleration radial component vector uniform circular motion

$$\vec{\mathbf{a}}_r(t) = -r\omega^2(t)\,\hat{\mathbf{r}}(t)$$

Because the speed $v = r|\omega|$ is constant, the amount of time that the object takes to complete one circular orbit of radius r is also constant. This time interval, T , is called the *period***.** In one period the object travels a distance $s = vT$ equal to the circumference,$s=2\pi r$; thus

$$s = 2\pi r = vT .$$

The period T is then given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{r\omega} = \frac{2\pi}{\omega} .$$

The *frequency* f is defined to be the reciprocal of the period

$$f = \frac{1}{T} = \frac{\omega}{2\pi} .$$

The SI unit of frequency is the inverse second, which is defined as the hertz, $[s^{-1}] = [Hz]$

The magnitude of the radial component of the acceleration can be expressed in several equivalent forms since both the magnitudes of the velocity and angular velocity are related by $v = r\omega$. Thus we have several alternative forms for the magnitude of the centripetalacceleration. Thefirstis thatin Equation (6.5.3).

Thesecond is in terms of the radius and the angular velocity

$$|a_r| = r\omega^2 .$$

The third form expresses the magnitude of the centripetal acceleration in terms of the speed and radius

$$|a_r| = \frac{v^2}{r}.$$

Recall that the magnitude of the angular velocity is related to the frequency by $\omega = 2\pi f$, so we have a fourth alternate expression for the magnitude of the centripetal acceleration in terms of the radius and frequency

$$|a_r| = 4\pi^2 r f^2 .$$

A fifth form commonly encountered uses the fact that the frequency and period are related by $f = \frac{1}{T} = \frac{\omega}{2\pi}$. Thus we have the fourth expression for the centripetal acceleration in terms of radius and period,

$$|a_r| = \frac{4\pi^2 r}{T^2} .$$

Often we decide which expression to use based on information that describes the orbit. A convenient measure might be the orbit's radius. We may also independently know the period, or the frequency, or the angular velocity, or the speed. If we know one, we can calculate the other three but it is important to understand the meaning of each quantity.[2]

2.2.5.Geometric Interpretation for Radial Acceleration for Uniform Circular Motion

An object traveling in a circular orbit is always accelerating towards the center. Any radial inward acceleration is called *centripetal acceleration.* Recall that the direction of the velocity is always tangent to the circle. Therefore the direction of the velocity is constantly changing because the object is moving in a circle, as can be seen in Figure 6.4. Because the velocity changes direction, the object has a nonzero acceleration.

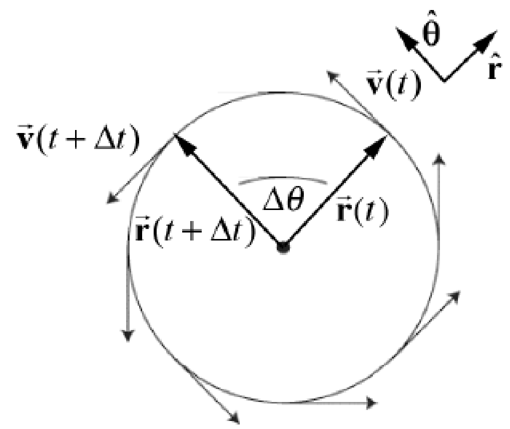


Figure. Direction of the velocity for circular motion.

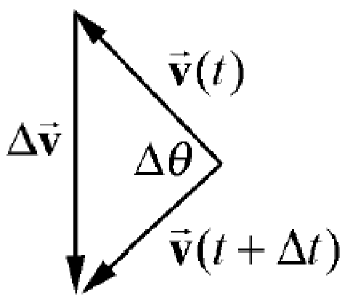


Figure . Change in velocity vector.

The calculation of the magnitude and direction of the acceleration is very similar to the calculation for the magnitude and direction of the velocity for circular motion, but the change in velocity vector, $\Delta\vec{v}$, is more complicated to visualize. The change in velocity $\Delta\vec{v} = \vec{v}(t + \Delta t) - \vec{v}(t)$is depicted in Figure 6.5. The velocity vectors have been given a common point for the tails, so that the change in velocity, Δυ , can be visualized. The length $|\Delta\vec{v}|$of the vertical vector can be calculated in exactly the same way as the displacement$|\Delta\vec{r}|$. The magnitude of the change in velocity is

$$\left|\Delta\vec{\mathbf{v}}\right| = 2v\sin(\Delta\theta / 2).$$

We can use the small angle approximation $\sin(\frac{\theta}{2}) \cong \frac{\Delta\theta}{2}$ to approximate the magnitude of the change of velocity,

$$\left|\Delta\vec{\mathbf{v}}\right| \cong v\left|\Delta\theta\right|.$$

The magnitude of the radial acceleration is given by

$$\left|a_r\right| = \lim_{\Delta t\to 0}\frac{\left|\Delta\vec{\mathbf{v}}\right|}{\Delta t} = \lim_{\Delta t\to 0}\frac{v\left|\Delta\theta\right|}{\Delta t} = v\lim_{\Delta t\to 0}\frac{\left|\Delta\theta\right|}{\Delta t} = v\left|\frac{d\theta}{dt}\right| = v\left|\omega\right|.$$

The direction of the radial acceleration is determined by the same method as the direction of the velocity; in the limit Δθ→0, $\Delta\vec{v} \perp \vec{v}$, and so the direction of the acceleration radial component vector $\mathbf{a}_r(t)$ at time t is perpendicular to position vector $\vec{v}(t)$ and directed inward, in the $-\hat{r}$ –direction

3. Dynamics

3.1. Newton's First Law

The First Law of Motion, commonly called the "Principle of Inertia," was first realized by Galileo. (Newton did not acknowledge Galileo's contribution.) Newton was particularly concerned with how to phrase the First Law in Latin, but after many rewrites Newton choose the following expression for the First Law (in English translation):

Law 1: Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

Projectiles continue in their motions, so far as they are not retarded by the resistance of air, or impelled downwards by the force of gravity. A top, whose parts by their cohesion are continually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by air. The greater bodies of planets and comets, meeting with less resistance in freer spaces, preserve their motions both progressive and circular for a much longer time

The first law is an experimental statement about the motions of bodies. When a body moves with constant velocity, there are either no forces present or the sum of all the forces acting on the body is zero. If the body changes its velocity, it has non-zero acceleration, and hence the sum of all the forces acting on the body must be non-zero as well. If the velocity of a body changes in time, then either the direction or magnitude changes, or both can change.

After a bus or train starts, the acceleration is often so small we can barely perceive it. We are often startled because it seems as if the station is moving in the opposite direction while we seem to be at rest. Newton's First Law states that there is no physical way to distinguish between whether we are moving or the station is moving, because there is nearly zero total force acting on the body. Once we reach a constant velocity, our minds dismiss the idea that the ground is moving backwards because we think it is impossible, but there is no actual way for us to distinguish whether the train is moving or the ground is moving.

3.2. Momentum, Newton's Second Law and Third Law

Newton began his analysis of the cause of motion by introducing the quantity of motion:

Definition: Quantity of Motion

The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.

The motion of the whole is the sum of the motion of all its parts; and therefore in a body double in quantity, with equal velocity, the motion is double, with twice the velocity, it is quadruple.

Our modern term for quantity of motion is *momentum* and it is a vector quantity

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \,,$$

where m is the inertial mass and $\mathbf{v}$ is the velocity of the body. Newton's Second Law states that

Law II: The change of motion is proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force is impressed altogether and at once or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added or subtracted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both.

Suppose that a force is applied to a body for a time interval Δt . The impressed force or *impulse* (a vector quantity $\vec{\boldsymbol{I}}$) produces a change in the momentum of the body,

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}} \,.$$

From the commentary to the second law, Newton also considered forces that were applied continually to a body instead of impulsively. The instantaneous action of the total force acting on a body at a time t is defined by taking the mathematical limit as the time interval Δt becomes smaller and smaller,

$$\vec{\mathbf{F}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{p}}}{\Delta t} \equiv \frac{d\vec{\mathbf{p}}}{dt} .$$

When the mass remains constant in time, the Second Law can be recast in its more familiar form,

$$\vec{\mathbf{F}} = m \frac{d\vec{\mathbf{v}}}{dt} .$$

Because the derivative of velocity is the acceleration, the force is the product of mass and acceleration,

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} .$$

Because we defined force in terms of change in motion, the Second Law appears to be a restatement of this definition, and devoid of predictive power since force is only determined by measuring acceleration. What transforms the Second Law from just a definition is the additional input that comes from *force laws* that are based on experimental observations on the interactions between bodies. Throughout this book, we shall investigate these force laws and learn to use them in order to determine the forces and accelerations acting on a body (left-hand-side of Newton's Second Law). When a physical body is constrained to move along a surface, or inside a container (for example gas molecules in a container), there are *constraint forces* that are not determined beforehand by any force law but are only determined by their effect on the motion of the body. For any given constrained motion, these constraint forces are unknown and must be determined by the particular motion of the body that we are studying, for example the contact force of the surface on the body, or the force of the wall on the gas particles.

The right-hand-side of Newton's Second Law is the product of mass with acceleration. Acceleration is a mathematical description of how the velocity of a body changes. Knowledge of all the forces acting on the body enables us to predict the acceleration. Eq. is known as the *equation of motion.* Once we know this equation we may be able to determine the velocity and position of that body at all future times by integration techniques, or computational techniques. For constrained motion, if we know the acceleration of the body, we can also determine the constraint forces acting on the body.[2]

3.3. Newton's Third Law

Newton’s third law states that for every action there is an equal and opposite reaction. This means that forces always exist in pairs that are equal in magnitude but directly opposed to each other. It is important to realize that in Newton’s third law the action and reaction forces act on different objects. If you push on the wall with a force equal to 10 N, then the wall pushes back on you with a force equal to 10 N. As a person walks across the ground, he exerts a force on the Earth, while simultaneously the Earth exerts a force on the person. The Earth’s force causes the person to accelerate forward according to Newton’s second law, $a = F/m$. At the same time the Earth is accelerated in a direction opposite of the person walking. Because of the Earth’s tremendous mass compared to a person, its acceleration can be considered to be zero.

There are many familiar examples of Newton’s third law. Jet and rocket engines work by expelling hot gases and propelling the vehicle forward. The Space Shuttle uses this same concept to make corrections in flight with short bursts from horizontal and vertical thrusters. Anyone who has fired a rifle experiences a certain degree of recoil with each shot. The recoil is the rifle’s reaction from the action of firing the bullet. As a boat is rowed, the oars exert a force on the water and the water exerts an equal and opposite force on the boat, moving it forward.

There are forces that don't change appreciably from one instant to another, which we refer to as constant in time, and forces that don't change appreciably from one point to another, which we refer to as constant in space. The gravitational force on an object near the surface of the earth is an example of a force that is constant in space.

There are forces that depend on the configuration of a system. When a mass is attached to one end of a spring, the spring force acting on the object increases in strength whether the spring is extended or compressed.

There are forces that spread out in space such that their influence becomes less with distance. Common examples are the gravitational and electrical forces. The gravitational force between two objects falls off as the inverse square of the distance separating the objects provided the objects are of a small dimension compared to the distance between them. More complicated arrangements of attracting and repelling interactions give rise to forces that fall off with other powers of r : constant, $1/r$, $1/r^2$, $1/r^3$, ...,.

A force may remain constant in magnitude but change direction; for example the gravitational force acting on a planet undergoing circular motion about a star is directed towards the center of the circle. This type of attractive central force is called a ***centripetal force***.

A ***force law*** describes the relationship between the force and some measurable property of the objects involved. We shall see that some interactions are describable by force laws and other interactions cannot be so simply described.

3.4. Hooke's Law

In order to stretch or compress a spring from its equilibrium length, a force must be exerted on the spring. Consider an object of mass *m* that is lying on a horizontal surface. Attach one end of a spring to the object and fix the other end of the spring to a wall. Let *10* denote the equilibrium length of the spring (neither stretched or compressed). Assume, that the contact surface is smooth and hence frictionless in order to consider only the effect of the spring force. If the object is pulled to stretch the spring or pushed to compress the spring, then by Newton's Third Law the force of the spring on the object is equal and opposite to the force that the object exerts on the spring. We shall refer to the force of the spring on the object as the *spring force* and experimentally determine a relationship between that force and the amount of stretch or compress of the spring.

Choose a coordinate system with the origin located at the point of contact of the spring and the object when the spring-object system is in the equilibrium configuration. Choose the **i** unit vector to point in the direction the object moves when the spring is being stretched. Choose the coordinate function *x* to denote the position of the object with respect to the origin.

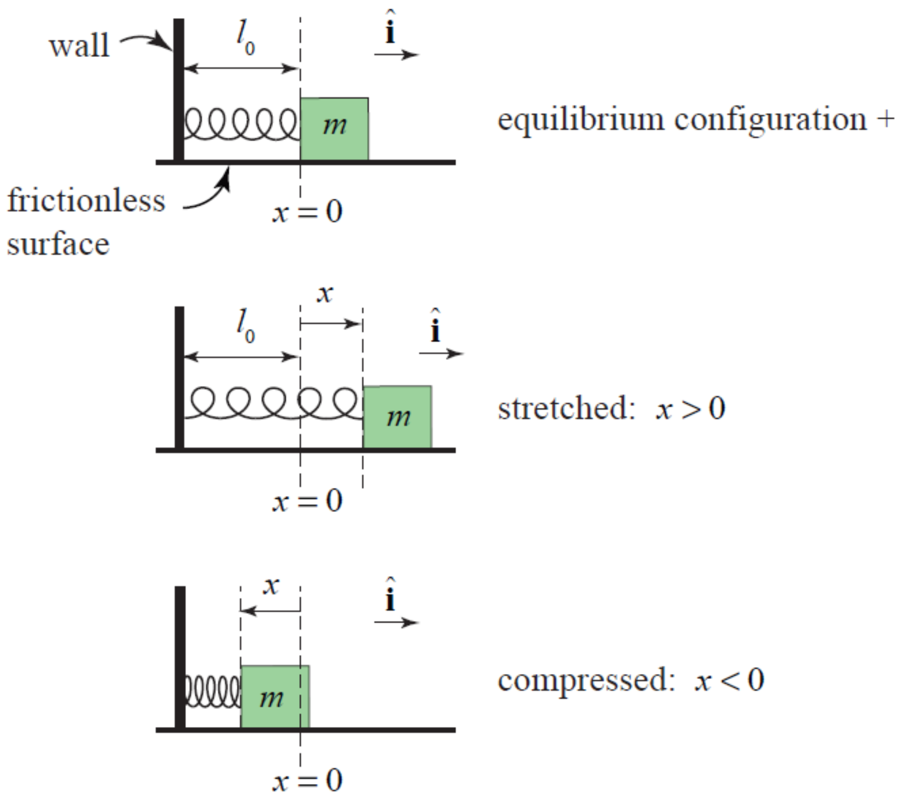


Figure . Spring attached to a wall and an object

Initially stretch thespring until the object is at position x . Then release the object and measure the acceleration of the object the instant the object is released. The magnitude of the spring force acting on the object is$|\vec{F}| = m|\vec{a}|$. Now repeat the experiment for a range of stretches (or compressions). Experiments show that for eachspring, there is a range of maximum values x_{max}>0 for stretching and minimum values x_{min}< 0 for compressing such that the magnitude of the measured force is proportional to the stretched or compressed length and is given by the formula.

$$\left|\vec{\mathbf{F}}\right| = k\left|x\right| ,$$

where the ***spring constant*** k has units N*m $^{-1}$. The free-body force diagram is shown in

Figure

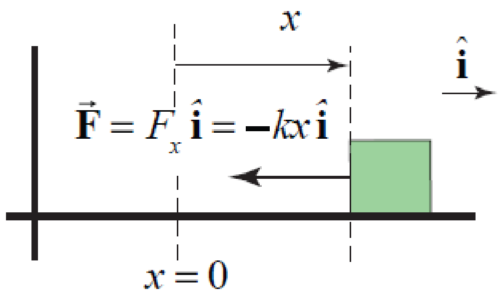


Figure Spring force acting on object

The direction of the acceleration is always towards the equilibrium position whether the spring is stretched or compressed. This type of force is called a ***restoring force***. Let F_x denote the x -component of the spring force. Then

$$F_x = -kx\,.$$

Now perform similar experiments on other springs. For a range of stretched lengths, each spring exhibits the same proportionality between force and stretched length, although the spring constant may differ for each spring.

It would be extremely impractical to experimentally determine whether this proportionality holds for all springs, and because a modest sampling of springs has confirmed the relation, we shall *infer* that all *ideal springs* will produce a restoring force, which is linearly proportional to the stretched (or compressed) length. This experimental relation regarding force and stretched (or compressed) lengths for a finite set of springs has now been *inductively* generalized into the above mathematical model for ideal springs, a force law known as a **Hooke's Law**.

This inductive step, referred to as ***Newtonian induction,*** is the critical step that makes physics a predictive science. Suppose a spring, attached to an object of mass m, is stretched by an amount Δx . Use the force law to predict the magnitude of the force between the rubber band and the object, $|\vec{F}| = k|\Delta x||$, without having to experimentally measure the acceleration. Now use Newton's Second Law to predict the magnitude of the acceleration of the object

$$\left|\vec{\mathbf{a}}\right| = \frac{\left|\vec{\mathbf{F}}\right|}{m} = \frac{k\left|\Delta x\right|}{m}.$$

Carry out the experiment, and measure the acceleration within some error bounds. If the magnitude of the predicted acceleration disagrees with the measured result, then the model for the force law needs modification. The ability to adjust, correct or even reject models based on new experimental results enables a description of forces between objects to cover larger and larger experimental domains.

Many real springs have been wound such that a force of magnitude *F* must be applied before the spring begins to stretch. The value of F_0 is referred to as the *pre-tension* of the spring. Under these circumstances, Hooke's law must be modified to account for this pretension,

$$\begin{cases} F_x = -F_0 - kx, & x > 0 \\ F_x = +F_1 - kx, & x < 0 \end{cases}.$$

Note the value of the pre-tension F_0 and *F1* mayiffer for compressing or stretching a spring.

3.5. Fundamental Laws of Nature

Force laws are mathematical models of physical processes. They arise from observation and experimentation, and they have limited ranges of applicability. Does the linear force law for the spring hold for all springs? Each spring will most likely have a different range of linear behavior. So the model for stretching springs still lacks a universal character. As such, there should be some hesitation to generalize this observation to all springs unless some property of the spring, universal to all springs, is responsible for the force law.

Perhaps springs are made up of very small components, which when pulled apart tend to contract back together. This would suggest that there is some type of force that contracts spring molecules when they are pulled apart. What holds molecules together? Can we find some fundamental property of the interaction between atoms that will suffice to explain the macroscopic force law? This search for ***fundamental forces*** is a central task of physics.

In the case of springs, this could lead into an investigation of the composition and structural properties of the atoms that compose the steel in the spring. We would investigate the geometric properties of the lattice of atoms and determine whether there is some fundamental property of the atoms that create this lattice. Then we ask how stable is this lattice under deformations. This may lead to an investigation into the electron configurations associated with each atom and how they overlap to form bonds between atoms. These particles carry charges, which obey Coulomb's Law, but also the Laws of Quantum Mechanics. So in order to arrive at a satisfactory explanation of the elastic restoring properties of the spring, we need models that describe the fundamental physics that underline Hooke's Law.

3.5.1.Universal Law of Gravitation

At points significantly far away from the surface of Earth, the gravitational force is no longer constant with respect to the distance to the center of Earth. ***Newton's Universal Law of Gravitation*** describes the gravitational force between two objects with masses, *m.* and *m*2 . This force points along the line connecting the objects, is attractive, and its magnitude is proportional to the inverse square of the distance, $r1_2$, between the two point-like objects (Figure a). The force on object 2 due to the gravitational interaction between the two objects is given by

$$\vec{\mathbf{F}}^{G}_{1,2} = -G\frac{m_1 m_2}{r^2_{1,2}}\hat{\mathbf{r}}_{1,2},$$

where $\vec{r}_{1,2} = \vec{r}_2 - \vec{r}_1$is a vector directed from object 1 to object 2, $r_{1,2} = |\vec{r}_{1,2}|$, and $\hat{\boldsymbol{r}}_{\mathbf{1},\mathbf{2}} = \frac{\vec{\boldsymbol{r}}_{\mathbf{1},\mathbf{2}}}{|\vec{\boldsymbol{r}}_{\mathbf{1},\mathbf{2}}|}$is a unit vector directed from object 1 to object 2 (Figure b). The constant of proportionality in SI units is $G = 6.67 \times 10^{-11}\ N \cdot m^2 \cdot kg^{-2}$.

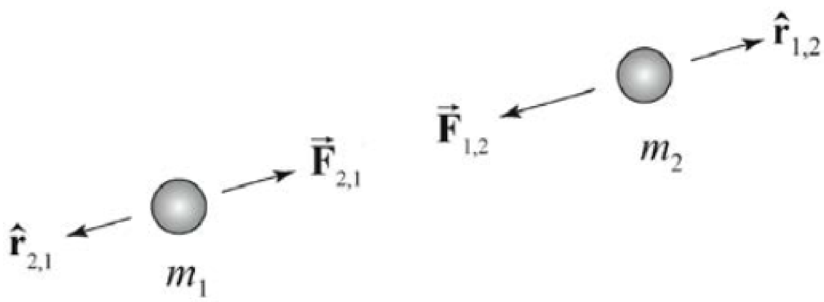


Figure a. Gravitational force between two point – like objects**.**

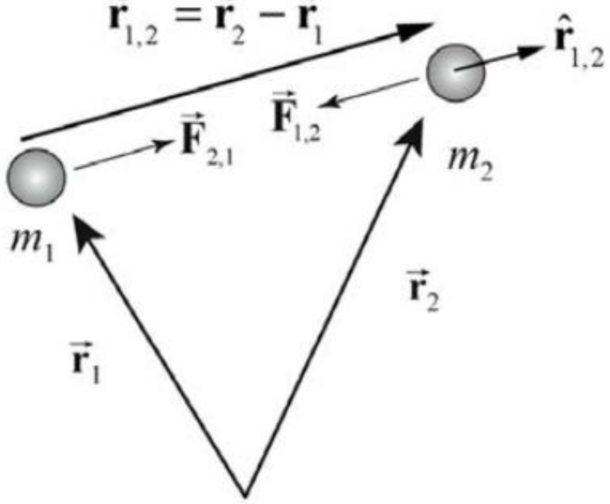


Figure b. Coordinate system for the two – body problem.

Principle of Equivalence: The Principle of Equivalence states that the mass that appears in the Universal Law of Gravity is identical to the inertial mass that is determined with respect to the standard kilogram. From this point on, the equivalence of inertial and gravitational mass will be assumed and the mass will be denoted by the symbol *m*.

3.5.2.Gravitational Force near the Surface of the Earth

Near the surface of Earth, the gravitational interaction between an object and Earth is mutually attractive and has a magnitude of

$$\left|\vec{\mathbf{F}}^{G}_{earth,object}\right| = mg$$

where g is a positive constant.

The International Committee on Weights and Measures has adopted as a standard value for the acceleration of an object freely falling in a vacuum $g = 9.80665\ \mathrm{m\cdot s^{-2}}$. The actual value of g varies as a function of elevation and latitude. If ϕ is the latitude and h the elevation in meters then the acceleration of gravity in SI units is

$$g = (9.80616 - 0.025928\cos(2\phi) + 0.000069\cos^2(2\phi) - 3.086\times 10^{-4} h)\ \mathrm{m\cdot s^{-2}}.$$

This is known as Helmert's equation. The strength of the gravitational force on the standard kilogram at 42° latitude is $9.80345\ \mathrm{N\cdot kg^{-1}}$, and the acceleration due to gravity at sea level is therefore $g = 9.80345\ \mathrm{m\cdot s^{-2}}$ for all objects. At the equator, $g = 9.78\ \mathrm{m\cdot s^{-2}}$ and at the poles $g = 9.83\ \mathrm{m\cdot s^{-2}}$. This difference is primarily due to the earth's rotation, which introduces an apparent (fictitious) repulsive force that affects the determination of g as given in Equation and also flattens the spherical shape of Earth (the distance from the center of Earth is larger at the equator than it is at the poles by about 26.5 km). Both the magnitude and the direction of the gravitational force also show variations that depend on local features to an extent that's useful in prospecting for oil, investigating the water table, navigating submerged submarines, and as well as many other practical uses. Such variations in g can be measured with a sensitive spring balance. Local variations have been much studied over the past two decades in attempts to discover a proposed "fifth force" which would fall off faster than the gravitational force that falls off as the inverse square of the distance between the objects.

3.5.3.Contact Forces

Pushing, lifting and pulling are ***contact forces*** that we experience in the everyday world. Rest your hand on a table; the atoms that form the molecules that make up the table and your hand are in contact with each other. If you press harder, the atoms are also pressed closer together. The electrons in the atoms begin to repel each other and your hand is pushed in the opposite direction by the table.

According to Newton's Third Law, the force of your hand on the table is equal in magnitude and opposite in direction to the force of the table on your hand. Clearly, if you push harder the force increases. Try it! If you push your hand straight down on the table, the table pushes back in a direction perpendicular (normal) to the surface. Slide your hand gently forward along the surface of the table. You barely feel the table pushing upward, but you do feel the friction acting as a resistive force to the motion of your hand. This force acts tangential to the surface and opposite to the motion of your hand. Push downward and forward. Try to estimate the magnitude of the force acting on your hand.

The force of the table acting on your hand, $\vec{F}^C \equiv \vec{C}$, is called the ***contact force.*** This force has both a normal component to the surface, $\vec{C}_\perp \equiv \vec{N}$, called the ***normal force***, and a tangential component to the surface, $\vec{C}_\parallel \equiv \vec{f}$, called the ***friction force*** (Figure).

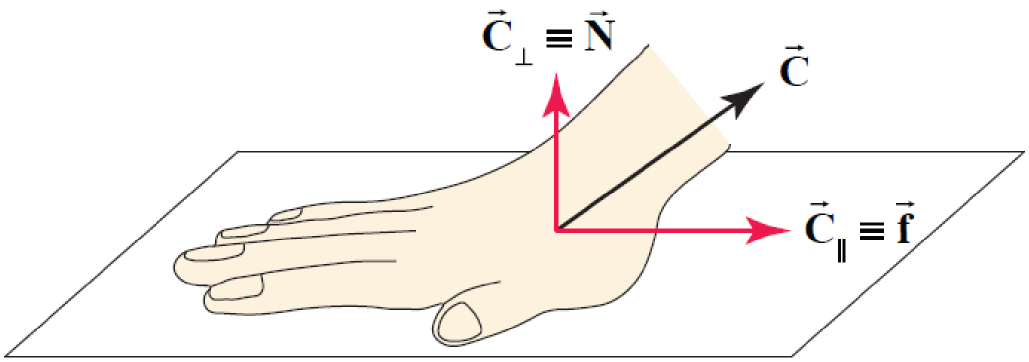


Figure. Normal and tangential components of the contact force

The contact force, written in terms of its component forces, is therefore

$$\vec{\mathbf{C}} = \vec{\mathbf{C}}_{\perp} + \vec{\mathbf{C}}_{\parallel} \equiv \vec{\mathbf{N}} + \vec{\mathbf{f}} \,.$$

Any force can be decomposed into component vectors so the normal component, **N**, and the tangential component, **f**, are not independent forces but the vector components of the contact force, perpendicular and parallel to the surface of contact. The contact force is a *distributed force* acting over all the points of contact between your hand and the surface.

For most applications we shall treat the contact force as acting at single point but precaution must be taken when the distributed nature of the contact force plays a key role in constraining the motion of a rigid body.

In next Figure, the forces acting on your hand are shown. These forces include the contact force, $\vec{\boldsymbol{C}}$, of the table acting on your hand, the force of your forearm, $\vec{\boldsymbol{F}}_{forearm}$, acting on your hand (which is drawn at an angle indicating that you are pushing down on your hand as well as forward), and the gravitational interaction, $\vec{\boldsymbol{F}}^{g}$, between the earth and your hand.

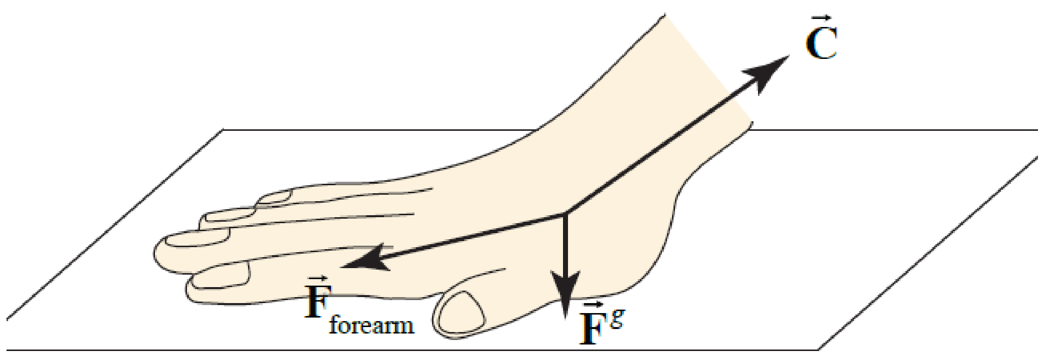


Figure. Forces on hand when moving towards the left

One point to keep in mind is that the magnitudes of the two components of the contact force depend on how hard you push or pull your hand and in what direction, a characteristic of constraint forces, in which the components are not specified by a force law but dependent on the particular motion of the hand.

3.5.4.Normal Component of the Contact Force and Weight

Hold a block in your hand such that your hand is at rest (Figure a). You can feel the "weight" of the block against your palm. But what exactly do we mean by "weight"?

Figure a Block resting in hand

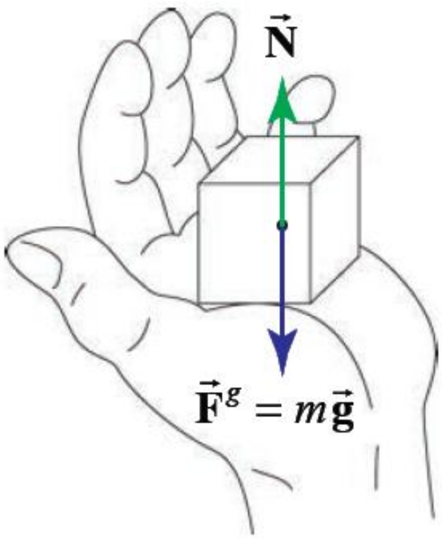


Figure b Forces on block

There are two forces acting on the block as shown in Figure b. One force is the gravitational force between the earth and the block, and is denoted by $\mathbf{F}^g = m\ \mathbf{g}$. The other force acting on the block is the contact force between your hand and the block. Because our hand is at rest, this contact force on the block points perpendicular to the surface, and hence has only a normal component, **N** . Let N denote the magnitude of the normal force. Because the object is at rest in your hand, the vertical acceleration is zero. Therefore Newton's Second Law states that

$$\vec{\mathbf{N}} + \vec{\mathbf{F}}^{g} = \vec{\mathbf{0}} .$$

Choose the positive direction to be upwards and then in terms of vertical components we have that

$$N - mg = 0 .$$

which can be solved for the magnitude of the normal force

$$N = mg .$$

When we talk about the "weight" of the block, we often are referring to the effect the block has on a scale or on the feeling we have when we hold the block. These effects are actually effects of the normal force. We say that a block "feels lighter" if there is an additional force holding the block up. For example, you can rest the block in your hand, but use your other hand to apply a force upwards on the block to make it feel lighter in your supporting hand.

The word "weight," is often used to describe the gravitational force that Earth exerts on an object. We shall always refer to this force as the *gravitational force* instead of "weight." When you jump in the air, you feel "weightless" because there is no normal force acting on you, even though Earth is still exerting a gravitational force on you; clearly, when you jump, you do not turn gravity off!

This example may also give rise to a misconception that the normal force is always equal to the mass of the object times the magnitude of the gravitational acceleration at the surface of the earth. The normal force and the gravitational force are two completely different forces. In this particular example, the normal force is equal in magnitude to the gravitational force and directed in the opposite direction because the object is at rest. The normal force and the gravitational force do not form a Third Law interaction pair of forces. In this example, our system is just the block and the normal force and gravitational force are external forces acting on the block.

3.5.5.Kinetic and Static Friction

When a block is pulled along a horizontal surface or sliding down an inclined plane there is a lateral force resisting the motion. If the block is at rest on the inclined plane, there is still a lateral force resisting the motion. This resistive force is known as dry friction, and there are two distinguishing types when surfaces are in contact with each other. The first type is when the two objects are moving relative to each other; the friction in that case is called ***kinetic friction*** or ***sliding friction***. When the two surfaces are non-moving but there is still a lateral force as in the example of the block at rest on an inclined plane, the force is called, ***static friction.***

Leonardo da Vinci was the first to record the results of measurements on kinetic friction over a twenty-year period between 1493-4 and about 1515. Based on his measurements, the force of kinetic friction, $\vec{f}^k$, between two surfaces, he identified two key properties of kinetic friction. The magnitude of kinetic friction is proportional to the normal force between the two surfaces

$$f_k = \mu_k N ,$$

where μ_k is called the ***coefficient of kinetic friction***. The second result is rather surprising in that the magnitude of the force is independent of the contact surface. Consider two blocks of the same mass, but different surface areas. The force necessary to move the blocks at a constant speed is the same. The block in Figure a has twice the contact area as the block shown in Figure b, but when the same external force is applied to either block, the blocks move at constant speed. These results of da Vinci were rediscovered by Guillaume Amontons and published in 1699. The third property that kinetic friction is independent of the speed of moving objects (for ordinary sliding speeds) was discovered by Charles Augustin Coulomb.

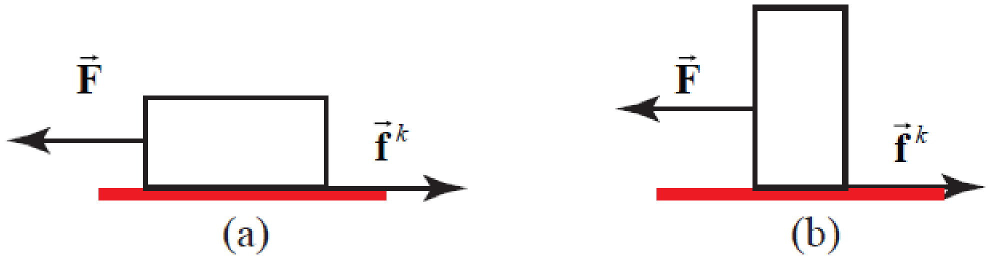


Figure (a) and (b): kinetic friction is independent of the contact area

The kinetic friction on surface 2 moving relative to surface 1 is denoted by, $\vec{f}^{\,k}_{1,2}$. The direction of the force is always opposed to the relative direction of motion of surface 2 relative to the surface 1. When one surface is at rest relative to our choice of reference frame we will denote the friction force on the moving object by $\vec{f}^{\,k}$

The second type of dry friction, static friction occurs when two surfaces are static relative to each other. Because the static friction force between two surfaces forms a third law interaction pair, will use the notation $\vec{f}^{\,s}_{1,2}$ to denote the static friction force on surface 2 due to the interaction between surfaces 1 and 2. Push your hand forward along a surface; as you increase your pushing force, the frictional force feels stronger and stronger. Try this! Your hand will at first stick until you push hard enough, then your hand slides forward. The magnitude of the static frictional force, f_s , depends on how hard you push.

If you rest your hand on a table without pushing horizontally, the static friction is zero. As you increase your push, the static friction increases until you push hard enough that your hand slips and starts to slide along the surface. Thus the magnitude of static friction can vary from zero to some maximum value, $(f_s)_{max}$, when the pushed object begins to slip

$$0 \leq f_s \leq (f_s)_{max} .$$

Is there a mathematical model for the magnitude of the maximum value of static friction between two surfaces? Through experimentation, we find that this magnitude is, like kinetic friction, proportional to the magnitude of the normal force

$$(f_s)_{max} = \mu_s N .$$

Here the constant of proportionality is μ_s , the ***coefficient of static friction.*** This constant is slightly greater than the constant μ_k associated with kinetic friction,$\mu_{s.} > \mu_k$ This small difference accounts for the slipping and catching of chalk on a blackboard, fingernails on glass, or a violin bow on a string.

The direction of static friction on an object is always opposed to the direction of the applied force (as long as the two surfaces are not accelerating). In Figure a, an external force, $\vec{F}$, is applied the left and the static friction, $\vec{f}^s$, is shown pointing to the right opposing the external force. In Figure b, the external force, **F** , is directed to the right and the static friction, $\vec{f}^s$, is now pointing to the left.

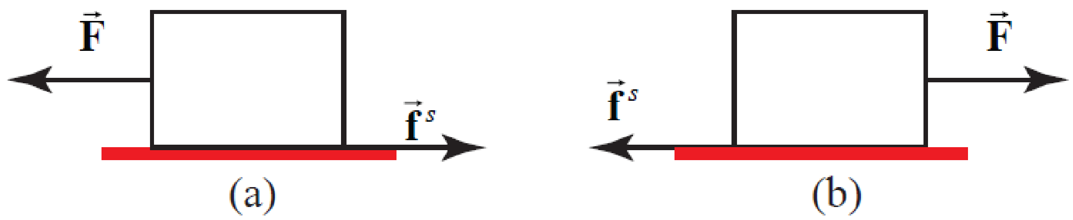


Figure (a) and (b): External forces and the direction of static friction.

Although the force law for the maximum magnitude of static friction resembles the force law for sliding friction, there are important differences:

1. The direction and magnitude of static friction on an object always depends on the direction and magnitude of the applied forces acting on the object, where the magnitude of kinetic friction for a sliding object is fixed.

2. The magnitude of static friction has a maximum possible value. If the magnitude of the applied force along the direction of the contact surface exceeds the magnitude of the maximum value of static friction, then the object will start to slip (and be subject to kinetic friction.) We call this the ***just slipping*** condition.

3.6. Rotational Dynamics

3.6.1.Center of Mass

Consider two point-like particles with masses *m1* and *m2*. Choose a coordinate system with a choice of origin such that body 1 has position **F**1 and body 2 has position $\mathbf{r}_2$ (Figure).

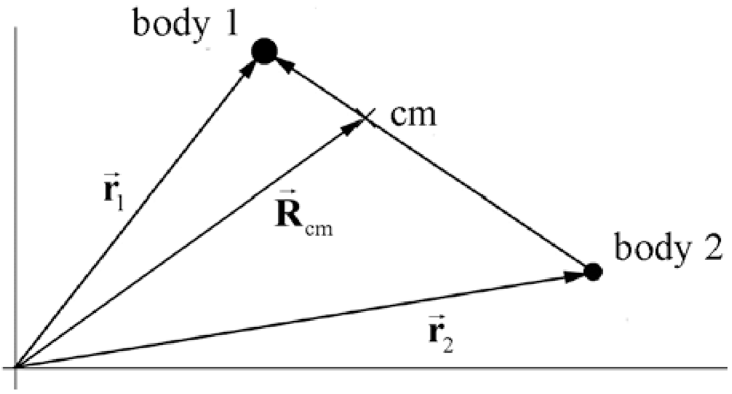


Figure Center of mass coordinate system

The center of mass vector, $\mathbf{R}_{cm}$, of the two-body system is defined as

$$\vec{\mathbf{R}}_{cm} = \frac{m_1 \vec{\mathbf{r}}_1 + m_2 \vec{\mathbf{r}}_2}{m_1 + m_2}.$$

We shall now extend the concept of the center of mass to more general systems. Suppose we have a system of N particles labeled by the index $i = 1, 2,3, \cdots, N$. Choose a coordinate system and denote the position of the i^{th} particle as $\vec{r}_i$. The mass of the system is given by the sum

$$m_{sys} = \sum_{i=1}^{i=N} m_i$$

and the position of the center of mass of the system of particles is given by

$$\vec{\mathbf{R}}_{cm} = \frac{1}{m_{sys}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i .$$

For a continuous rigid body, each point-like particle has mass *dm* and is located at the

$$\vec{\mathbf{R}}_{\text{cm}} = \frac{\int_{\text{body}} dm\,\vec{\mathbf{r}}'}{\int_{\text{body}} dm}.$$

3.6.2.Moment of Inertia

$$I_{cm} = \int_{body} dm\, r_{dm}^2 \,.$$

is called the ***moment of inertia*** *of the rigid body about a fixed axis passing through the center of mass, and is a physical property of the body. The SI units for moment of inertia are [kg·m*^{2}*].*

Moment of Inertia of a Rod of Uniform Mass Density

Consider a thin uniform rod of length L and mass m . In this problem, we will calculate the moment of inertia about an axis perpendicular to the rod that passes through the center of mass of the rod. A sketch of the rod, volume element, and axis is shown in Figure . Choose Cartesian coordinates, with the origin at the center of mass of the rod, which is midway between the endpoints since the rod is uniform. Choose the x -axis to lie along the length of the rod, with the positive x -direction to the right, as in the figure.

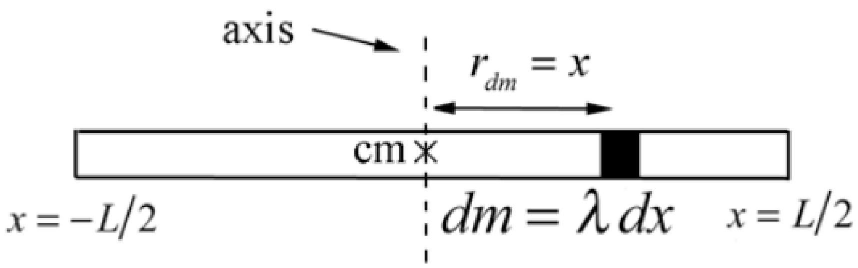


Figure. Moment of inertia of a uniform rod about center of mass.

Identify an infinitesimal mass element $dm = \lambda dx$, located at a displacement x from the center of the rod, where the mass per unit length $\lambda = m/L$ is a constant, as we have assumed the rod to be uniform. When the rod rotates about an axis perpendicular to the rod that passes through the center of mass of the rod, the element traces out a circle of radius $r_{dm} = x$. We add together the contributions from each infinitesimal element as we go from $x = -L/2$ to $x = L/2$. The integral is then

$$I_{\text{cm}} = \int_{\text{body}} r_{dm}^2 \, dm = \lambda \int_{-L/2}^{L/2} (x^2)\, dx = \lambda \frac{x^3}{3}\bigg|_{-L/2}^{L/2}$$

$$= \frac{m}{L}\frac{(L/2)^3}{3} - \frac{m}{L}\frac{(-L/2)^3}{3} = \frac{1}{12} m L^2.$$

By using a constant mass per unit length along the rod, we need not consider variations in the mass density in any direction other than the x - axis. We also assume that the width is the rod is negligible. (Technically we should treat the rod as a cylinder or a rectangle in the x-y plane if the axis is along the z - axis. The calculation of the moment of inertia in these cases would be more complicated.)

3.6.3.Parallel Axis Theorem (Huygens-Steiner theorem)

Consider a rigid body of mass m undergoing fixed-axis rotation. Consider two parallel axes. The first axis passes through the center of mass of the body, and the moment of inertia about this first axis is I_{cm} . The second axis passes through some other point S in the body. Let $d_{S,\text{cm}}$ denote the perpendicular distance between the two parallel axes (Figure).

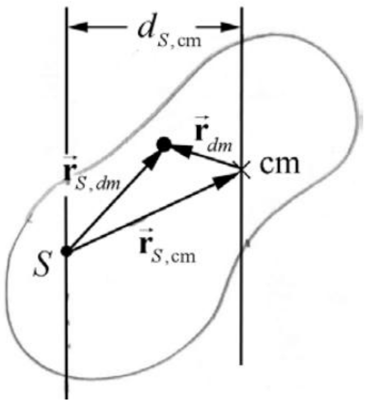


Figure Geometry of the parallel axis theorem

Then the moment of inertia I_S about an axis passing through a point S is related to I_{cm} by

$$I_S = I_{\text{cm}} + m\, d_{S,\text{cm}}^2 .$$

3.6.4.Definition of Torque about a Point

In order to understand the dynamics of a rotating rigid body we will introduce a new quantity, the torque. Let a force $\vec{F}_P$ with magnitude $F = |\vec{F}_P|$ act at a point P . Let $\vec{r}_{S,P}$ point S to a point P , with magnitude $r = |\vec{r}_{S,P}|$. The angle between the vectors $\vec{r}_{S,P}$ and $\vec{F}_P$ is θ with $[0 \le \theta \le \pi]$ (Figure).

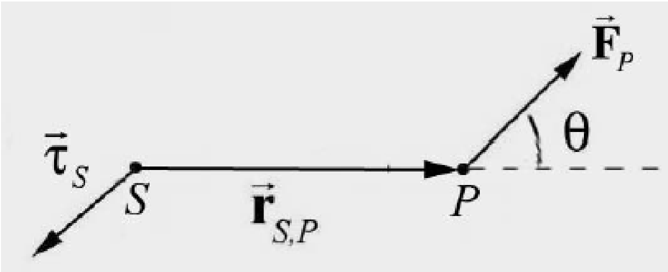
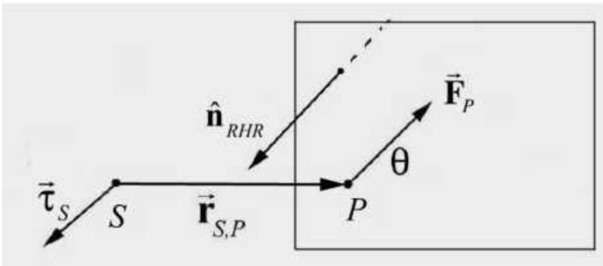


Figure. Torque about a point *S* due to a force acting at a point *P*

The torque about a point S due to force $\vec{F}p$ *acting at P, is defined by*



$$\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P .$$

The magnitude of the torque about a point S due to force $\vec{F}p$ *acting at P, is given by*

$$\tau_S \equiv |\vec{\tau}_S| = r\,F\sin\theta .$$

The SI units for torque are [N·m]. The direction of the torque is perpendicular to the plane formed by the vectors $\vec{r}_{S\,P}$ and $\vec{F}_P$ (for $[0 < \theta < \pi]$), and by definition points in the direction of the unit normal vector to the plane $\hat{n}_{RHR}$ as shown in Figure.

Figure shows the two different ways of defining height and base for a parallelogram defined by the vectors $\vec{r}_{S\,P}$ and $\vec{F}_P$

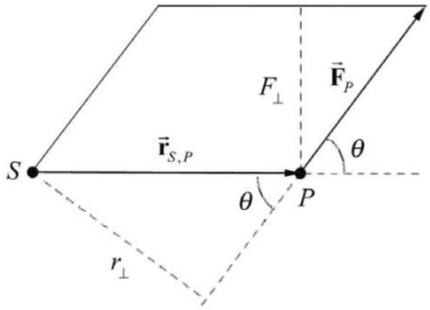


Figure. Area of the torque parallelogram.

Let $r_\perp = r\sin\theta$and let$F_\perp = F\sin\theta$ be the component of the force $\vec{F}_p$ that perpendicular to the line passing from the point S to P . (Recall the angle θ has a range of values $0 \leq\theta \leq \pi$ so both $r_\perp \geq 0$ and $F_\perp \geq 0$.) Then the area of the parallelogram defined by $\vec{r}_{S,P}$ and $\vec{F}_p$ is given by

$$\text{Area} = \tau_S = r_\perp F = r\,F_\perp = r\,F\sin\theta\ .$$

We can interpret the quantity $r_\perp$ as follows.

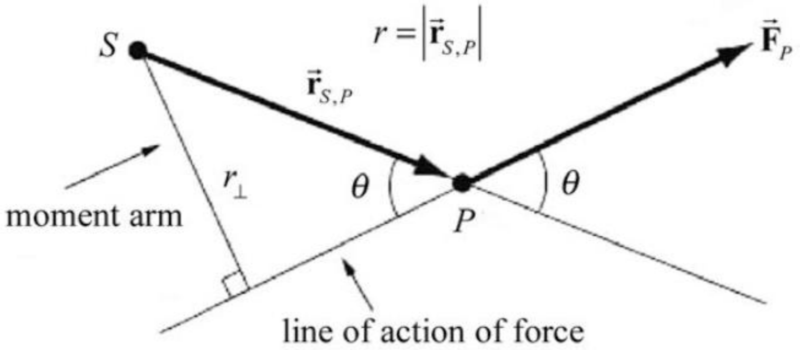


Figure The moment arm about the point *S* and line of action of force passing through the point *P*

We begin by drawing the ***line of action of the force*** $\mathbf{F}_p$. This is a straight line passing through P, parallel to the direction of the force $\mathbf{F}_p$. Draw a perpendicular to this line of action that passes through the point S (Figure). The length of this perpendicular, $r_\perp = r\sin\theta$ is called ***the moment arm about the point S of the force*** $\mathbf{F}_p$.

You should keep in mind three important properties of torque:

1. The torque is zero if the vectors $\mathbf{r}_{S,\,P}$ and $\mathbf{F}p$ are parallel ($\theta = 0$) or anti-parallel ($\theta = \pi$).

2. Torque is a vector whose direction and magnitude depend on the choice of a point S about which the torque is calculated.

3. The direction of torque is perpendicular to the plane formed by the two vectors, $\mathbf{F}_p$ and $r = |\vec{r}_{S,P}|$. (the vector from the point S to a point P).

3.6.5.Angular Momentum

When we consider a system of objects, we have shown that the external force, acting at the center of mass of the system, is equal to the time derivative of the total momentum of the system,

$$\vec{\mathbf{F}}^{\text{ext}} = \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt} .$$

We now introduce the rotational analog of Equation.We will first introduce the concept of angular momentum for a point-like particle of mass *m* with linear momentum $\vec{p}$ about a point *S* , defined by the equation

$$\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_S \times \vec{\mathbf{p}} ,$$

where $\mathbf{r}_S$ is the vector from the point *S* to the particle. We will show in this lecture that the torque about the point *S* acting on the particle is equal to the rate of change of the angular momentum about the point *S* of the particle,

$$\vec{\boldsymbol{\tau}}_S = \frac{d\vec{\mathbf{L}}_S}{dt} .$$

This equation generalizes to any body undergoing rotation

We shall concern ourselves first with the special case of rigid body undergoing fixed axis rotation about the z-axis with angular velocity$\vec{\omega} = \omega_z \hat{k}$. We divide up the rigid body into *N* elements labeled by the index *i*, *i* = 1,2,...*N*, the i^{th} element having mass m_i, and position vector $\vec{r}_{S,P}$. The rigid body has a moment of inertia I_S about some point *S* on the fixed axis, (often taken to be the *z*-axis, but not always) which rotates with angular velocity $\vec{\omega}$ to about this axis. The angular momentum is then the vector sum of the individual angular momenta

$$\vec{\mathbf{L}}_S = \sum_{i=1}^{i=N} \vec{\mathbf{L}}_{S,i} = \sum_{i=1}^{i=N} \vec{\mathbf{r}}_{S,i} \times \vec{\mathbf{p}}_i$$

When the rotation axis is the *z*-axis the *z*-component of the angular momentum, $L_{S\,z}$, about the point *S* is then given by

$$L_{S,z} = I_S \omega_z .$$

We shall show that the z-component of the torque about the point S, $\tau_{S\,z}$, is then the time derivative of the z-component of angular momentum about the point S,

$$\tau_{S,z} = \frac{dL_{S,z}}{dt} = I_S \frac{d\omega_z}{dt} = I_S \alpha_z .$$

3.6.6.Angular Momentum for a Point Particle

Consider a point-like particle of mass *m* moving with a velocity **v** (Figure) with momentum **p** = *m***v** .

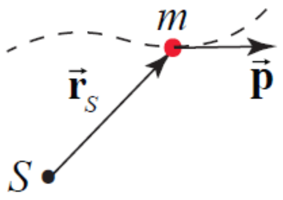


Figure A point-like particle and its angular momentum about *S*

Consider a point *S* located anywhere in space. Let $\vec{r}_S$ denote the vector from the point *S* to the location of the object.

Define the angular momentum $\vec{L}_S$ *about the point S of a point-like particle as the vector product of the vector from the point S to the location of the object with the momentum of the particle*

$$\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_S \times \vec{\mathbf{p}} .$$

The derived SI units for angular momentum are [kg·m^2·s^{-1}] = [N·m·s]. There is no special name for this set of units.

Because angular momentum is defined as a vector, we begin by studying its magnitude and direction. The magnitude of the angular momentum about *S* is given by

$$\left|\vec{\mathbf{L}}_S\right| = \left|\vec{\mathbf{r}}_S\right|\left|\vec{\mathbf{p}}\right| \sin\theta ,$$

where θ is the angle between the vectors and **p** , and lies within the range [0 <θ < π] Analogous to the magnitude of torque, there are two ways to determine the magnitude of the angular momentum about *S*.

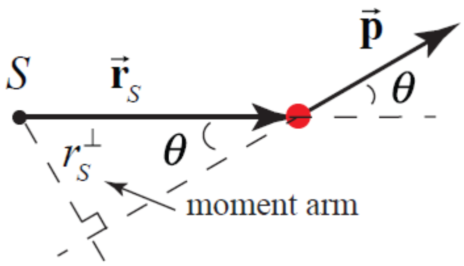


Figure (a) Moment arm

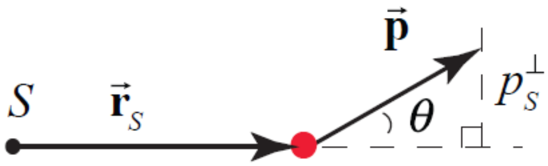


Figure (b) Perpendicular component of momentum.

Define the *moment arm*, $r_S^{\perp}$, (Figure (a)), as the perpendicular distance from the point *S* to the line defined by the direction of the momentum. Then

$$r_S^{\perp} = |\vec{\mathbf{r}}_S| \sin\theta$$

Hence the magnitude of the angular momentum is the product of the moment arm with the magnitude of the momentum,

$$|\vec{\mathbf{L}}_S| = r_S^{\perp} |\vec{\mathbf{p}}| .$$

Denote the magnitude of the component of the momentum perpendicular to the line defined by the direction of the vector $\vec{r}_S$ .From the geometry shown in Figure (b).

$$p_S^{\perp} = |\vec{\mathbf{p}}| \sin\theta .$$

Thus the magnitude of the angular momentum is the product of the distance from *S* to the particle with $r_S^{\perp}$,

$$|\vec{\mathbf{L}}_S| = |\vec{\mathbf{r}}_S| \, p_S^{\perp} .$$

Right-Hand-Rule for the Direction of the Angular Momentum

We shall define the direction of the angular momentum about the point *S* by a right hand rule. Draw the vectors $\vec{r}_s$ and $\vec{\boldsymbol{p}}$ so their tails are touching. Then draw an arc starting from the vector $\vec{r}_S$. and finishing on the vector $\vec{\boldsymbol{p}}$. (There are two such arcs; choose the shorter one.) This arc is either in the clockwise or counterclockwise direction. Curl the fingers of your right hand in the same direction as the arc. Your right thumb points in the direction of the angular momentum.

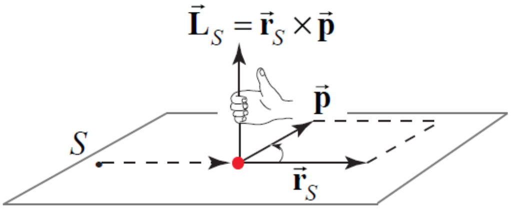


Figure The right hand rule for determining the direction of angular momentum about S.

Remember that, as in all vector products, the direction of the angular momentum about S is perpendicular to the plane formed by $\vec{r}_S$ and $\vec{\boldsymbol{p}}$**.**

4. Energy and Work

4.1. Kinetic Energy

The first form of energy that we will study is an energy associated with the coherent motion of molecules that constitute a body of mass *m*; this energy is called the *kinetic energy* (from the Greek word *kinetikos* which translates as *moving)* Let us consider a car moving along a straight road (along which we will place the x -axis). For an observer at rest with respect to the ground, the car has velocity$\vec{v} = v_x \vec{i}$. The speed of the car is the magnitude of the velocity, $v \equiv v_x$.

The ***kinetic energy*** *K of a non-rotating body of mass m moving with speed v is defined to be the positive scalar quantity*

$$K \equiv \frac{1}{2} m v^2$$

The kinetic energy is proportional to the square of the speed. The SI units for kinetic energy are [kg • m^2• s^{-2}] . This combination of units is defined to be a joule and is denoted by [J] , thus 1J = 1 kg • m^2• s^{--2} . (The SI unit of energy is named for James Prescott Joule.) The above definition of kinetic energy does not refer to any direction of motion, just the speed of the body.

Let's consider a case in which our car changes velocity. For our initial state, the car moves with an initial velocity $\vec{v}_i = v_{i,x} \vec{i}$ along the x -axis. For the final state (at some later time), the car has changed its velocity and now moves with a final velocity $\vec{v}_f = v_{f,x} \vec{i}$. Therefore the change in the kinetic energy is

$$\Delta K = \frac{1}{2} m v_f^{\,2} - \frac{1}{2} m v_i^{\,2}.$$

Constant Accelerated Motion

Let's consider a constant accelerated motion of a *rigid body* in one dimension in which we treat the rigid body as a point mass. Suppose at t = 0 the body has an initial x - component of the velocity given by v_x. . If the acceleration is in the direction of the displacement of the body then the body will increase its speed. If the acceleration is opposite the direction of the displacement then the acceleration will decrease the body’s speed. The displacement of the body is given by

$$\Delta x = v_{x,i}\, t + \frac{1}{2} a_x t^2 .$$

The product of acceleration and the displacement is

$$a_x \Delta x = a_x (v_{x,i}\; t + \frac{1}{2} a_x t^2) .$$

The acceleration is given by

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_{x,f} - v_{x,i})}{t} .$$

Therefore

$$a_x \Delta x = \frac{(v_{x,f} - v_{x,i})}{t}\left(v_{x,i}\; t + \frac{1}{2}\frac{(v_{x,f} - v_{x,i})}{t} t^2 \right).$$

$$a_x \Delta x = (v_{x,f} - v_{x,i})(v_{x,i}) + \frac{1}{2}(v_{x,f} - v_{x,i})(v_{x,f} - v_{x,i}) = \frac{1}{2} v_{x,f}{}^2 - \frac{1}{2} v_{x,i}^2 .$$

If we multiply each side of Equation by the mass m of the object this kinematical result takes on an interesting interpretation for the motion of the object. We have

$$m\, a_x \Delta x = \frac{1}{2} m v_{x,f}^2 - m \frac{1}{2} v_{x,i}^2 = K_f - K_i .$$

Recall that for one-dimensional motion, Newton’s Second Law is $F_x = ma_x$, for the motion considered here, Equation becomes

$$F_x\, \Delta x = K_f - K_i .$$

Non-constant Accelerated Motion

If the acceleration is not constant, then we can divide the displacement into N intervals indexed by j=1 to N . It will be convenient to denote the displacement intervals by Δx_j , the corresponding time intervals by Δt_j and the x -components of the velocities at the beginning and end of each interval as $v_{x,j-1}$ and $v_{x,j}$ Note that the x -component of the velocity at the beginning and end of the first interval j=1 is then $v_{x,1} = v_{x,i}$. and the velocity at the end of the last interval, $j = N$ is $v_{x,N} = v_{x,j}$. Consider the sum of the products of the average acceleration $\left(a_{x,j}\right)_{ave}$ and displacement Δx_j in each interval,

$$\sum_{j=1}^{j=N}(a_{x,j})_{\text{ave}}\,\Delta x_j\,.$$

The average acceleration over each interval is equal to

$$(a_{x,j})_{\text{ave}} = \frac{\Delta v_{x,j}}{\Delta t_j} = \frac{(v_{x,j+1} - v_{x,j})}{\Delta t_j}\,,$$

and so the contribution in each integral can be calculated as above and we have that

$$(a_{x,j})_{\text{ave}}\,\Delta x_j = \frac{1}{2}v_{x,j}{}^2 - \frac{1}{2}v_{x,j-1}{}^2\,.$$

When we sum over all the terms only the last and first terms survive, all the other terms cancel in pairs, and we have that

$$\sum_{j=1}^{j=N}(a_{x,j})_{\text{ave}}\,\Delta x_j = \frac{1}{2}v_{x,f}^2 - \frac{1}{2}v_{x,i}^2\,.$$

In the limit as $N\to\infty$ *and* $\Delta x_j \to 0$ for all j (both conditions must be met!), the limit of the sum is the definition of the definite integral of the acceleration with respect to the position,

$$\lim_{\substack{N\to\infty \\ \Delta x_j\to 0}}\sum_{j=1}^{j=N}(a_{x,j})_{\text{ave}}\,\Delta x_j \equiv \int_{x=x_i}^{x=x_f} a_x(x)\,dx\,.$$

Therefore In the limit as $N\to\infty$ *and* $\Delta x_j \to 0$ for all j , with $v_{x,N} \to v_{x,f}$ Eq. becomes

$$\int_{x=x_i}^{x=x_f} a_x(x)\,dx = \frac{1}{2}(v_{x,f}^2 - v_{x,i}^2)$$

This integral result is consequence of the definition that $a_x = dv_x\ I\ dt$. The integral is an integral with respect to space, while our previous integral

$$\int_{t=t_i}^{t=t_f} a_x(t)\,dt = v_{x,f} - v_{x,i}\,.$$

requires integrating acceleration with respect to time. Multiplying both sides of Eq. by the mass m yields

$$\int_{x=x_i}^{x=x_f} ma_x(x)\,dx = \frac{1}{2}m(v_{x,f}^2 - v_{x,i}^2) = K_f - K_i\,.$$

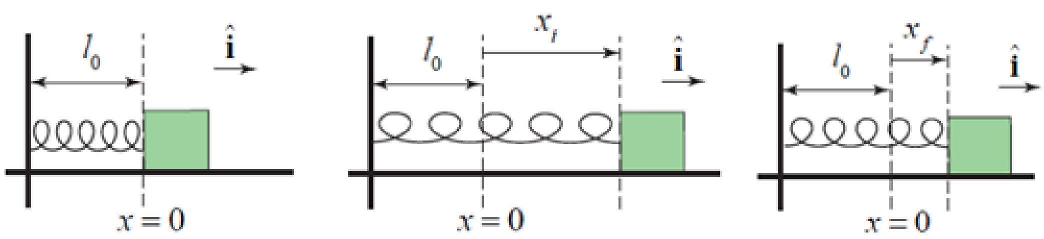
When we introduce Newton's Second Law in the form $F_x = ma_x$, then Eq. becomes

$$\int_{x=x_i}^{x=x_f} F_x(x)\,dx = K_f - K_i.$$

The integral of the x -component of the force with respect to displacement in Eq. applies to the motion of a point-like object. For extended bodies, Eq. applies to the center of mass motion because the external force on a rigid body causes the center of mass to accelerate.

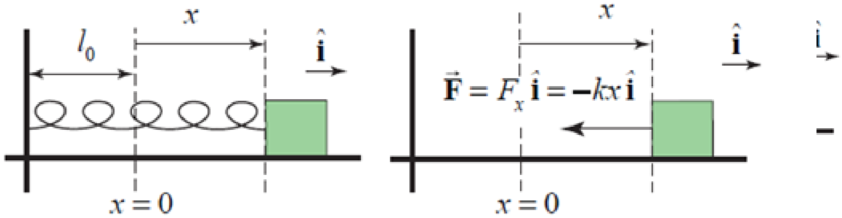
Work done by the Spring Force

Connect one end of an unstretched spring of length l_0 with spring constant k to an object resting on a smooth frictionless table and fix the other end of the spring to a wall. Choose an origin as shown in the figure. Stretch the spring by an amount x_i and release the object. How much work does the spring do on the object when the spring is stretched by an amount x_f?



Equilibrium, initial and final states for a spring

Solution: We first begin by choosing a coordinate system with our origin located at the position of the object when the spring is unstretched (or uncompressed). We choose the $\hat{\mathbf{i}}$ unit vector to point in the direction the object moves when the spring is being stretched. We choose the coordinate function x to denote the position of the object with respect to the origin. We show the coordinate function and free-body force diagram in the figure below.

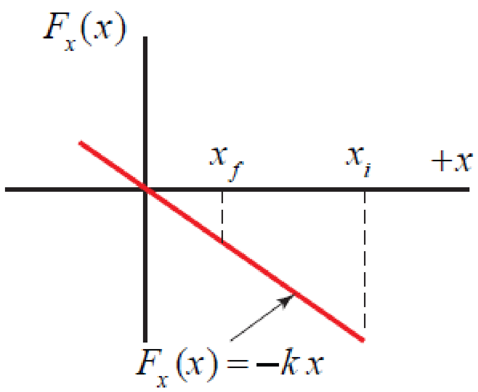


Spring force

The spring force on the object is given by (Figure 13.6a)

$$\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} = -k\,x\,\hat{\mathbf{i}}$$

In Figure show the graph of the x -component of the spring force, $F_x\,(x)$, as a function of x .



Plot of spring force $F_x(x)$ vs. displacement x

The work done is just the area under the curve for the interval x_i to x_f,

$$W = \int_{x'=x_i}^{x'=x_f} F_x(x')\,dx' = \int_{x'=x_i}^{x'=x_f} -kx'\,dx' = -\frac{1}{2}k(x_f^2 - x_i^2)$$

This result is independent of the sign of x_i and x_f because both quantities appear as squares. If the spring is less stretched or compressed in the final state than in the initial state, then the absolute value, $|x_f| < |x_i|$, and the work done by the spring force is positive. The spring force does positive work on the body when the spring goes from a state of “greater tension” to a state of “lesser tension.”

4.2. Work-Kinetic Energy Theorem

There is a direct connection between the work done on a point-like object and the change in kinetic energy the point-like object undergoes. If the work done on the object is nonzero, this implies that an unbalanced force has acted on the object, and the object will have undergone acceleration. For an object undergoing one-dimensional motion the left hand side of Equation (13.3.16) is the work done on the object by the component of the sum of the forces in the direction of displacement,

$$W = \int_{x=x_i}^{x=x_f} F_x \, dx = \frac{1}{2} m v_f^{\,2} - \frac{1}{2} m v_i^{\,2} = K_f - K_i = \Delta K$$

When the work done on an object is positive, the object will increase its speed, and negative work done on an object causes a decrease in speed. When the work done is zero, the object will maintain a constant speed. In fact, the work-energy relationship is quite precise; the work done by the applied force on an object is identically equal to the change in kinetic energy of the object.

Power Applied by a Constant Force

Suppose that an applied force F^a acts on a body during a time interval Δt , and the displacement of the point of application of the force is in the x -direction by an amount

Δx . The work done, ΔW^a , during this interval is

$$\Delta W^a = F_x^a \, \Delta x .$$

where F_x^a is the x -component of the applied force. (Equation (13.7.1) is the same as Equation (13.4.2).)

The ***average power*** of an applied force is defined to be the rate at which work is done,

$$P_{ave}^a = \frac{\Delta W^a}{\Delta t} = \frac{F_x^a \, \Delta x}{\Delta t} = F_x^a v_{\text{ave},x} .$$

The average power delivered to the body is equal to the component of the force in the direction of motion times the component of the average velocity of the body. Power is a scalar quantity and can be positive, zero, or negative depending on the sign of work. The nits of power are called watts [W] and $[1 \text{ W}] = [1 \text{ J} \cdot \text{s}^{-1}]$

The ***instantaneous power*** at time t is defined to be the limit of the average power as the time interval $[t, t + \Delta t]$ approaches zero,

$$P^a = \lim_{\Delta t \to 0} \frac{\Delta W^a}{\Delta t} = \lim_{\Delta t \to 0} \frac{F_x^a \Delta x}{\Delta t} = F_x^a \left(\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right) = F_x^a v_x .$$

The instantaneous power of a constant applied force is the product of the component of the force in the direction of motion and the instantaneous velocity of the moving object.

5. Electrostatics.

5.1. Ostrogradsky-Gauss theorem. Voltage and potential.

5.1.1.The law of conservation of electric charge. The electric field. The strength of an electric field

Electrostatics is a branch of physics that deals with the interactions and properties of electric charges stationary in the coordinate system in which these charges are studied.

There are two types of electric charges in nature - ***positive and negative. It was*** agreed that the charge that appears, for example, on glass rubbed with silk is positive, and the charge on amber rubbed with fur is negative. Similarly charged bodies repel each other, and differently charged bodies attract each other.

When bodies are electrified by friction, ***both bodies*** are always simultaneously electrified, with one of them receiving a positive charge and the other a negative charge.

The positive charge of the first body is always exactly equal to the negative charge of the second body, if both bodies were not charged before electrification. Numerous experiments have established the *law of conservation of electric charges*:

In an electrically isolated system, the total algebraic sum of electric charges remains unchanged. Charges can only be transferred from one body of the system to another, or shifted within a given body.

Electric charges can disappear and reappear, but two electric charges of opposite signs always disappear or appear.

In 1909, the American scientist R. Miliken established the multiplicity of an electric charge to a certain elementary charge *e:*

$$q = \pm n \cdot e,$$

where n = 1,2,3...

This elementary charge was found to have a value of $1.6 \cdot 10^{-19}$ coulomb.

The unit of electric charge in the SI system is the charge that passes in one second through the cross-section of a conductor with a constant current of 1 ampere.

In 1785, the French scientist S. Coulomb experimentally established the basic law of interaction of fixed point electric charges using torsion scales.

A point charge is a charge concentrated on a body whose linear dimensions are small compared to the distance to other charged bodies with which it interacts.

Coulomb's law:

the strength of electrostatic interaction between two point electric charges in a vacuum is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

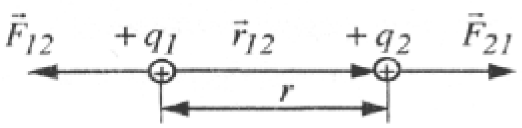
$$F = k\frac{|q_1||q_2|}{r^2},$$

where *k* is the proportionality coefficient *(k>0).*

The forces acting on the charges are ***central***, *i.e.* they are directed along the line connecting the charges. For charges of the same name, the force *F>0* corresponds to the case of mutual repulsion of charges of the same name, and the force *F<0 corresponds to the* case of mutual attraction of charges of different names.

Coulomb's law can be written in vector form. If r12 is a radius vector that connects the charge *q1* with the charge *q2* (Fig.) and$|\vec{r}_{12}| = r$, then

$$\vec{F}_{12} = -k\frac{q_1 q_2}{r^3}\vec{r}_{12}, \quad \vec{F}_{21} = k\frac{q_1 q_2}{r^3}\vec{r}_{12}.$$



In the SI system, for charges, the coefficient *k* in the Coulomb's law formula is taken to be equal to

$$k = \frac{1}{4\pi\varepsilon_o\varepsilon},$$

where.

$$\varepsilon_0 = 8.854\ 187\ 8188(14) \times 10^{-12}\ F \cdot m^{-1}$$

ε is the relative permittivity of the medium (for vacuum $\varepsilon = 1$)

Multiplier 4π in the expressi on k equal reflects the spherical symmetry of the electrostatic field of a point charge, since the value of 4π is numerically equal to the total solid angle in steradian.

An electric field is a specific type of matter that exists around electric charges and through which electrical interactions are transmitted. It manifests itself in the fact that an electric charge placed in it is subject to a force. Experiments show that this force, ceteris paribus, is proportional to the magnitude of the charge. Therefore, this force cannot be a characteristic of the field itself.

But the value is equal to the ratio of the field characteristics.

$$\frac{F}{q_0} = const$$

can serve as a power- is the test charge. A test charge is a unit positive charge that does not The vector value is the eigen field. q_0 - a test charge is a single positive charge that does not creates its own fields.

is called the *electric field strength.*

The vector quantity

$$\vec{E} = \frac{\vec{F}}{q_0}$$

The strength of an electric field is numerically equal to the force acting on a single positive test charge at a given point in the field.

The direction of the stress vector *E* is the direction of the force with which the field acts on a test positive charge placed a t a specific point in the field (Fig.). In the SI system, the unit of intensity is electric field of 1 *N/m* is the intensity of such field, which at a point 1 a charge of *1 C* acts with a force of *1 N.*

a

Electric fields are represented by ***lines of intensity,*** which are drawn so that the tangents to these lines at each point coincide with the directions of the vector *E* .Tension lines have a beginning and an end or go on forever, they are directed from positive to negative charge, i.e. they come out of a positive charge and enter a negative charge. The lines of intensity never intersect. These lines are drawn with such a density that the number of lines penetrating a unit area perpendicular to the intensity vector is numerically equal to the intensity of the electric field at the location of the area.

A field at all points of which the magnitude and direction of the stress vector are unchanged is called homogeneous.

A homogeneous field is represented by parallel lines of stress that have the same density.

If the field is created by a system of *N* stationary charges, the resulting force acting on the test charge by the system of charges is equal to the vector sum of the forces with which the individual charges act on the test charge.

It follows:

F = Σ *F i* = *F* 1 + *F* 2 + ... + *F n*

i=1 .

The field strength of a system of point charges is equal to the vector sum of the field strengths that would be produced by each of the charges in the system individually:

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n$$

.

This statement is called *the principle of independent action of* electric fields, or the *principle of superposition of fields.*

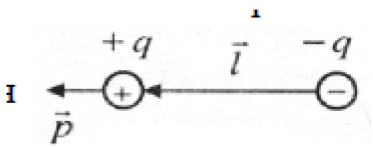
$$\vec{E}=\frac{\vec{F}}{q_{np}}=\frac{\sum_{i=1}^{n}\vec{F}_i}{q_{np}}=\frac{\vec{F}_1}{q_{np}}+\frac{\vec{F}_2}{q_{np}}+\ldots+\frac{\vec{F}_n}{q_{np}},\quad \vec{E}=\vec{E}_1+\vec{E}_2+\vec{E}_3+\ldots+\vec{E}_N.$$

Taking into account Coulomb's law, the field strength of a point charge in a vacuum at a distance *r* from the charge is:

$$\vec{E}=\frac{1}{4\pi\varepsilon_o}\frac{q}{r^3}\vec{r}_{12},\ (\varepsilon=1)$$

This shows that the point charge field is centrally symmetric.

The principle of superposition makes it possible to calculate the field strength of any system of charges. By mentally dividing, for example, a charged body of finite dimensions into point charges, we find the components of the intensity at a particular point created by individual elements of the charged body. Then, according to the principle of superposition, we determine the resulting stress. *An electric dipole* is a system of two electric charges of equal magnitude and opposite sign +*q* and -*q*, the distance *l* between which is small compared to the distance to the points of the field under consideration (Fig.).



The dipole arm is the vector *l* directed along the axis dipole from negative to positive charge; it is numerically equal to the distance between by them. The product of the positive charge of the dipole *q* on the arm *l* is called the *electric dipole moment*:

$$\vec{p}=q\vec{l}.$$

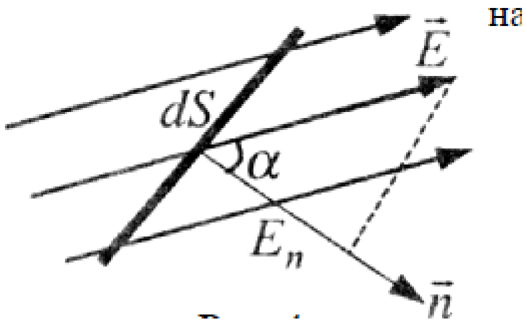
The vector *p* coincides in direction with the dipole arm *l* .[3]

5.1.2.The flow of the tension vector.

The main task of electrostatics is to find the magnitude and direction of the intensity vector *E a t* each point of the field, given the distribution in space and the magnitude of electric charges. The use of the superposition principle to determine the The calculation of electric fields involves considerable mathematical difficulties.

A much simpler method of calculating the fields is based on the use of the Osztrogradsky- Gauss theorem.

Suppose that an arbitrary plane *dS* is drawn in a uniform electric field (*E=const). A* single vector *n* normal to the plane forms an angle *α* with the vector *E* (Fig.).



The elementary flux of the stress vector is the value of

Or

$$d\Phi_E = EdS\cos\alpha$$

$$d\Phi_E = E_n dS = \left(\vec{E}, d\vec{S}\right),$$

where E_n is the projection of the vector *E* on the direction of the vector

are normal, and the vector

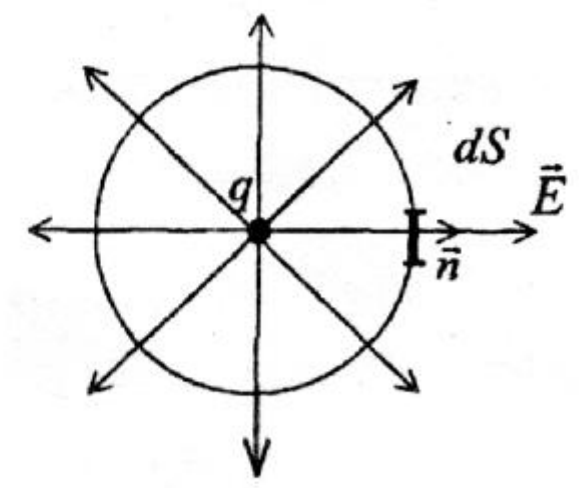
$$d\vec{S} = dS\vec{n}.$$

The total flow of the stress vector through an arbitrary the surface *S* will be

$$\Phi_E = \int_S E_n dS = \int_S E\cos\alpha dS$$

5.1.3.The Ostrogradsky-Gauss theorem

Let a point charge *q be* described around a point charge is a spherical surface of radius *r*, in the centre of which is where this charge is located (Fig.).



The projection of the stress vector E_n on the normal direction will be:

$$E_n = \left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2},$$

then the flow of the stress vector through the closed spherical surface will be:

$$\Phi_\Gamma = \oint_S E_n dS = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\oint_S dS = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}4\pi r.$$

So.

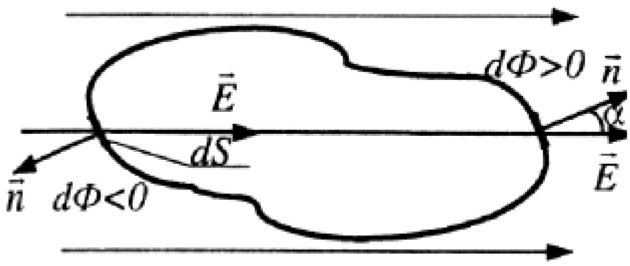
$$\Phi_E = \oint_S E_n dS = \frac{q}{\varepsilon_o}.$$

This equation is called the ***Ostrogradsky-Gauss** theorem.* It is valid not only for spherical surfaces, but also for any closed surface and for any number of charges enclosed by it. In general, this theorem is written as follows:

$$\oint_S \vec{E}d\vec{S} = \oint_S \vec{E}_n d\vec{S} = \frac{1}{\varepsilon_o}\sum_{i=1}^{N} q_i .$$

The flux of the electric field intensity vector through a closed surface is equal to the algebraic sum of electric charges covered by this surface divided by the electric constant.

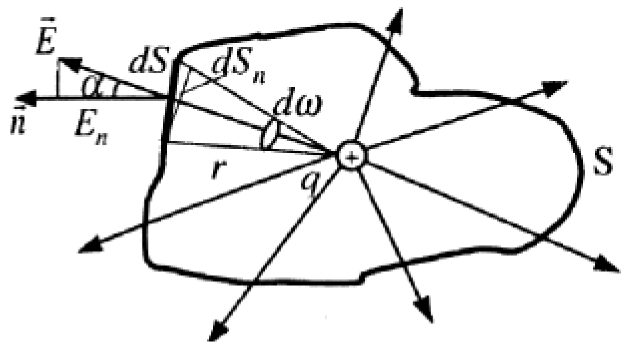
The flux of the electric field strength vector through an arbitrary closed surface that does not contain a charge is zero.



The flow sign depends on the choice of normal direction. For closed surfaces, the normal extending outwards is assumed to be positive. Then where the vector *E* is directed outwards, E_n and Φ_E are positive, and when *E* is inside the surface, E_n and

$$\Phi_E = \oint_S E_n dS = \oint_S \left(\vec{E}, d\vec{S}\right).$$

Suppose that a spherical surface of radius *r is* described around a point charge *q, with* this charge in its centre (Fig.).



Suppose that an arbitrary closed surface S is described around a point charge + *q* in vacuum The lines of stress come from this surface. Let's select an arbitrary elementary site S***, the*** normal n to which is is the angle α with the vector E. Let us project the element ***ds of the*** surface d*S* onto a surface of radius *r* centred at the location of the charge q.

Then

$dS_n = dS\cos\alpha.$

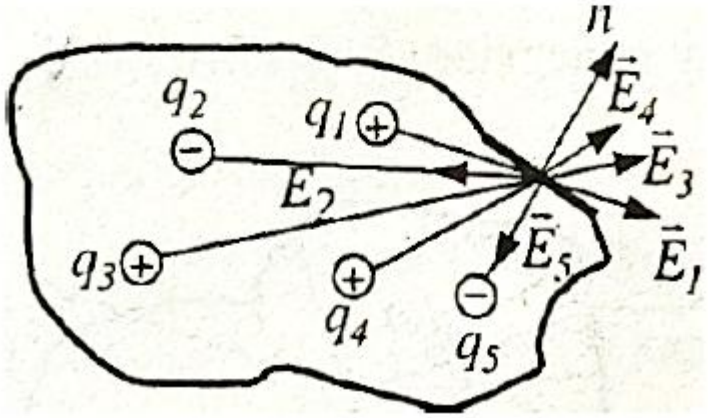
The elementary flow

$$d\boldsymbol{\Phi}_E = E\cos\alpha dS = \frac{q}{4\pi\varepsilon_0 r^2} dS_n = \frac{q}{4\pi\varepsilon_0 r^2} r^2 d\omega = \frac{q}{4\pi\varepsilon_0} d\omega$$

and $d\omega$ is the solid angle at which the elementary site *ds is* visible from the point charge **q.** By integrating over the angle, we obtain

$$\Phi_E = \oint_S E_n dS = \int_0^{4\pi} \frac{q}{4\pi\varepsilon_0} d\omega = \frac{q}{\varepsilon_0}.$$

If there is a negative charge inside the closed surface, the angle between the normal and the vector E will be obtuse (the lines of stress are inside the closed surface)
So, cos$\alpha < 0$. Then $d\Phi_E < 0$. This means that the flow through the closed surface
Let there be N positive and negative charges inside a closed surface S (Fig.).



According to the principle of superposition, the strength E of the field created by by all charges, is equal to the sum of the intensities E created by each charge in particular.
Therefore, the projection of the vector E onto the direction of the normal to the site dS is

$$\vec{E} = \sum_{i=1}^{N} \vec{E_i}$$

equal to the algebraic sum of the projections of all vectors E_i onto this direction:

The flux of the resultant field intensity vector through an arbitrary closed surface S, covering charges q_1, q_2,...q_n is equal to

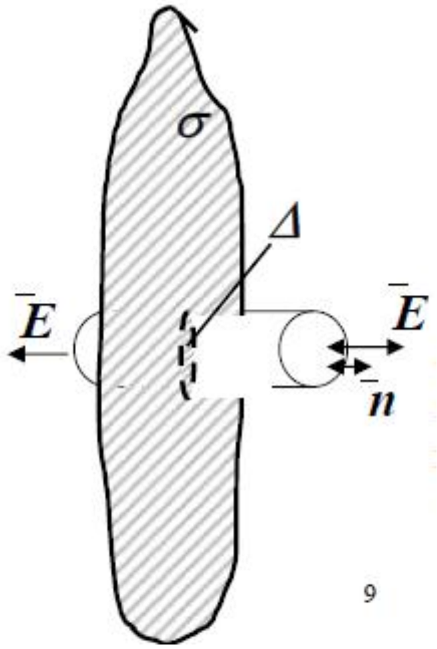
$$\Phi_E = \oint_S E_n dS = \oint_S \left(\sum_{i=1}^{n} E_{in} \right) dS = \sum_{i=1}^{n} \oint_S E_{in} dS$$

$$\Phi_E = \oint_S E_n dS = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i \,.$$

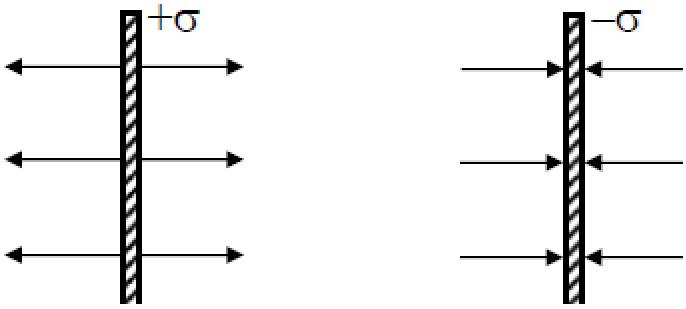
So, the *flow of the tension vector in vacuum through an arbitrary closed surface enclosing electric charges is equal to the algebraic sum of these charges divided by the electric constant.*

5.1.4.Application of the Ostrogradsky-Gauss theorem to the calculation of electrostatic field intensity

Let the field be created infinite, uniformly charged plane with a surface charge density σ (Fig.).



The strength of the electrostatic field at any point has direction perpendicular to the plane (Fig.)



. At points that are seventh relative to the plane, the field strength is equal in magnitude and opposite in direction.

Choose a closed surface in the form of a cylinder with area ΔS.

By virtue of symmetry *E*\`=*E*\`\`=*E*. Let's apply Flow *E* through the side surface of the cylinder is absent , since E_n =0. Only the flux of the vector *E* through the surfaces of the base remains, which is equal to:

$$\Phi_e = 2E \cdot \Delta S .$$

Inside the closed surface there is an electric charge equal to:

$$q = \sigma \cdot \Delta S .$$

So,

$$2E \cdot \Delta S = \frac{\sigma \cdot \Delta S}{\varepsilon_o} ,$$

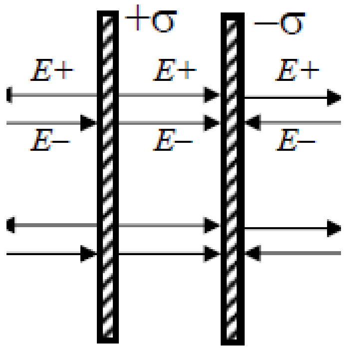
So where

$$E = \frac{\sigma}{2\varepsilon_o}$$

This means that at any distance from the plane, the field strength is the same.

A field that is created by by two infinite charged planes:

As can be seen from Fig., the electric field has a deterministic intensity only between the plates:

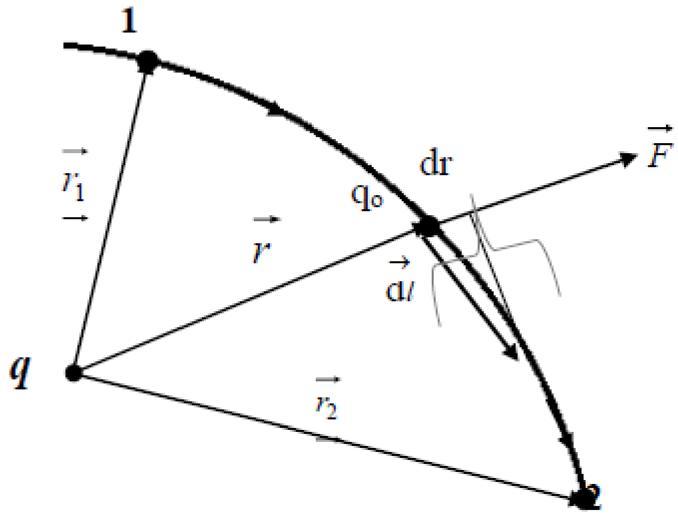


$$E = \frac{\sigma}{\varepsilon_o} .$$

5.2. Electrostatic field potential

The electrostatic field of a point charge is **potential,** and the electrostatic forces are **conservative**. Let's prove it.

Consider the motion of a test charge q0 in the field created by the charge q (Fig.)



When the charge q_0 is moved to *dl, the* field performs elementary work:

$$\delta A = (\vec{F}\vec{dl}) = Fdl\cos(\vec{F}\vec{dl}).$$

Taking into account that *dl cos(Fdl)=dr* , and the force F is determined by Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon\varepsilon_o}\frac{qq_o}{r^2}$$

then the expression defining the elementary work takes the form :

$$dA = \frac{1}{4\pi\varepsilon\varepsilon_o}\cdot\frac{qq_o}{r^2}dr.$$

When the charge q_0 is finally moved from point 1 to point 2, the total work done by the field is equal to :

$$A_{1\to 2} = \frac{qq_o}{4\pi\varepsilon\varepsilon_o}\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{qq_o}{4\pi\varepsilon\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

As follows from the last expression, the work to move the charge q0 does not depend on the trajectory! This means that the forces acting on the charge are conservative, and the electrostatic field is potential. For a potential field,

$$\oint dA = 0$$

A body that is in a potential field of forces has a potential energy, according to due to which the field forces perform work. Therefore, the work of electrostatic field forces can be represented as the difference of potential energies possessed by a point charge q_0 at the initial and final points of the field created by the charge q.

$$A_{1\to 2} = \frac{qq_o}{4\pi\varepsilon\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = W_{\Pi_1} - W_{\Pi_2}.$$

The potential energy of a charge q_0 in the charge field q can be expressed as follows :

$$W_{\Pi} = \frac{qq_o}{4\pi\varepsilon\varepsilon_o}.$$

For charges of the same name, the value of the potential energy is positive ($W_п>0$), for charges of different names, it is negative ($W_п <0$).

From the formulas obtained, it follows that the ratio $W_п/q_0$ is independent of the charge q_0 and is the energy characteristic of the electrostatic field. This value is called the **potential:**

$$\varphi = \frac{W_{\Pi}}{q_o}.$$

The potential φ at any point in an electrostatic field is a physical quantity determined by the potential energy of a unit positive charge placed at that point in the field.

Let's compare the expressions for :

$$A_{1\to 2} = \frac{qq_o}{4\pi\varepsilon\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = W_{\Pi_1} - W_{\Pi_2} = q_o(\varphi_1 - \varphi_2)\ ;\ \ A_{1\to 2} = \int_1^2 \left(\vec{F}\,\vec{dl}\right) = \int_1^2 \left(q_o\,\vec{E}\,\vec{dl}\right) = \int_1^2 \left(\vec{E}\,\vec{dl}\right).$$

We get

$$\varphi_1 - \varphi_2 = \int_1^2 \left(\vec{E}\,\vec{dl}\right).$$

If you move the charge q_0 from an arbitrary point to infinity, the work of the forces is equal to :

$$A_\infty = q_0\varphi,$$

from

$$\varphi = \frac{A_\infty}{q_o}$$

Therefore, the potential φ of the electrostatic field can be defined as follows. Potential is a physical quantity determined by the work to move a unit positive charge from a given point in the field to infinity.

5.2.1.Tension as a potential gradient

Consider the case of moving a unit positive point charge q from point 1 to point 2 along the axis *x*.

The elementary work to move this charge is equal to :

$$\delta A = (\vec{F} d\vec{x}) = q_o E_x dx = E_x dx\ , \quad (q_o = 1).$$

The same work is equal to the potential difference :

$$\delta A = \varphi_1 - \varphi_2 = -d\varphi.$$

By equating the right-hand sides of both expressions, we get :

$$E_x = -\frac{\partial \varphi}{\partial x},$$

i.e., the projection of the electrostatic field intensity vector onto the x-axis is determined by the rate of change of the potential in the x-direction.

Similarly, you can get that:

$$E_y = -\frac{\partial \varphi}{\partial y} \quad \text{та} \quad E_z = \frac{\partial \varphi}{\partial z}.$$

By adding the right and left hand sides of these equations and multiplying them by unit vectors *i*, *j*, *k* (orth) , we obtain :

$$\underbrace{E_x \vec{i} + E_y \vec{j} + E_z \vec{k}}_{\vec{E}} = -\underbrace{\left(\frac{\partial \varphi}{\partial x} \vec{i} + \frac{\partial \varphi}{\partial y} \vec{j} + \frac{\partial \varphi}{\partial z} \vec{k} \right)}_{\overrightarrow{grand}\ \varphi}.$$

Or :

$$\vec{E} = -\overrightarrow{grand}\ \varphi\ .$$

grand φ - is the vector of the potential gradient numerically equal to the rate of change of the potential fields per unit length.

The minus sign means that the vector *E is* directed in the direction of decreasing potential. An imaginary surface with all points having the same potential is called a surface of equal potential, or **equipotential** surface. Its equation is φ (x,y,z)= const. When you move tangentially to the equipotential surface by segment *dl, the* potential does not change (dφ=0). Since

$$E_1 = \frac{\partial \varphi}{\partial l}$$

then the projection of the vector *E* onto tangent line is zero. This means that the *vector E* is perpendicular to the equipotential surface.

Thus, the lines of electrostatic field intensity at each point are perpendicular to the equipotential surfaces.

By by means of formula

$$\vec{E} = -\overrightarrow{grand}\ \varphi \ .$$

by known value. The potential difference between two arbitrary points in the field can be found by calculating the field strength:

a) A field of infinite uniform charge in a plane:

$$E = \frac{\sigma}{2\varepsilon_o}$$

σ- is the surface charge density; $E \neq f(x)$

$$\Delta\varphi = \varphi_1 - \varphi_2 = \int_{\varphi_1}^{\varphi_2} d\varphi = -\Delta\varphi \ .$$

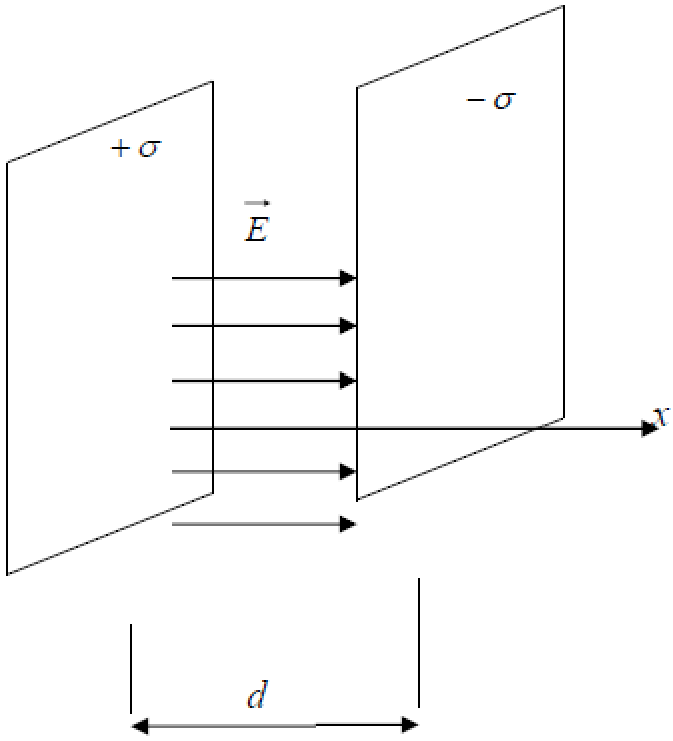
$$\Delta\varphi = \int_{x_1}^{x_2} Edx = \int_{x_1}^{x_2} \frac{\sigma dx}{2\varepsilon_o} = \left(\sigma \big/ 2\varepsilon_o\right)(x_2 - x_1)$$

$\Delta\varphi =$

If.

$$E = \frac{\sigma}{\varepsilon_o}$$

(see Fig.)



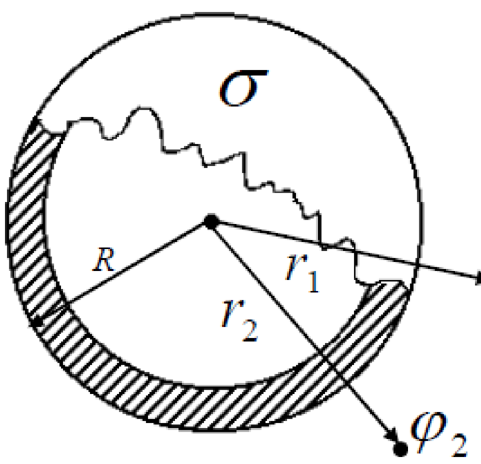
, $\varphi_1 - \varphi_2 = \int_0^d E dx = \frac{\sigma d}{\varepsilon_o}$.

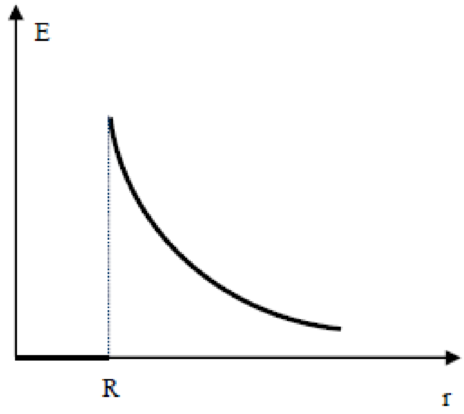
б) Let the field be created by a hollow spherical surface of radius *R* (Fig.). At r<R E=0 (according to the Ostrogradsky-Gauss theorem)

At *r*≥*R*

$$E = \frac{q}{4\pi\varepsilon_o r^2}.$$

$$\varphi_1 - \varphi_2 = \int_{r_1}^{r_2} E dr = \frac{q}{4\pi\varepsilon_o} \int_{r_1}^{r_2} \frac{dr}{r^2} = \left(\frac{q}{4\pi\varepsilon_o}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$





If $r_1= R$, and $r_2 = \infty$, then the potential of the charged sphere is

$$\varphi = \frac{q}{4\pi\varepsilon_0 R}$$

c) Suppose that a field is created by a charged sphere of radius R.

If z`<R :

a)

$$E = \frac{q}{4\pi\varepsilon_o R^3}\cdot r'$$ (inside the ball);

$$\varphi_1' - \varphi_2' = \int_{r_1'}^{r_2'} E dr' = \left(\frac{q}{4\pi\varepsilon_o R^3}\right)\frac{1}{2}\left[\left(r_2'\right)^2 - (r_1')^2\right].$$

b) If

$$E = \frac{q}{4\pi\varepsilon_o r^2}$$

$r \geq R$, then

$$;\ \varphi_1 - \varphi_2 = \left(\frac{q}{4\pi\varepsilon_o}\right)\cdot\int_{r_1}^{r_2}\frac{dr}{r^2} = \left(\frac{q}{4\pi\varepsilon_o}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$;

(we do the same for a point charge or a hollow sphere!).[4]

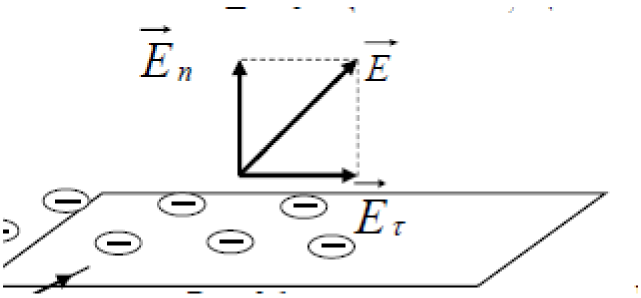
5.3. Conductors and dielectrics in an electrostatic field. Electric capacity. Capacitors

5.3.1.Conductors in an electrostatic field

The conductor differs from semiconductor or dielectric in that it has a it has a large number of free charge carriers (electrons) $\boldsymbol{n > 10^{-21}\ m^{-3}}$**.** For free charges in electrostatic field acts force $\vec{F} = e\vec{E}$. Under the the action of this force charges are set in motion and move as long as the field strength in the conductor will not$\vec{E} = 0$. This means that the potential inside the conductor should be the same. Then the field strength vector near the surface of the conductor at each point must be perpendicular to the surface and

$$\vec{E} = \vec{E}_n, \ E_\tau = 0$$

since the displacemen charges does not occur (*Fig.)*.



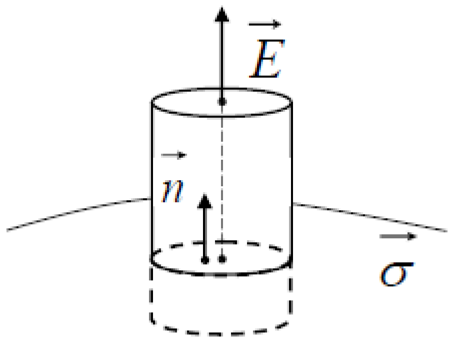
This means that the surface of the conductor is equipotential!

The **excess** charge q in the conductor is **distributed** over the surface of the conductor!

Because if there were charges in the conductor, they would either start at power lines end. But in the conductor $E = 0$!

Let us find the relationship between the field strength E near the surface of a charged conductor and the surface charge density σ .

Let's apply the Ostrogradsky-Gauss theorem.



Consider the surface element of a charged conductor (*Fig.)*. The flow of the *vectorE* through a closed surface (surface of the cylinder) is determined only by the by the flow through the outer base of the cylinder, since there is no field inside the conductor.

$$d\Phi_e = (\vec{E}d\vec{s}) = Eds = \frac{\sigma ds}{\varepsilon\varepsilon_o}, \text{ звідки } E = \frac{\sigma}{\varepsilon\varepsilon_o}.$$

The strength of the electrostatic field near a charged surface is proportional to the surface charge density. The surface charge density is different for different points on the surface and depends on the curvature of the conductor surface.

Let's explain this with the example of two charged spheres with radii *R1* and *R2* that are in contact. The charges on the conductors are always redistributed in this way, so that their potentials are equal, i.e.

$\varphi_1 = \varphi_2$

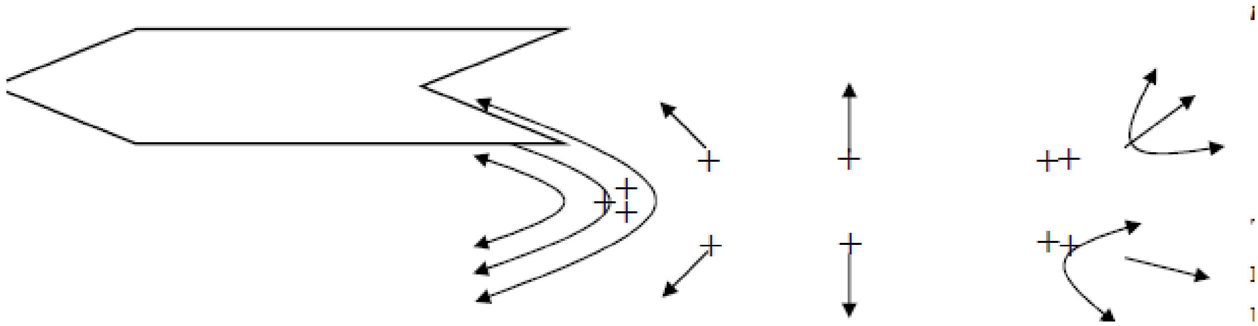
$$\varphi_1 = \varphi_2 = k\frac{q_1}{R_1} = k\frac{q_2}{R_2}; \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}.$$

Let us define the charges on the spheres through their surface densities :

$$q_1 = 4\pi R_1^2\sigma_1 ; \quad q_1 = 4\pi R_2^2\sigma_2 .$$

$$\frac{4\pi R_1^2\sigma_1}{4\pi R_2^2\sigma_2} = \frac{R_1}{R_2}; \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2}.$$

Thus, the ***ratio of surface density charges is inversely proportional to curvature surface.***



In places here the surface curvature is less, the surface charge density is higher and vice versa. And since $E \approx \sigma$, the intensity fields near points of small curvature are larger than near relatively flat surfaces (*Fig.)*.

As can be seen from the above, the stress near the tip will be very high. It can be so high that the air is "broken" and the charge flows rapidly from the tip into the air. That is why parts of electrical devices that are under high voltage and carry a large charge are made with surfaces with large radii of curvature.

If an uncharged conductor is placed in an external electrostatic field, the charges in it will redistribute s o that the field strength in each point of the conductor was zero

$(\vec{E} = 0)$.

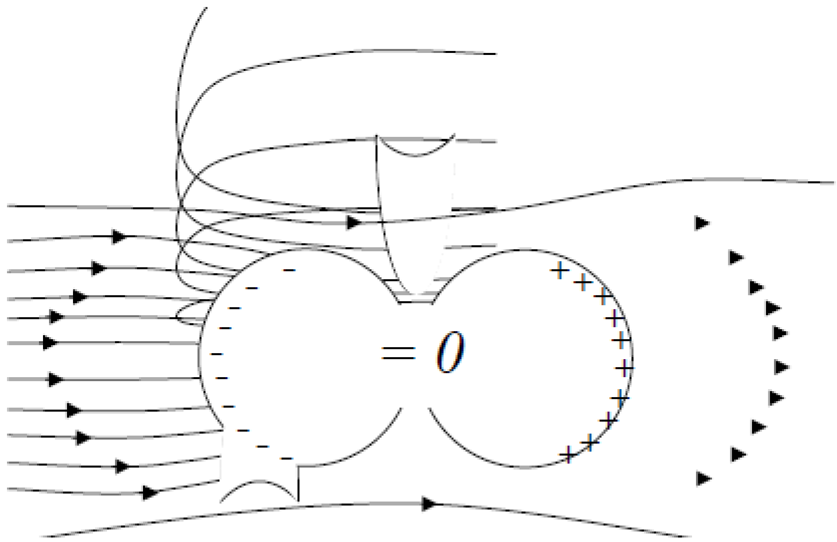
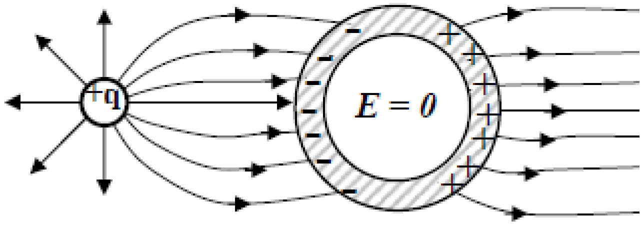


Fig.

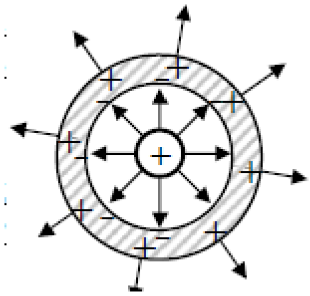
The phenomenon of redistribution of surface charges on a conductor in an external electrostatic field is called **electrostatic induction,** and the redistributed charges are called **induced charges**.

If y external electric field is placed closed hollow conductor (*Fig.)*, then inductive charges arise only on the outer surface of such conductor, and the electric field inside the cavity will be zero.

Thus, a hollow metal conductor shields the electric field of external charges. This is whats essence electrostatic protection.



Note that the metal shield protects only against the external field. If a charge is placed *inside* the metal shell, no shielding occurs (*Fig.)*.



5.3.2. Electrical capacity

Excess electric charges on a separated conductor are distributed depending only on the shape of the conductor with different σ. Each new portion of charges is distributed over the surface similarly to the previous portion. σ = *kq* for any point on the surface,

$$k = f(x, y, z)$$

The potential of a charged conductor

$$\varphi = \frac{1}{4\pi\varepsilon\varepsilon_0}\oint\frac{\sigma ds}{r} = \frac{q}{4\pi\varepsilon\varepsilon_0}\oint\frac{kds}{r} \Rightarrow \varphi \approx q$$

The ratio of the charge of a separate conductor to its potential is called the *electrical capacity of the* conductor.

$$C = \frac{q}{\varphi}$$

As follows from the formula, $C = \frac{4\pi\varepsilon\,\varepsilon_0}{\oint\frac{kds}{r}}$ the electrical capacitance C depends on :

1. shape and size conductor

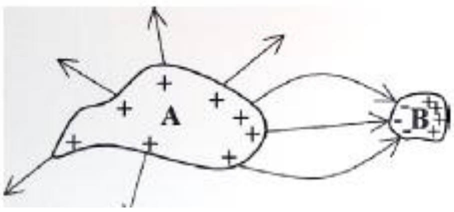
2. the dielectric property of the medium (ε).

The electrical capacitance of an isolated conductor is numerically equal to the electric charge that must be applied to the conductor to increase its potential by one.

[C]=1F (Farad)

5.3.3. Mutual electrical capacitance

If other conductors are present near a charged conductor *A, the* electrical capacitance C of such a conductor will be greater than that of an isolated conductor. This is due to the fact that the induced charges weaken the field generated by conductor A and reduce the potential of the conductor, thereby *increasing its capacitance* (Fig.).



For a system of two oppositely charged conductors (charges are the same), a potential difference $\varphi_1 = \varphi_2$ arises. Then the mutual capacitance of the two conductors is determined:

$$C = \frac{q}{\varphi_1 - \varphi_2}$$

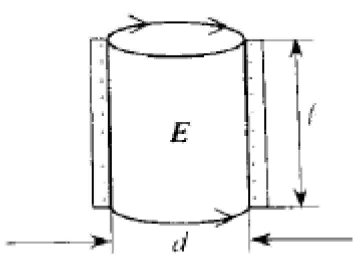
C = f(ε, conductor shape, relative position). A system of two conductors charged with charges of equal magnitude and

opposite sign is called a **capacitor** if the shape of the conductors ensures localisation of the electric field in a limited area of space.

1) Flat capacitor.

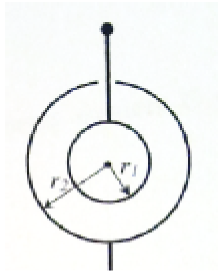
The field between the covers is homogeneous. Homogeneity is

broken near the edges (ignored when $d << l$).



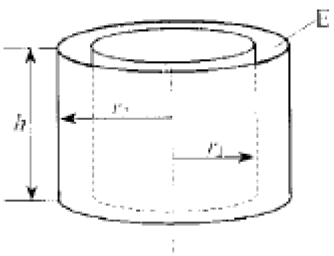
$$C = \frac{\varepsilon\varepsilon_0 S}{d}$$

2) Spherical capacitor



$$\varphi_1 - \varphi_2 = \frac{q}{4\pi\varepsilon\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right); \qquad C = \frac{4\pi\varepsilon\varepsilon_o r_1 r_2}{r_2 - r_1}.$$

3) Cylindrical capacitor.



$$\varphi_{1,2} = \frac{\tau}{2\pi\varepsilon\varepsilon_o}\ln\frac{r_1}{r_2};$$

$$\tau = \frac{q}{h} \text{ (лінійна густина);}$$

$$C = \frac{q}{\varphi_{1,2}} = \frac{q \cdot 2\pi\varepsilon\varepsilon_o}{\frac{q}{h}\ln\frac{r_2}{r_1}}.$$

$$C = \frac{4\pi\varepsilon\varepsilon_o h}{\ln\frac{r_1}{r_2}}.$$

This formula can be used to calculate the capacitance of a coaxial cable.

5.3.5. Energy charged separated conductor, capacitor. The energy of an electrostatic field. Volumetric energy density

Suppose there is an isolated conductor whose charge, capacitance and potential are *q, C, φ,* respectively. Increase the charge of this conductor by *dq*. To do this, we need to transfer the charge *dq* from infinity to the surface of the conductor, expending work

$$dA = \varphi dq = C\varphi d\varphi\,, \quad \left(C = \frac{dq}{d\varphi}\right).$$

To charge a body from zero potential to φ , *we* must do the work

$$A = \int_0^{\varphi} C\varphi d\varphi = C\frac{\varphi^2}{2}.$$

*The energy of a charged conductor is numerically equal to the work that must be done to charge this conductor, i.*e.

$$W_e = \frac{C\varphi^2}{2} = \frac{d\varphi}{2} = \frac{q^2}{2C}.$$

When a capacitor is charged, work is expended to transfer electrical charges from one plate
to another. The energy of a charged capacitor is determined by the formula:

$$W_e = \frac{C\Delta\varphi^2}{2} = \frac{q\Delta\varphi}{2} = \frac{q^2}{2C},$$

where

dφ- the potential difference between the covers.

Let's find the energy of the electric field.

$$W_e = \frac{C\Delta\varphi^2}{2} = \frac{1}{2}\frac{\varepsilon\varepsilon_o S}{d}(Ed)^2 = \frac{1}{2}\varepsilon\varepsilon_o SE^2 d = \frac{\varepsilon\varepsilon_o E^2}{2}V,$$

where *V =Sd*- is the volume of the capacitor.

It is known that

$$\varepsilon\varepsilon_o E = D.$$

Then

$$W_e = \frac{1}{2} D \cdot E \cdot V.$$

These formulas show that the energy of a capacitor is expressed in terms of a quantity characterising the electric field, the intensity *E*. This means that the electrostatic field has energy and we can speak of the energy of the electrostatic field. Energy is one of the characteristics of the state of matter. Therefore, energy is inextricably linked to its material carrier, the electric field.

The energy of an electric field is localised in the space where the field exists.

Expression.

$$W_e = \frac{C\Delta\varphi^2}{2} = \frac{q\Delta\varphi}{2}$$

corresponds to the provisions of the long-range theory, where *W* is considered as the potential energy of charged bodies attracted by either are repelled by each other. Formula

$$W_e = \frac{\varepsilon\varepsilon_o E^2}{2} V$$

meets the expectations field theory (proximity theory), where it is believed that energy, like substances, is distributed in space with a ***bulk density***

$$w_e = \frac{W_e}{V} = \frac{\varepsilon\varepsilon_o E^2}{2} = \frac{(\vec{E}, \vec{D})}{2}.$$

The energy *dWe* of an infinitesimal volume *dV of the* field is equal:

$$dW_e = w_e dV = \frac{\varepsilon\varepsilon_o E^2}{2} dV$$

By interpolating this equation over the entire volume *V* of the field, we find the total energy *We* of the electrostatic field:

$$W_e = \int_V \frac{1}{2} \varepsilon\varepsilon_o E^2 dV.$$

5.4. Dielectrics in an electrostatic field. Types of dielectrics.

5.4.1. Electronic and orientational polarisation

Dielectrics (or insulators) are substances that are unable to conduct electricity.

There are no perfect insulators in nature, they conduct current 10^{15} -10^{20} times worse than conductors.

The resistivity of dielectrics is $\rho \approx 10^{15} \Omega \cdot m$

Dielectrics do not have free electrical charges (electrons), as metals or other conductors do.

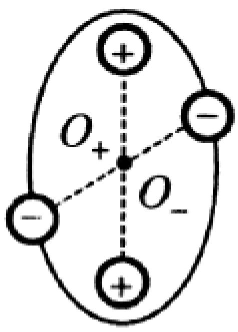
Each molecule (or atom) of a dielectric has positively charged nuclei and negatively charged electrons that move around the nuclei. The positive charges of all nuclei are equal to the absolute value of the charge of all electrons, and therefore the molecule of a substance is generally electrically neutral.

When studying the electrical properties of dielectrics, dielectric molecules can be represented as a system consisting of two point charges.

Let us replace all positive charges of the molecule's nuclei with a single total charge +q, located in the centre of mass of the positive charges, and all negative charges with a single total negative charge -q, located in the centre of mass of the negative charges. Then the dielectric molecule can be considered as a dipole consisting of +q and -q charges.

Dielectrics are divided into three types.

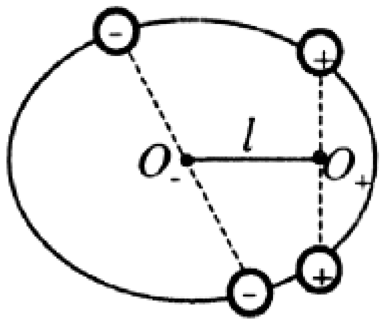
I. Nonpolar dielectrics are dielectrics composed of molecules whose centres of mass of positive and negative charges coincide in the absence of an electric field



. Examples of nonpolar dielectrics are the gases N_2 , H_2 , O_2 , CO_2 , CH_4

Molecules of such dielectrics are called ***nonpolar.*** The dipole moment of such molecules is zero in the absence of an external electric field.

***II. Polar dielectrics** are* dielectrics in which the centres of mass of positive and negative charges do not coincide, i.e., they have an asymmetric structure (Fig.).



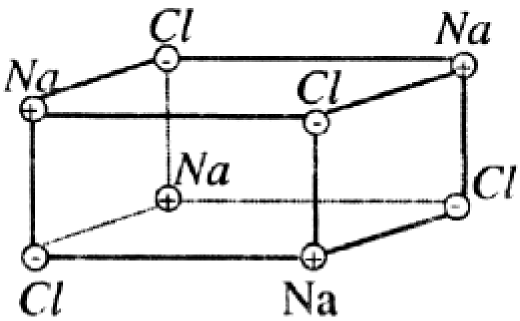
Polar dielectrics include gases SO_2 , H_2 *S*, NH_3 , etc., liquids - water H_2O, hydrochloric acid *HCl*, benzene C_6H_6, etc.

The molecules of such dielectrics are called ***polar.*** In the absence of an external field, these molecules have dipole moments

$$\vec{p}_o = q\vec{l}$$

their are called rigid dipoles.

***III. Ionic dielectrics** are* substances whose molecules have an ionic structure. An example of such dielectrics is lectrics are *NaCl*, *KCl* and others.

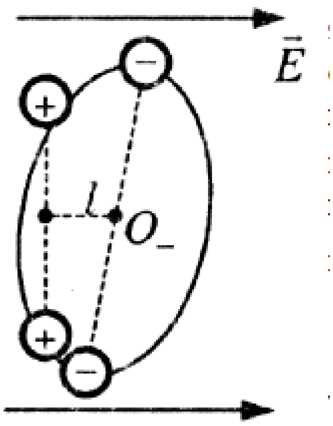


Ionic crystals are spatial lattices with a regular alternation of ions of different signs (Fig.. Individual molecules cannot be distinguished in these crystals. Ionic crystals should be viewed as a system of ionic sublattices nested within each other. In these dielectrics, each pair of neighbouring ions of different names is like a dipole.

Consider what happens to dielectrics when they are placed in a homogeneous electric field.

5.4.2. Nonpolar dielectrics. Electronic polarisation

The forces with which an electric field acts on the positive and negative charges of molecules are opposite and therefore push them apart. In an electric field, the centres of mass of the positive and negative charges of each molecule do not coincide, but are shifted by a distance *l* between them (Fig)



. The greater the field strength *E*, the greater the distance *l between* charges of opposite signs. A molecule with nonpolar turns into a polar one with a dipole moment *p* .

Since

$l \sim E$ and $p = pl$,

then

$p \sim E$

or

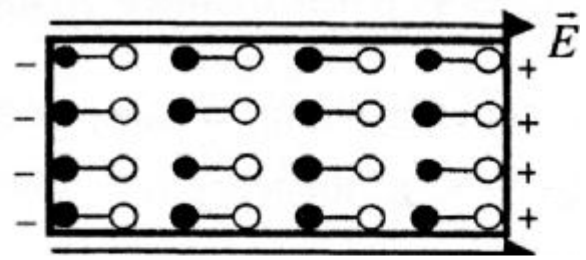
$\vec{p} = \beta\varepsilon_o\vec{E}$

where

ε_o is the electrical constant,

β is the polarisation of a single dielectric molecule.

The value of β has different values for atoms and molecules of different substances. The polarizability β characterizes the ability of electrons in an atom or The molecule is displaced by the electric field.

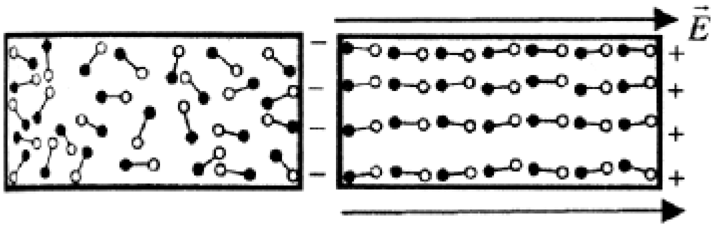


The dipole moments *p* of molecules of unpolarised dielectrics are called *induced* or *quasi-elastic*.

When a nonpolar dielectric is introduced into an electromagnetic field, all the induced dipole moments are placed in chains along the lines of intensity *E* (Fig.), where ● arenegative charges and ○ are positive charges. As a result, the faces of the dielectric acquire different charges - the dielectric is polarised. This kind of dielectric polarisation is called ***electronic.***

5.4.3. Polar dielectrics. Dipole or orientation polarisation

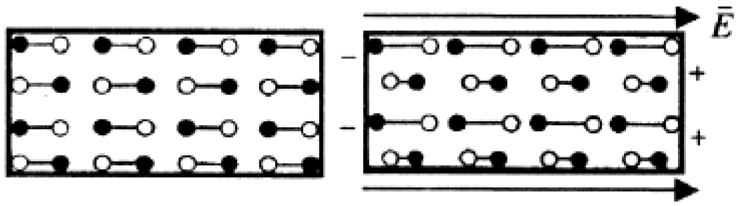
If the dielectric with polar molecules is not in an external electric field, then due to the chaotic thermal motion of the molecules, the vectors of their dipole moments are oriented randomly (Fig.*)*.



Therefore, the vector sum of the dipole moments of all molecules in an arbitrary volume ΔV dielectric, is zero. If a dielectric with polar molecules are introduced into an electric field, then under the action of the field, the polar molecules dielectric are trying to rotate so that the vectors of their dipole moments coincided with the direction of the field strength vector *E* (Fig.*)*. However, the thermal motion of the molecules randomly scatters the dipoles and interferes with the orientation of all *p* vectors (dipole moments) along the field. As a result of the joint action of these two factors in the dielectric is dominated by the orientation of the dipole moments of the molecules along the field. This orientation will be The more complete the electric field in the dielectric and the weaker the thermal motion of the molecules, i.e., the lower the temperature. This process is called the ***orientational polarisation of the*** dielectric.

5.4.4. Ionic dielectrics. Ionic polarisation

In crystalline dielectrics with cubic crystal lattices (*NaCl*, *KCl*, *Naj,* etc.), under the action of an electric field, all positive ions are displaced in the direction of the field strength E , and all negative ions are displaced in the opposite direction (Fig). In his case. in each unit volume of the crystal is the same number of positive and negative ions, and on each of the two opposite faces of the crystal, perpendicular to the vector of of the electric field elasticity, are contained in ions of any one sign.



The charges that appear on the faces of the dielectric are not free, they are bound to the atoms and molecules of the substance.

The phenomenon of limited charge displacement in atoms and molecules or directional orientation of the dipole moments of rigid molecules in an external electric field, which results in bound electric charges on the surface of a dielectric, is called dielectric polarisation.

The degree of polarisation of a dielectric is characterised by the *polarisation vector*, or *polarisation*.

The polarisation vector is the limit of the ratio of the electric moment of a certain volume of a dielectric to this volume when the volume tends to zero:

$$\vec{P} = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{n} \vec{p}_i}{\Delta V},$$

where $\vec{p_i}$ dipole moment of the *i*-th dipole, *n* is the number of dipoles in the ΔV.

Thus, the vector *P* is the dipole moment of a unit volume of a dielectric that occurs when it is polarised.

For a homogeneous dielectric in a homogeneous electric field,

fair equality

: $\vec{P} = n \cdot \vec{p}_i$,

where n is the number of molecules per unit volume, $\vec{p_i}$ is the dipole moment of the molecule. Since

$$\vec{p} = \beta\varepsilon_o \vec{E}$$

so

$$\vec{P} = n\beta \cdot \varepsilon_o \vec{E}.$$

Let

$$n\beta = \chi$$

So

$$\vec{P} = \chi\varepsilon_o \vec{E}$$

The coefficient χ is called the *dielectric the susceptibility of a substance* or the *polarisation of a unit volume of a dielectric*. X - is a dimensionless quantity($\chi \approx 80$ for water, $\chi \approx 25$ for alcohols).

5.5. Mechanical effects in dielectrics. Electrostriction and piezoelectric effect. Segnothelectrics.

Experiments show that in an electric field, even an initially uncharged dielectric is subject to a mechanical force. The origin of this force has already been explained: polarisation charges (both surface and bulk) appear on the dielectric, and therefore certain forces act on each element of the dielectric's surface and bulk. These forces lead to deformation of the dielectric - n an electric field, the dielectric deforms. This phenomenon is called **electrostriction. As a result of electrostriction, mechanical stresses** arise inside the dielectric, the magnitude of which depends in a complex way on the strength of the electric field. Along with electrostriction, there is another mechanical effect, the inverse piezoelectric effect.

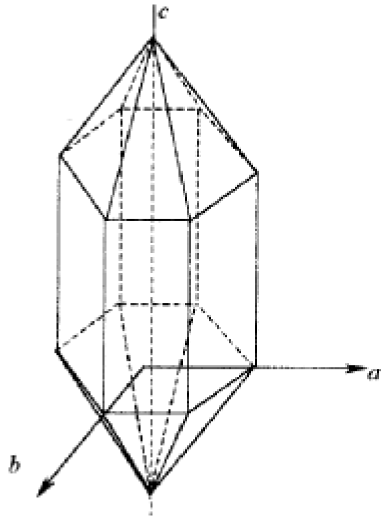
Electrostriction is distinguished by the following features:

• Electrostriction occurs in every dielectric placed in an electric field.

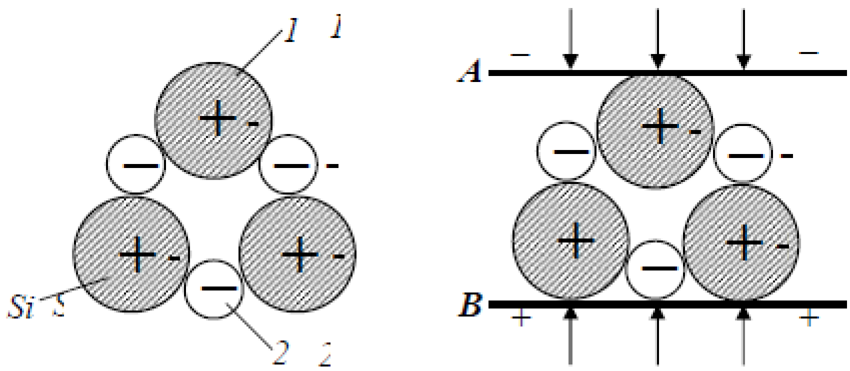
• Electrostriction is independent of the direction of the electric field.

• Electrostriction is a macroscopic reflection of the fact that individual molecules-dipoles are subjected to a electricfield that tries to pull them into areas of higher intensity.

Direct and inverse piezoelectric effect. So far, we have considered the polarisation of dielectrics caused by an external electric field. It turns out **that in some crystalline dielectrics, polarisation can occur under the influence of mechanical deformation. This phenomenon is called the direct piezoelectric effect**. A classic example of such a dielectric is a quartz crystal, i.e. a SiO_2 crystal . These crystals belong to the hexagonal syngonics, a quartz crystal has the shape of a hexagonal prism bounded by two hexagonal pyramids (Fig.).

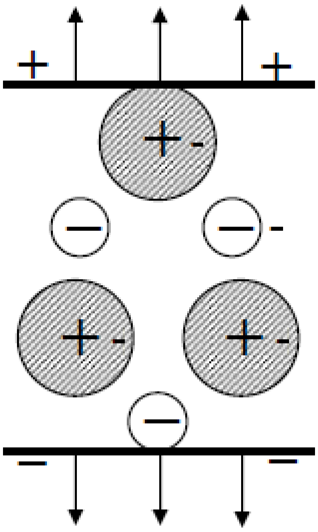


When the crystal is compressed or stretched perpendicular to the *caxis*, which is called the optical axis of the crystal, polarization crystals appear on the surface of the crystal. The piezo effect is explained as follows. At mechanical deformation of a crystal, two types of changes occur in ts lattice. First, each elementary lattice. For example, under compression, the regular hexagon at the base of the quartz lattice turns into a low-symmetry hexagonal prism. When a crystal is deformed, the positive and negative lattice ions are displaced relative to each other (Fig.).

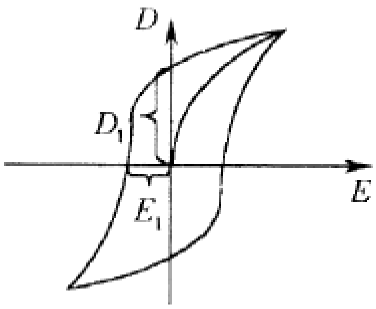


When deformation is positive ion *1* and negative ion *2* are "squeezed" inside the cell, which causes the protruding charges to decrease, which is equivalent to the appearance of a negative charge on the plane *A* and positive on the plane *B*. When the crystal is stretched: An additional positive charge is generated on the *A-side*, and an additional negative charge is generated on the *B-side.* Secondly, deformation of a crystal can cause a shift of the ionic sublattices that make up the lattice relative to each other. This is manifested in a change in the electric moment of the lattice.

Calculations show that the piezoelectric effect should only occur in dielectrics whose unit cell lattice does not have a centre of symmetry, as is clearly observed in the case of the SiO_2 crystal . The piezoelectric effect is even more pronounced in crystals of sodium sulfate, but this salt has very low mechanical properties and is unstable when the temperature rises above 40-45°C.



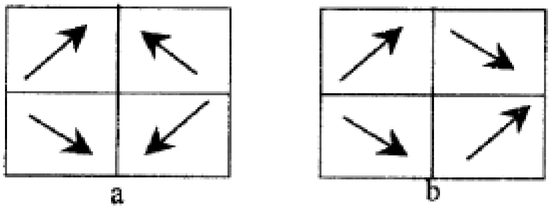
The inverse piezoelectric **effect** means that the polarisation of a dielectric by an external field causes mechanical deformations. It is easy to guess that the existence of the inverse piezoelectric effect is a consequence of the law of energy conservation. If a plate cut from quartz is placed between the plates of a capacitor so that the *a-axis* is perpendicular to the capacitor plates, then applying an external voltage to the capacitor will cause compression (or tension) of this plate in the direction perpendicular to the a-axis and in the direction coinciding with the a-axis. The direct piezoelectric effect, for example, is the basis for extremely highquality microphones and sound pickups. Ultrasonic generators and stabilisers of reference frequencies in radio engineering devices are based on the inverse effect. **Ferroelectrics.** Some chemical compounds in their solid state have unusual and interesting dielectric properties. These properties were initially discovered in crystals of selenium salt, and then all dielectrics with such properties were called **ferroelectrics.** Segnetite salt - chemical formula $NaKC_4H_4O_6 \cdot 4H_2O$ crystallises in low-symmetric rhombic unit cell lattice. Segnetite salt crystals exhibit a strongly pronounced anisotropy of physical properties. As for electrical properties, the first striking feature is the abnormally high value of the dielectric constant ε . If chemically pure deionised water has the highest dielectric constant among conventional dielectrics ε= 81, then in the room temperature range, *s* for sennet salt reaches **several thousand!** The second property is that the value of the induction vector depends in a complex way on the field strength, i.e., the dielectric permeability is a function of the external field. The third feature is that the magnitude of the induction when the stress vector changes forms a hysteresis loop - see Fig.



.

The value of D_1 at E_o = 0 is called the final polarisation, and the magnitude of the field *E* that depolarises the dielectric is called the coercive force.

Furthermore, the ferroelectric properties are very much dependent on temperature. There is a temperature point (and for some ferroelectrics, even two temperature points) above which (and below which) the ferroelectric properties disappear. These points are called the *Curie temperature*. For example, for sodium sulfate, the ferroelectric properties occur in the temperature range between -*15°C* and +22.*5°C*. It is believed that the reason for the unusual properties of ferroelectrics is the existence of spontaneous polarisation in microregions of the volume due to particularly strong interactions between the particles of the substance. In these micro-volumes, called **domains,** th**e** dipole moments of the particles are oriented in the same way, and therefore the domain has a large dipole moment. In general, the dipole moments of neighbouring domains are randomly arranged and therefore the total dipole moment is small. This arrangement of dipole moments corresponds to a minimum energy, otherwise an electric field with a certain energy would arise around the ferroelectricity. In an external electric field, the polarisation direction in individual domains changes - see Fig.



. These changes are such that the polarization vectors of the domains .The electric moment of the whole ferroelectric material is changed and is perceived as polarisation. Therefore, the electric moment of the entire ferroelectric changes and this change is perceived as its polarisation. The presence of regions of spontaneous polarisation - domains - is the most characteristic feature of ferroelectrics.

5.6. Laws of direct electric current

5.6.1. Direct electric current

Electric current is the orderly movement of free electric charges.

The electric current that occurs under the influence of an electric field is called **conduction current.**

The orderly movement of charged macroscopic bodies in space is called **convection current**. Electric current in conductors exists until all points of the conductor become equipotential.

The conditions necessary for the appearance and existence of an electric conduction current:

1. The presence of ***free charge carriers*** (electrons, holes, ions) in a given medium.
2. The presence of an external electric field whose energy would be used for the orderly movement of charges. This requires a source of electrical energy, i.e. a device that converts any kind of energy into the energy of the electric field.

The direction of the electric current is assumed to be the direction of movement of positive charges.

The current is a quantitative measure of electric current.

Current is a scalar physical quantity that is numerically equal to the electric charge passing through the cross-section of a conductor per unit time:

$$I = \frac{dq}{dt}$$

If the current strength and its direction do not change over time, then such a current is called constant. For direct current strength current is defined as follows:

$$I = \frac{q}{t}.$$

Current direction and distribution of the current I across the crosssection of the conductor is determined by the vector value j, called the *current density*:

The vector $\vec{j}$ is directed along the current and is numerically equal to the current strength that passes through a unit cross-sectional area of the conductor, which is drawn perpendicular to the current direction:

$$\vec{j} = \frac{dI}{dS_{\perp}}. \qquad dI = \left(\vec{j} \cdot \vec{dS}\right); \qquad d\vec{S} = dS \cdot \vec{n};$$

The current in a conductor is determined by :

$$I = \oint_S \left(\vec{j}d\vec{S}\right) = \int_S j_n \cdot dS$$

Where

$$j_n = j \cdot \cos\left(\widehat{\vec{j}\vec{n}}\right)$$

Express the current strength and density in terms of velocity $\langle \upsilon \rangle$ of the ordered motion charges in the conductor. In time *dt*, *a* charge is transferred through the cross section *S*

$$dq = ne < \upsilon_e > Sdt$$

Current strength :

$$I = \frac{dq}{dt} = n \cdot e < \upsilon_e > \cdot S.$$

Then the current density is defined as

$$\vec{j} = n \cdot e \cdot < \vec{\upsilon}_e > .$$

For metals, $n \sim 10^{28} \ldots 10^{29}\ m^{-3}$, $\langle \upsilon \rangle \sim 10^{-4}\ m/s$; $j(Cu) = 107\ A/m^2$.

For comparison, $< U_{sq} > \sim 105\ m/s$ at T=273°K.

Speed propagation current y conductor is not equal to velocity orderly movement of electrons $\langle \upsilon \rangle$, a is determined by by the velocity is the electric field propagation, which is equal to

$C = 3 \cdot 10^8\ m/s$.

5.6.2. Ohm's law in differential form

Let y metal conductor there exists electric field intensity

$$\vec{E} = Const$$

A force acts on each electron from the field:

$$\vec{F} = e \cdot \vec{E} \;;\; F = e \cdot E .$$

According to Newton's second law

$$a = \frac{F}{m} = \frac{eE}{m} \;;$$

The electron acquires a velocity

$$\upsilon_{\max} = a < t >= \frac{e \cdot E}{m} < t >$$

where

$$< t >= \frac{< \lambda >}{< \upsilon_{ел} >},$$

($< \lambda >$ - average free run length; $\langle \upsilon \rangle$ - average speed electron)

$$< \upsilon_{ел} >=< \upsilon_T > + < \upsilon_в >; \quad \text{але} \; \langle \upsilon_T \rangle >> \langle \upsilon_в \rangle$$

(10^5 m/s >> 10^{-4} m/s),

There fore

$$< \upsilon_{ел} >=< \upsilon_T >$$

$$\upsilon_{\max} = \frac{e \cdot E}{m} < t >= \frac{e \cdot E}{m} \cdot \frac{< \lambda >}{< \upsilon_T >} \;; \qquad < \upsilon_в >= \frac{\upsilon_{\max} + 0}{2} = \frac{e \cdot E < \lambda >}{2m < \upsilon_T >} .$$

Then the current density is defined

as :

$$\vec{j} = n \cdot e \cdot < \vec{\upsilon_в} > .$$

$$\vec{j} = n \cdot e \cdot \frac{e \cdot \vec{E} < \lambda >}{2m < \upsilon_T >} = \frac{ne^2 < \lambda >}{2m < \upsilon_T >} \cdot \vec{E} \, .$$

$$\gamma = \frac{ne^2 < \lambda >}{2m < \upsilon_T >}$$

γ- specific electrical conductivity.

Finally :

$$\vec{j} = \gamma \cdot \vec{E}$$

- ***Ohm's law in differential form.***

5.6.3. Joule-Lenz law

The speed of movement of electrons relative to crystal lattice in the the presence of an orderly movement of the fold:

$$<\vec{\upsilon}> = <\vec{\upsilon}_T> + <\vec{\upsilon}_в>.$$

Before an electron collides with a lattice node, it has kinetic energy:

$$W_к = \frac{m<\upsilon^2>}{2}.$$

$$<\left(\vec{\upsilon}_T + \vec{\upsilon}_в\right)^2> = <\vec{\upsilon}_T^{\,2} + 2\left(\vec{\upsilon}_T \cdot \vec{\upsilon}_в\right) + \vec{\upsilon}_в^{\,2}> = \underbrace{<\vec{\upsilon}_T^{\,2}>}_{<\upsilon_T^2>} + \underbrace{2<\upsilon_T \cdot \upsilon_в \cdot Cos\,\alpha>}_{0} + \underbrace{<\vec{\upsilon}_в^{\,2}>}_{<\upsilon_в^2>}.$$

So,

$$<\left(\vec{\upsilon}_T + \vec{\upsilon}_{ву}\right)^2> = <\upsilon_T^2> + <\upsilon_в^2>.$$

Then the kinetic energy of the electron consists of:

$$W_к = \frac{m<\upsilon_T^2>}{2} + \frac{m<\upsilon_в^2>}{2};$$

$$\Delta W_к = \frac{m<\upsilon_в^2>}{2}.$$

As you can see , if presence of current in a conductor (ordered motion electrons), each electron acquires additional kinetic energy

$$\Delta W_к = \frac{m<\upsilon_в^2>}{2}$$

which it transmits to the nodes of the crystal lattice during the collision and which is released in the conductor in the form of heat - the **Joule-Lenz heat.**

But at the moment of collision, the electron has a maximum velocity that can be determined

by :

$$\upsilon_{\max} = \frac{eE}{m} \cdot \frac{<\lambda>}{<\upsilon_T>}.$$

The number of electron collisions per second is equal to

$$Z = \frac{<\upsilon_T>}{<\lambda>}.$$

That's why в unit of volume of a conductor per unit of time will stand out is the subsequent amount of heat, which is called the specific heat:

$$Q_{num.} = n \cdot Z \cdot \Delta W_{\kappa} = n \frac{<\upsilon_T>}{<\lambda>} \frac{e^2 <\lambda^2>}{2m <\upsilon_T^2>} E^2 = \underbrace{\frac{ne^2 <\lambda>}{2m <\upsilon_T>}}_{\gamma} \cdot E^2 = \gamma \cdot E^2.$$

$Q_{num.} = \gamma \cdot E^2$ specific thermal power of the current (Joule-Lenz law).

Heat released in the volume *V of the* conductor during time *t*:

$$Q = Q_{num.} V \cdot t \;\; ; \;\; E = \frac{U}{\ell} = \frac{IR}{\ell} \;\; ; \;\; V = S \cdot \ell \;\; ; \;\; Q = \gamma \frac{U^2}{\ell^2} S \cdot \ell \cdot t = \frac{U^2}{R} t = I^2 R t.$$

$$Q = I^2 R \cdot t.$$

5.6.4. Ohm's law in its integral form

External forces are *non-electrostatic* forces that act on the current carriers in a conductor to cause their orderly movement and maintain a constant electric current in the circuit.

External forces, unlike Coulomb forces, do not connect charges of different names, but cause their separation and maintain a potential difference at the ends of the conductor. The field of third-party forces *E p* is created *by sources of electrical energy*, e.g., ex, a galvanic cell, an electric generator, etc. In this case, at each point inside the conductor, an electric field of intensity *E is* created, which consists of :

$$\vec{E} = \vec{E}_{кул.} + \vec{E}_{стор.}.$$

Then law Ohm's law y differential form it is necessary be written down as follows In this way:

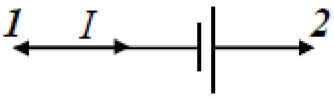
$$\vec{j} = \gamma \cdot \vec{E} = \gamma\left(\vec{E}_{кул} + \vec{E}_{стор}\right) = \frac{1}{\rho}\left(\vec{E}_{кул} + \vec{E}_{стор}\right).$$

Multiply right and left part of the last equation by *dl (the* element current), and note that

$$\vec{j} = \frac{I}{S};$$

$$\frac{I}{S}\rho\vec{dl} = \left(\vec{E}_{кул.} \cdot \vec{dl}\right) + \left(\vec{E}_{стор.} \cdot \vec{dl}\right).$$

Integrate both parts of the equation в the range of the point 1 to point *2* 2 of the conductor.



$$I\int_1^2 \rho\frac{dl}{S} = \int_1^2\left(\vec{E}_{кул} \cdot \vec{dl}\right) + \int_1^2\left(\vec{E}_{стор} \cdot \vec{dl}\right).$$

$$\int_1^2 \frac{\rho dl}{S} = \frac{\rho \cdot l}{S} = R_{12}$$

R_{12} - is the electrical resistance of the section of the circuit between points 1 and 2.

$$\int_1^2 \left(\vec{E}_{кул.} \cdot \vec{dl}\right) = A_{1-2}$$

A_{1-2}- numerically is equal to the work that performed by Coulomb is the force to move a unit positive charge from point **1** to point **2.**

But this work is equal to the potential difference according to the definition of :

$$A_{1\to 2} = \varphi_1 - \varphi_2 .$$

The second integral of the righthand side

$$\int_1^2 \left(\vec{E}_{стор.} \cdot \vec{dl}\right)$$

is also numerically equal to work, which performed by outside forces by displacement positive charge from point 1 in point 2 and is called the electromotive force (EFS).

Thus, Ohm's law takes the following form**:**

$$IR_{12} = (\varphi_1 - \varphi_2) + \varepsilon_{12} .$$

It is called **Ohm's law in its integral form.**

The total amount of work that performed by the Coulomb and third-party forces is called **is the electrical voltage** on the section 1 → 2 and is denoted by U 12 :

$$(\varphi_1 - \varphi_2) + \varepsilon_{12} = U_{12}$$

- electrical voltage, or drop voltage on the circuit section 1 → 2 So,

$$IR_{12} = U_{12} .$$

That is why this law is also called Ohm's law for a section of a circle.

5.7. Calculation of electrical circuit parameters

A knot is a point of the circle at which possible more than two possible current directions.

The first Kirchhoff's rule:

the algebraic sum of currents in the node is zero.

$$\sum_{k=1}^{n} I_k = 0 .$$

Currents flowing to a node are considered positive, and currents flowing from a node are considered negative.

The second Kirchhoff's rule:

in any arbitrarily chosen closed circuit of an electric circuit, the algebraic sum of voltage drops in the sections of the circuit is equal to the algebraic sum of EFS in this circuit.

$$\sum_{i=1}^{n} I_k R_i = \sum_{j=1}^{m} \varepsilon_j .$$

The procedure for calculating t h e parameters of a DC circuit:

1. The directions of currents in all parts of the circuit are randomly selected.

2. For *n* nodes, we write down the *(n-1)* independent equation according to the first Kirchhoff rule

3. Closed loops are selected and, after choosing the direction of their traversal, a system of equations is written according to the second Kirchhoff rule. If there are *n* nodes and *p* contours in a circle, there must be *(p-n+1)* independent equations.

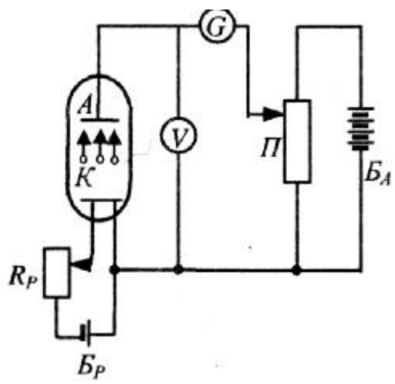
5.8.Electric current in a vacuum.

Thermoelectric phenomena. Electric current in gases

Electric current in a vacuum

Emission is the release of electrons from a metal under the influence of external factors.

The phenomenon of thermoelectronic emission is the fact that heated metals emit electrons. A conduction electron can escape from any metal when its energy (kinetic) exceeds the work required to release the electron from the metal. Thermoelectric emission results in a thermoelectric current. In practice, the phenomenon of thermoelectronic emission can be observed using a vacuum diode lamp with two electrodes, cathode *K* and anode *A*, soldered together (Fig.).



The cathode is heated by an electric current from an incandescent battery *Bp* . By adjusting the strength of the incandesce n t current using a rheostat *Rp* , the temperature of the cathode can be changed. The battery supplies the electrodes with a voltage *Ua* , the value of which can be changed adjusted with a potentiometer *P* and measured with a voltmeter *V* . *The* thermoelectronic current *I*a is measured by a galvanometer *G.*

The strength of the thermoelectronic current *I*a depends on the voltage *Ua* , which is applied between cathode and anode, the temperature of the cathode and the material of the cathode.

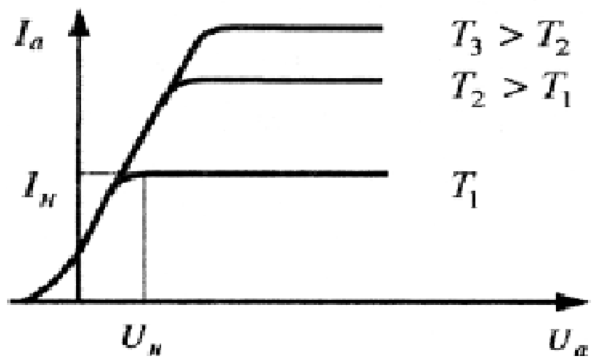


Figure shows the dependence of the thermoelectronic current *Ia* on the voltage *Ua* at different cathode temperatures. This curve is called the *voltampere characteristic of the diode.* At low values of *Ua* , the curves at different temperatures are the same. At low anode voltages, the current initially increases slowly with increasing voltage. This is because at low values of *Ua* not all electrons that have left the cathode reach the anode. Some of the electrons between the cathode and the anode form an electron cloud (spatial charge), which prevents electrons from moving to the anode, which again electrons fly out of the cathode. As the voltage *Ua* increases, the electron cloud gradually dissipates and the current *Ia* increases. At $U = U_{ан}$, the current growth stops. This is due to the fact that the number of electrons reaching the anode per unit time is equal to the number of electrons leaving the cathode during the same time.

The maximum thermoelectronic current possible at a given cathode temperature is called the ***saturation current In***.

At small values of B$U_a \ll U_н$, the dependence of the thermoelectronic current on the anode voltage is described by the Boguslavsky-Langmuir law

$$I_a = BU_a^{3/2}.$$

where B - is the coefficient, which depends on on the shape of the electrodes and their their relative positioning.

The mathematical dependence of the saturation current density is described by the classical

electron theory using the ***Risardson formula***:

$$j_н = A\sqrt{T}e^{-\frac{e\Delta\varphi}{kT}},$$

where *e, m, n are*, respectively, the charge, mass and concentration of electrons in metal, *k is* the Boltzmann constant.

Thus, according to classical electronic theory, the coefficient *A* depends on the electron concentration *n* and is different for different metals.

According to quantum theory density of current saturation is determined by ***by the Richardson-Dashman formula:***

$$j_н = BT^2e^{-\frac{e\Delta\varphi}{kT}},$$

where.

eφ

$$B = \frac{4\pi emk^2}{h^3}$$

.

$$B \approx 120\frac{A}{sm^2K}$$

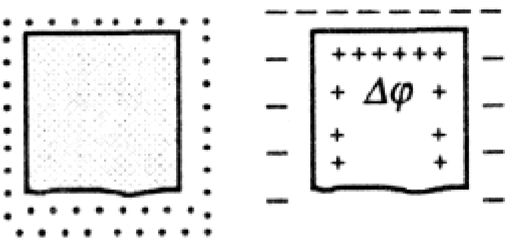
Thus, the constant *B* is the same for all metals. However, no such *B has been* found for any of the metals. The discrepancy is explained by the fact that theoretical calculations use a model of an ideal electron gas.

Both theories correctly describe the exponential dependence of the current density *jн* on temperature. The factors $T^{1/2}$ and T^2 play a secondary role, since the exponent function changes much more strongly than the power function.

5.8.1. The work of electrons leaving the metal. Contact potential difference

Conducting electrons in a metal are constantly in chaotic thermal motion. The fact that free electrons are trapped inside the metal indicates that a trapping electric field occurs in the surface layer of the metal, which prevents electrons from leaving the metal for the surrounding vacuum. To leave the metal, the electron must perform some work, called the ***work of escape***.

One of the reasons for the output work is as follows. When an electron is released from a metal by thermal motion, it induces a charge on the metal surface called the specular reflection charge. An attractive force, called the electric image force, arises between the electron and this charge, which tries to return the electron back to the metal. Another reason i s that there is a n electron cloud near the metal surface in a vacuum, which is negatively charged (Fig.).

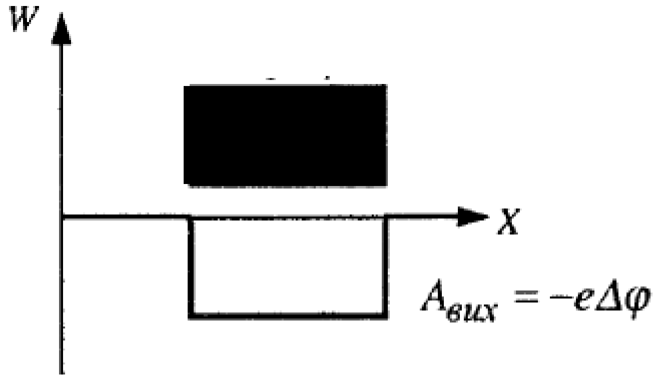


The size of this cloud is of the same order as the size of atoms *(10^{-10} m). In this case, the* metal covered by the negative electron cloud relative to vacuum is positively charged relative to the vacuum (Fig.). The positive potential of the metal interior relative to vacuum is called *internal potential* $\Delta\varphi$.

The potential energy *W* of free electrons *-e* in vacuum is considered to be is equal to zero (because $\varphi = 0$ *).* Then inside the of the metal with positive internal potential $\Delta\varphi$ the potential energy of conduction electrons is negative:

$$W = (-e)\Delta\varphi = -e\Delta\varphi .$$

Thus, free electrons in metals are located in a "flat-bottomed potential well" (Fig.) (flat because the surface the new double layer creates an electric field similar to that of a flat capacitor).



For an electron to escape from a metal into a vacuum, it must overcome a potential barrier - the double surface layer field. This requires additional energy, which must be at least as deep as the depth of the potential well.

The output work *is the value of Aвих , which is equal to the smallest pre-data energy with the opposite sign, that must be transferred to the electron conduction in the metal for it to escape into the vacuum.*

Thus, the numerical work of the output is equal:

$$A_{вих} = e\Delta\varphi .$$

If an electron in a metal is given additional energy, its kinetic energy increases. The condition for an electron to leave the metal can be written as follows:

$$\frac{m\upsilon_n^2}{2} \geq e\Delta\varphi .$$

Where υ_n the projection of the electron velocity on the normal to the metal surface.

Conduction electrons can gain additional energy when a metal is illuminated (external photoelectric effect), heated (thermoelectronic emission), exposed to a strong electric field (auto-electronic emission), or bombarded with a stream of electrons in a vacuum (secondary electron emission).

The operation of the exit depends on the chemical nature of the metal and the condition of its surface.

Dirt, moisture residues, etc., change the amount of work.

By selecting a certain surface coating, it is possible to significantly reduce *Авых* . If a layer of alkaline earth metal oxide *(Ca,Ba) is* applied to the surface of tungsten *(Авых = 4.5 eV), the* work of output is reduced to *2 eV*.

If we consider two contacting metals with work of output $A1 > A2$, then electrons will transfer more to the first metal. A potential difference $\Delta\varphi 12 = \varphi 1 - \varphi 2$ arises between the metals, which is called the **internal contact potential difference**.

This phenomenon was discovered by A. Volta and is described by Volta's laws.

The first Volta's law

When two conductors made of different metals are connected, a contact potential difference arises between them, which depends only on their chemical properties and temperature:

$$\Delta\varphi_{12} = \varphi_1 - \varphi_2 = \frac{kT}{e}\ln\frac{n_1}{n_2},$$

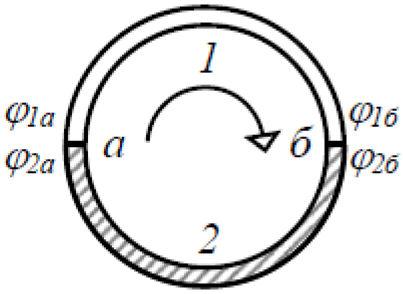
where. *k is* theBoltzmann's ;*e is* thecharge of electron;n_1 , n_2 are the concentrations of free electrons in metals.

The secondVolta's law

The potential difference between the ends of a circuit consisting of seriesconnected metal conductors at the same temperature T does not depend on the chemical composition of the intermediate conductors. It is equal to the contact potential difference that occurs when the outermost conductors are directly connected.

5.8.2. Thermoelectric phenomena

Consider an electrical circuit consisting of two mechanical conductors (Fig.).



Let's choose direction bypass contour using the arrow. On the section*a1b* there is a decrease of the voltage

$U_1=IR_1$

On the section a2b

$U_2=IR$

$$U_1=\varphi_{1a}-\varphi_{1б};\ U_2=\varphi_{2б}-\varphi_{2a}.$$

Then

$$(\varphi_{1a}-\varphi_{2a})+(\varphi_{2б}-\varphi_{2б})=\varepsilon.$$

At

$$T_a=T_б=T$$

$$\varepsilon=\frac{kT}{e}\ln\frac{n_1}{n_2}+\frac{kT}{e}\ln\frac{n_2}{n_1}=0,$$

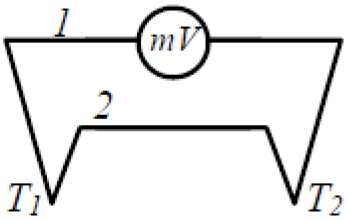
i.e., the efficiency of the circle is equal to zero . At $T_a \neq T_b$ (let $Ta>Tb$) в circle appears ***is the thermoelectromotive force*** (thermo-e.m.f.), which is equal to:

$$\varepsilon=\frac{k}{e}(T_a-T_б)\ln\frac{n_1}{n_2}=\alpha(T_a-T_б),$$

where α is the specific thermal efficiency.

This thermoelectric energy generates a thermoelectric current in the circuit. This phenomenon is used in thermocouples.

The inverse phenomenon to the thermocurrent was discovered in 1834 and is called the **Paltier effect**.



When a current flows through a circuit consisting of two different welded metals, *heat is generated* at one junction, while the other junction is *cooled*.

5.8.3. Electric current in gases

Gases, which consist of electrically neutral molecules and atoms, are ***insulators***. For electrical conductivity to occur, the gas must be ***ionised***.

Ionisation - detachme n t or attachment to neutral atom of one or more electrons to a neutral atom. The reverse process is called ***recombination***.

Methods of ionisation of gases.

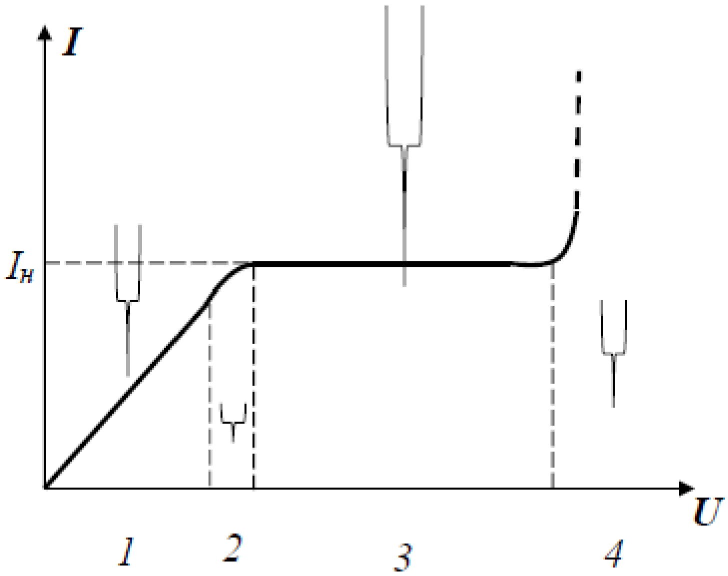
Increase in temperature.

Exposure to radiation (ultraviolet, X-ray, radioactive). Bombardment with fast electrons or ions (impact ionisation).

The minimum electron energy for impact ionisation is determined:

$$\frac{m\upsilon^2}{h^{32}} \approx A_i\left(1+\frac{m}{M}\right),$$

where *Ai* is the work of insulation, *m is the* mass of the electron, and *M is the* mass of the atom.



The process of current flow through a gas is called a **gas discharge**. If the electrical conductivity of a gas is created by an external ioniser, this phenomenon is called a *non-independent gas discharge.*

The dependence of the current *I* during an independent gas discharge on the applied voltage *U* is shown in Fig.

At site *1*, *the following is* observed the linear dependence of *I* on *U* and the current density is determined:

$$j = q_+ n_+ (U_+ + U_-) E,$$

where q_+ , n_+ are the charge and concentration of positive charges; U_+ , U_- *are the* mobility of positive and negative ions; *E is the* electric field strength.

In section *2, the* proportionality is broken b e c a u s e the concentration of ions in the gas

decreases.

At site *4, shock ionisation* occurs, accompanied by the formation of *secondary electrons* and *ions*.

The impact ionisation created by electrons is not sufficient for self-discharge. It is necessary for positive ions to ionise gas molecules or knock electrons out of the cathode metal. This is possible at high voltages *U*.

The transition of a non-self-sustaining gas discharge to a self-sustaining one is called **an electrical gas breakdown,** and the corresponding voltage $U_З$ **is called the ignition voltage**.

Gas discharges are accompanied by gas glow. Light emission occurs when gas molecules change from an excited state to a normal state, as well as when positive ions recombine with electrons (recombination luminescence).

6. Electromagnetism

6.1. Magnetic field in a vacuum. Ampere's law. The law of total current. Lorentz force.

6.1.1.Magnetic field. Magnetic induction. Ampere's law

Experiments have shown that a *magnetic field* exists around current-carrying conductors and permanent magnets, which can be easily detected by the force it exerts on other current-carrying conductors or permanent magnets.

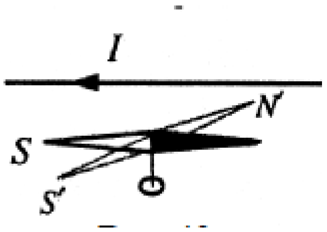
To study the basic properties of a magnetic field and how it is created, let's look at two experiments.

1. Interaction between stationary electric charges and a magnetic arrow

Suspend a dielectric ball from a thread near the magnetic arrow and give it an electric charge. We will not notice any effect of the stationary electric charges of the ball on the magnetic arrow. In turn, the magnetic field of the arrow has no effect on the charged ball. Therefore, the stationary electric charges do not produce a magnetic field and a permanent magnetic field does not act on stationary electric charges.

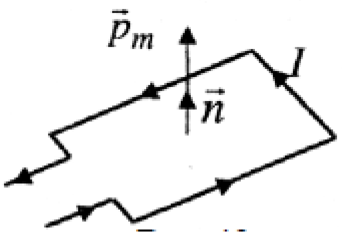
2. *Interaction between direct electric current and magnetic arrow*

If a constant current *I is* passed through the conductor, the magnetic arrow will rotate around its axis to become perpendicular to the conductor with the current (Fig.). This phenomenon G.Ersted. He discoveredthat the direction of rotation of the north pole of the arrow is reversed if the direction of current in the conductor is changed. The current in a conductor is anordered movementelectric charges. A magnetic field exists around any moving charge. At the same time The material of the conductor and the nature of its conductivity, as well as the processes taking place in it, do not play any role. Therefore, *around any moving charge, whether it i s a n electron, ion or charged body, there is a magnetic field in addition to an electric field.*



The electric field acts on both moving and stationary electric charges.

The magnetic field acts on only on electric charges moving in this field. To characterise a magnetic field, we need to consider its effect on a certain current. Consider a closed flat circuit with a current whose dimensions are small compared to the distance to the currents forming the field. The positive direction of the normal is the direction oftranslational movementdrill bit, the head of which rotates in the direction of the current flowing in the circuit (Fig.)



. A circuit with a current is characterised by a *magnetic moment* $\overrightarrow{p_m}$ *which is equal toproduct current I flowing in the circuit and the surface area of the circuit S*

$$\vec{p}_m = IS\vec{n},$$

where $\vec{n}$ *is a* unit vector of the normal to the surface of the frame. The direction of the vector$\overrightarrow{p_m}$ coincides with the direction of the positive normal of the frame.

A current loop can also be used to quantify a magnetic field. A pair of forces acts on a circuit in a magnetic field. The torque of the forces *M* depends on the properties of the circuit:

$$M \sim p_m.$$

If a circuit with currentrotated by*90°* from its equilibrium position, it will be subjected t o a maximum torque *Mmax* .

If contours with different magnetic moments are placed in a given location of the magnetic field, they will be subjected to different rotational moments, but the ratio

Mmax / pm , is the same for all circuits and is a quantitative characteristic of the magnetic field:

$$B = \frac{M_{\max}}{p_m}.$$

***The magnetic induction** at a given location in the magnetic field is determined by the maximum torque acting on a circuit with a unit magnetic moment. The* unit of magnetic induction is the tesla: *1 T is the* magnetic induction of a magnetic field in which a frame with a magnetic $1A \cdot m^2$ maximumforce o f 1 *Nm.*

The direction of induction *B of a* magnetic field is the direction of the magnetic moment of a field that is in equilibrium in that field.

To graphically represent magnetic fields, it is convenient to use magnetic induction lines.

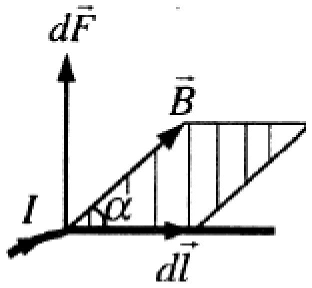
Magnetic induction lines are those lines whose tangents at each point coincide with direction the *vectorB at these points in the field.*

***The direction of the** magnetic field **induction lines is** determined by the drill bit rule:* if you screw the drill bit in the direction of current flow in the conductor, the direction of movement of its handle will show the direction of the magnetic induction lines.

Magnetic induction lines can be observed using small metal filings, which behave like small magnetic arrows in a magnetic field. Summarising the results of the study of the effect of a magnetic field on various conductors with current, Ampère found that the **force** *d F* **, with which the magnetic field acts on the element the length dl of a conductor with a current in a magnetic field is** directly proportional to the *current* strength I в the conductor i to the vector product of an element of length *dl on the* magnetic induction B :

$$\vec{d}\vec{F} = I\left[d\vec{l}, \vec{B}\right].$$

The direction of the *forcedl* can be found by the rule of vector product and by the rule of E of the left hand: if the palm of the left hand is placed so that it has lines of magnetic induction, and four outstretched fingers are pointed in the direction of the electric current in the conductor, then thumb will show the direction of the force acting on the conductor from the field. This rule is useful when a conductor element with a current is perpendicular to the direction of magnetic field.



B general case for determine direction force Amps *dl* should use the vector product rule:

the vector *dl* is directed perpendicular to the plane formed by the vectors *dl* and *B* so that from the end of the of the vector *d F* rotation of vector *dl* to vector *B* The shortest route was in an anti-clockwise direction (Fig.). The modulus of the Ampere force is calculated by the formula:

$$dF = IBdl \sin\alpha ,$$

where α - is the angle between the vectors *dl* and *B* .

Suppose that a conductor element *dl* with a current *I* is perpendicular to the nap row of the magnetic field (sin α = 1), then Ampere's law can be written as :

$$B = \frac{1}{I}\frac{dF_{\max}}{dl}$$

Thus, ***magnetic induction is a power characteristic of a magnetic field.***

6.1.2. Bio-Savar-Laplace law

In 1920, the French scientists J. Bio and F. Savard studied the magnetic fields created in the air by a straight current, a circular current, a coil with a current, etc. On the basis of numerous experiments, they came to the following conclusions:

a) in all cases, the induction *B of the* magnetic field of an electric current is proportional to the current *I*;

b) magnetic induction depends on the shape and size of the conductor carrying the current;

c) the magnetic induction *B at* any point in the field depends on the location of this point relative to a conductor with a current.

Bio and Savar tried to find a general law that would allow them to calculate the magnetic induction at every point of the field created by an electric current flowing through a conductor of any shape. However, they failed to do so.

This problem was solved by P . Laplace.

Laplace summarised the results of Bio and Savard's experiments in the form of a differential law called the ***Bio-Savard-Laplace*** law*:*

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I[d\vec{l},\vec{r}]}{r^3}.$$

where *dl* is a vector numerically equal to the length *dl of the* element conductor and coincides with the direction of the electric current, *r is* the radius vector drawn from the conductor element *dl* to are the points of the field *A* under consideration (Fig.), μ_o *is the* magnetic constant.

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I[d\vec{l},\vec{r}]}{r^3}.$$

Thus, the induction modulus *d B of the* magnetic field of a small element *dl* of a conductor with a current is directly proportional to the current *I*, the length of the element *dl of the* conductor, inversely proportional to of the square of the *distancer* from the conductor element to the considered point of the field, and also depends on the angle α between the directions current and the radius vector *r (*Fig.):

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I \cdot dl \cdot \sin\alpha}{r^2}.$$

The direction of the vector *d B* is perpendicular to *dl* and *r*, i.e. perpendicular to the plane in which they lie and coincides with the tangent to the magnetic line of induction. The direction *d B is* determined from the vector product[*dl*, *r*] and can be found by the drill rule.

The experiment shows that the ***principle of superposition is*** true for a magnetic field***:***

the induction of a magnetic field created by several currents or moving charges is equal to the vector sum of the magnetic fields created by each current or moving charge separately.

According to the principle of superposition, the magnetic induction $\vec{B}$ at any point of the magnetic field of a conductor with a current *I* is equal to the vector sum of Inductions $\Delta\overrightarrow{Bi}$ elementary magnetic fields created by individual sections of $\Delta\vec{l_i}$, this conductor:

$$\vec{B} = \sum_{i=1}^{n} \Delta\vec{B_i}.$$

By increasing the number of sections *n* indefinitelyand moving to the limit at *n*, which goes to infinity, can replace the sum with an integral:

$$\vec{B} = \int_l d\vec{B}.$$

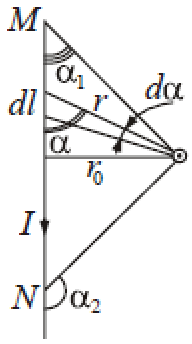
Thus, the magnetic induction of a field created in a vacuum by a *current I* flowing through a conductor of finite length and arbitrary shape is equal to

$$d\vec{B} = \frac{\mu_o}{4\pi}\int_l \frac{[d\vec{l},\vec{r}]}{r^3}.$$

In general, the calculation of magnetic field characteristics using the above formulas is quite complicated. However, if the current distribution has a certain symmetry, then the application of the Bio-Savar-Laplace law together with the superposition principle makes it possible to calculate the magnetic induction of specific fields quite simply.

6.1.3 Magnetic field of a straight conductor with current. The magnetic field of a circular current

Consider a ***straight conductor*** of arbitrary shape through which a current I flows, for example, from top to bottom (Fig.);



$$dB = \frac{\mu_0 \mu}{4\pi} \cdot \frac{Idl \sin\alpha}{r^2},$$

According to the Bio-Savar-Laplace law, the magnetic induction vector dB of a field in vacuum, created at point *A* by the element *dl of the* conductor with current, is numerically equal to

where α is the angle between the vectors *dl* ' and r.

At point *A*, which is located at a distance r_0 from the axis of the conductor, all the vectors *dB*, which characterise the magnetic fields created by individual sections of this conductor, are directed perpendicular to the plane of the figure. The vector B is numerically equal to the algebraic sum of the modules of the *dB* vectors:

$$B = \int_l dB = \int_l \frac{\mu_0 I dl \sin\alpha}{4\pi r^2}.$$

Let us replace *dl* and r by by one independent variable α

$$r = \frac{r_0}{\sin\alpha},$$

$$dl = \frac{r d\alpha}{\sin\alpha} = \frac{r_0 d\alpha}{\sin^2\alpha}.$$

As a result, the magnetic field induction of a straight conductor MN at point *A is* equal to

$$B = \int_{\alpha_1}^{\alpha_2} \frac{\mu_0 I}{4\pi} \cdot \frac{r_0 d\alpha \sin^3 \alpha}{\sin^2 \alpha\, r_0^2} = \int_{\alpha_1}^{\alpha_2} \frac{\mu_0 I}{4\pi r_0} \sin \alpha\, d\alpha .$$

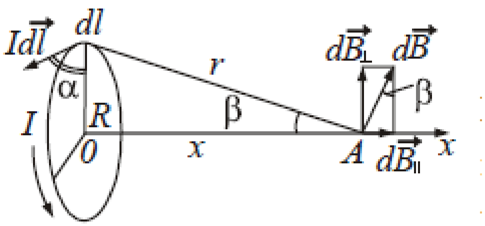
$$B = \frac{\mu_0 I}{4\pi r_0}\left(\cos \alpha_1 - \cos \alpha_2\right).$$

If the conductor MN is infinitely long, then $\alpha_1 = 0$ and $\alpha_2 = \pi$. Thus, the magnetic induction of an infinitely long conductor with a current is (cos0 = *1, cos* π *=-1)*

$$B = \frac{\mu_0 I}{4\pi r_0}\left(\cos 0 - \cos \pi\right) = \frac{\mu_0 I}{4\pi r_0} 2 ,$$

$$B = \frac{\mu_0 I}{2\pi r_0} ,$$

Let us find the induction of the magnetic field at the centre *O, a circular current of* radius R, through which the current I flows (Fig.):



$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin \alpha}{r^2} .$$

$$\vec{B} = \int_l d\vec{B}_\parallel + \int_l d\vec{B}_\perp .$$

All the vectors *dB* of the magnetic fields created at point O by different sections, *dl of the circular* current, are directed perpendicular to the plane of the figure "away from us". Then

$$B = \int_l dB_\parallel = \int_l dB \sin \beta .$$

$$r^2 = R^2 + x^2, \qquad \sin\beta = \frac{R}{r} = \frac{R}{\sqrt{R^2 + x^2}}.$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl\,R}{\left(R^2 + x^2\right)^{3/2}}.$$

$$B = \int_l \frac{\mu_0}{4\pi} \cdot \frac{Idl\,R}{\left(R^2 + x^2\right)^{3/2}} = \frac{\mu_0\, I\, 2\pi R^2}{4\pi\left(R^2 + x^2\right)^{3/2}},$$

$$B = \frac{\mu_0\, I R^2}{2\left(R^2 + x^2\right)^{3/2}}.$$

Thus, the magnetic field induction of the circular current is equal to

$$B = \frac{\mu_0\, I}{2R}.$$

6.1.4. The law of total current for magnetic fields in a vacuum. The vortex nature of the magnetic field

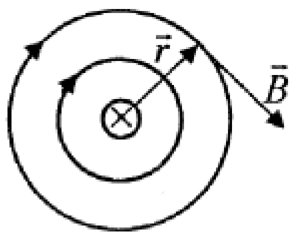
Let's introduce circulationvector magnetic induction vector.The circulation of the of the vector *B* along a closed circuit is called the integral

$$\oint_L \left(\vec{B}, d\vec{l}\right) = \oint_L B_l dl ,$$

where *dl* - is the vector of the contour length element directed along the contour traversal,

$B_l = B \cdot \cos\alpha$ *is the* projection of the vector *B* on the tangent to the contour, *α* - is the angle between the vectors *B and dl* .

Consider the magnetic field of an infinite straight conductor with current *I* in a vacuum (Fig. 5).



The lines of magnetic induction of this field are circles whose planes are perpendicular to the conductor and whose centres lie on the axis of the conductor. Find the circulation of the vector *B* along a circle of radius *r*. At all points of the circle, the vector *B is* numerically equal to

$$B = \frac{\mu_o}{4\pi}\frac{2I}{r}$$

directed along the tangent to the circle, so cos *α* = 1 .

Then

$$\oint_L Bdl\cos\alpha = \int_0^{2\pi r} \frac{\mu_o}{2\pi}\frac{I}{r}dl = \frac{\mu_o}{2\pi}\frac{I}{r}\int_0^{2\pi r} dl = \mu_o I .$$

Two conclusions can be drawn from this:

1) *the magnetic field of a direct current is a vortex field, because the circulation of the vector B along the induction lines is not equal to 0*;

2) *vector circulation B magnetic induction of a rectilinear current field is one along any induction line and is equal to μo I* .

This formula can be used to a closed circuit L of arbitrary shape, which covers infinitely long straight conductor from current *I*. If the circuit L_1 does not cover a conductor with current, then

$$\oint_{L_1} Bdl\cos\alpha = 0 .$$

In all cases, that have been discussed above, the angle *α* is acute, i.e. from the end of the of the vector of the density of the current *j* directed along the axis of the conductor в the direction of the current, by passalong the contour *L* , occurs against the arrow of the clock.

At opposite direction bypass contour *L* or in the opposite direction of current в conductor we get

$$\oint_{L_1} Bdl\cos\alpha = -\mu_o I .$$

From now on, we will use the following **rule of current signs**: *the current whose direction is related t o t h e direction of the contour traversal by the drill rule is considered* ***positive; the*** *current of the opposite direction is considered* ***negative****.*

In practice, the magnetic field is mainly created by several conductors through which currents flow I_1 , I_2 , I_3 , etc. Based on the principle of superposition, the magnetic induction *B* of the resulting field is equal to

Then

$$\vec{B} = \vec{B_1} + \vec{B_2} + \ldots + \vec{B_n} = \sum_{i=1}^{n} \vec{B_i} .$$

$$\oint_L Bdl\cos\alpha = \oint_L \sum_{i=1}^{n} B_i dl\cos\alpha = \sum_{i=1}^{n} \oint_L B_i dl\cos\alpha \,.$$

Each of the integrals under the sum sign is equal to either $\mu o\ I$, if the current is covered by the loop, or *0* if the current is not covered by the l o o p . So,

$$\oint_L Bdl\cos\alpha = \mu_o \sum_{i=1}^{n} I_i \,.$$

where *n is the* number of conductors with currents covered by a *loop L of* arbitrary shape. This equation , is the mathematical expression of the *full-current law* for the currents conductivity:

the circulation of the vector $\vec{B}$ along an arbitrary closed loop is equal to of the product of the magnetic constant μ_o , by the algebraic sum of the currents that are covered by this outline.

The obtained expression of the total current law is valid only for a magnetic field in a vacuum, since for a field in matter, molecular currents must be taken into account.

6.1.5. The power of Lorenz

The appearance of the macroscopic Ampere force acting on a conductor with a current in a magnetic field can be explained as follows. As the current flows, the charge carriers in the conductor move in a directional manner. Therefore, the magnetic field deflects them to one side. At the same time, they come into contact with the crystal lattice of the metal and transfer a certain momentum to it, which they acquired under the influence of the magnetic field. The macroscopic result of the elementary processes of collision of individual charge carriers with the crystal lattice of a conductor is the emergence of the Ampere force.

The magnetic field acts on free electrons in metal and without electric current in the conductor. Since the electrons in this ase move only randomly, the total momentum imparted by them to the crystal lattice of the conductor is zero and the conductor remains stationary.

To calculate the force acting on a moving charge in a magnetic field, consider is a conductor element *dl* with current *I* in a magnetic field with induction $\vec{B}$. For this element is subjected to an Ampere force

$$dF = B \cdot I \cdot dl \sin\alpha$$

. If the element *dl* contains *dN* free charge carriers, the force *F* per electron is equal:

$$F_Л = \frac{dF}{dN},$$

where $F_Л$ is the Lorentz force.

The number of charge carriers *dN* in the conductor element *dl* is written in terms of their concentration *n* and the volume *dV of the* element:

$$dN = ndV = nSdl,$$

S - is the cross-sectional area of the conductor. Then

$$F_Л = \frac{BIdl\sin\alpha}{nSdl} = \frac{B}{n}\frac{I}{S}\sin\alpha = \frac{Bj\sin\alpha}{n}.$$

Since according to the electronic theory

$$j = ne\upsilon$$,

then

$$F_{Л} = Be\upsilon \sin\alpha$$

or

$$F_{Л} = e[\vec{\upsilon}, \vec{B}].$$

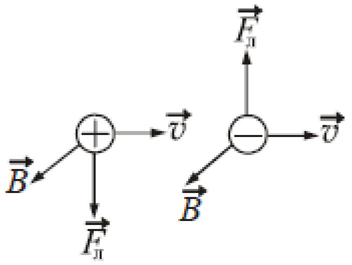
where α is the angle between the vectors $\vec{v}$ and $\vec{B}$. In the general case

$$F_{Л} = q[\vec{\upsilon}, \vec{B}].$$

Direction forces Lorentz force is determined by the **by the rule of vector product or the rule of of the left hand rule**:

if palm of the left hand is placed so that the vector $\vec{B}$*, and the four extended fingers are directed along the velocity vector v of the positive charges, then the bent 90° thumb will show the direction of the force acting on the positive charge.*

The force acts on a negative charge in the opposite direction (Fig.).



Thus, a magnetic field does not act on electric charges that do not move.

The Lorentz force is always perpendicular to t h e velocity of a charged particle, so it changes only the direction of this velocity without changing its modulus. Consequently, the Lorentz force does no work and the kinetic energy of the particle does not change when moving in a magnetic field.

If a moving electric charge is affected by a magnetic field with induction B , there is also an electric field with a strength E, the resulting force F applied to the charge:

$$\overrightarrow{F} = q\overrightarrow{E} + q[\overrightarrow{\upsilon}, \overrightarrow{B}]$$

is the Lorentz formula.

If a charged particle moves in a magnetic field with velocity *v* along the lines of magnetic induction or in the opposite direction to the direction of magnetic induction, then $\alpha = 0$ or $\alpha = \pi$. In this case, $F_L = 0$, the magnetic field does not act on the particle and it moves *evenly* and *straight*.

If a charged particle moves in a magnetic field with a velocity *v* perpendicular to the dicoupled to the vector *B,* the Lorentz force is constant modulo and normal to the particle's trajectory. The particle will move in a circle, because the Lorentz force, according to Newton's second law, will create centripetal acceleration. So,

$$q\upsilon B = \frac{m\upsilon^2}{r}.$$

From here.

$$r = \frac{m}{q}\frac{\upsilon}{B},$$

where *r is the* radius of the circle.

Using the relationship

$\upsilon = \omega \cdot r$, we find the cyclic frequency ω and the period *T* rotation of a particle around induction lines in a magnetic field:

$$\omega = \frac{\upsilon}{r} = \frac{q}{m}B,$$

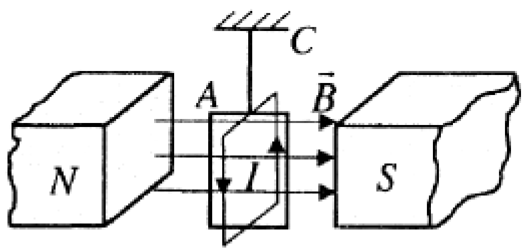
$$T = \frac{2\pi \cdot r}{\upsilon} = \frac{2\pi}{B}\frac{m}{q}.$$

.

The period of rotation of a particle in a homogeneous magnetic field does not depend on its velocity (at $v << c$). This is the basis for the operation of cyclic accelerators of charged particles.

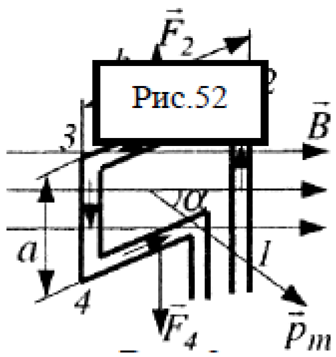
6.1.6. Circuit with current in a magnetic circuit

Consider behaviour в magnetic field of closedconductors with a current. Let us place в homogeneous magnetic field electromagnetconductor, which bent у the form of a rectangular frame *A,* which is suspended on a elastic thread *C* (Fig. 8).



If в frame there is no of current, it is in state of indifferent equilibrium. If through the frame is passed through the a constant electric current then it rotates around axis of the thread *C* so that its plane is located is perpendicular to to vector *B* of magnetic induction of the field. The frame with current Fig is set у external homogeneous magnetic field в that position, at which own magnetic moment $\overrightarrow{p_m}$ *of the* frame coincides with direction $\vec{B}$. *3* the end of this vector we see that the current у frame is going against hand of the clock.

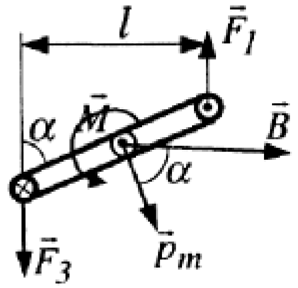
Let's find the expression for the moment of forces, that are acting on a rectangular frame *1-2-3-4* with current I, which is in a homogeneous magnetic field, the vector of magnetic induction vector *B* forms angle *α* з with the vector *pm* of the intrinsic magnetic moment frame (Fig.).



The sides of the frames *2-3* і *4-1* lie in planes, parallel to the induction of the external magnetic field induction *B* .

The forces F_2 and F_4 are directed along the vertical axis of the frame in opposite sides. They deform the frame in the vertical direction.

The sides of the frame *1-2* and *3-4* are perpendicular to the vector *B* magnetic field induction (Fig.).



The forces F_1 and F_3, which are applied to n Fig. of conductors *1-2* and *3-4 are* numerically equal:

$$F_1 = F_3 = IaB\sin(\vec{a}\,\hat{,}\,\vec{B}) = IaB\sin\frac{\pi}{2} = IaB.$$

The resulting torque *M* acting on the frame is equal to the moment of the bunk forces F_1 and

$$\vec{F}_2 = -\vec{F}_1,$$

i.e

$$M = F_1 l,$$

, where

$$l = b\sin\alpha$$

Then.

$$M = IabB\sin\alpha = ISB\sin\alpha,$$

where. *S* = *ab*-. *is the* area of the frame, *IS* = *pm is the* numerical value of the magnetic moment vector of the frame with current. Then

$$M = p_m B\sin\alpha = p_m B\sin(\vec{p}_m\,\hat{,}\,\vec{B})$$

The frame rotates under the action of a pair of forces F1 and F2 around the vertical axis, which is perpendicular to both *the* vector *B* and the vector *pm* . *The* vector *M* is directed towards the observer perpendicular to the drawing plane.

The vector of rotational torque $\vec{M}$ acting on a frame with a current in a magnetic field is equal to the vector product of the magnetic moment pm of the frame and the magnetic induction $\vec{B}$ of the external field:

$$\vec{M} = [\vec{p}_m, \vec{B}].$$

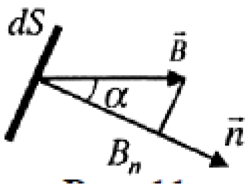
The torque *M* is zero and the circuit is in equilibrium if the magnetic moment of the circuit is parallel or antiparallel to the direction of the external *field (sinα* = 0). Only such a position of the circuit is stable when vectors $\overrightarrow{p_m}$ and $\vec{B}$ are parallel to each other.

6.2. Magnetic flux. The Ostrogradsky-Gauss theorem

The flux of the magnetic induction vector (magnetic flux) through the plane dS is a scalar physical quantity equal to

$$d\Phi_B = B_n dS = (\vec{B}, d\vec{S}).$$

where $B_n = B\cos\alpha$ is the projection of the vector B in the direction of the normal to the plane dS (α - is the angle between the vectors n and B) (Fig.)



$$dS = dS\vec{n}$$

I s the vector, the modulus which is equal to *dS*, and the direction coincides with the normal to the plane.

The flow of vector *B* can be either positive or negative depending on the *cosα* (determined by the choice of the positive direction of the normal *n*).

The flux of the magnetic induction vector Φ_B *through* an arbitrary surface *S* is equal

$$d\Phi_B = \int_S B_n dS = \int_S (\vec{B}, d\vec{S}).$$

For a homogeneous field and a flat surface placed perpendicular to the vector *B* , *Bn* = *B* = *const* and $\Phi_B = BS$.

Let us calculate the flux of the vector B *through the* solenoid . Inside the solenoid, the induction of a uniform field in vacuum is equal to

$$B = \frac{\mu_o NI}{l}$$

Magnetic flux through one turn of a menoid of area *S*:

$\Phi_1 = BS.$

The total magnetic flux through the solenoid, called flux cohesion T, is equal to:

$$\Psi = \Phi_1 N = NBS = \mu_o \frac{N^2 I}{l} S$$

In electrodynamics, ***the Ostrogradsky-Gauss theorem for the magnetic field is*** proved*: the magnetic flux through an arbitrary closed surface is zero:*

$$\oint_S B_n dS = 0$$

This theorem is a consequence of the fact that there are no magnetic "charges" in nature and the lines of induction of any magnetic field are closed curves.

6.3. Work of moving a conductor and a circuit with a current in a magnetic field

An ampere force acts on a conductor carrying a current in a magnetic field. If the conductor is not fixed, then under the influence of the Ampere force it will move in a magnetic field.

Let's calculate the work *dA* done by the Ampere force when moving an element *dl of a* conductor with a current *I* in a magnetic field (Fig. 12). The element of the conductor moves in the direction of the force *dF* acting on it. The work *dA is* equal to:

$$dA = dFdx.$$

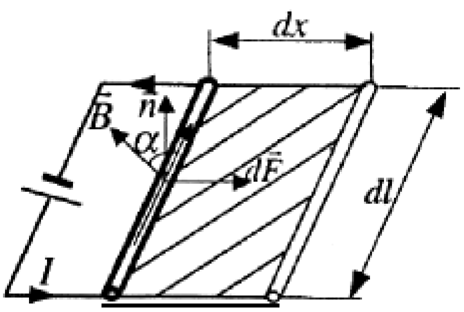
According to Ampere's law, then

$$dF = IBdl\sin\alpha.$$

$$dA = IB\sin\alpha dldx.$$

Force *dF* and displacement *dx* are directed perpendicular to the conductor element *dl* .

The product dl ·dx= dS is the surface area described by the conductor element *dl* at its movement by *dx.*



From Fig. shows that $B\sin\alpha = B_n$ *is the* projection of the vector *B* onto the direction of the normal *n* to the plane *dS.*

The product BdS= dFB_n is the magnetic flux through the surface *dS.* Then

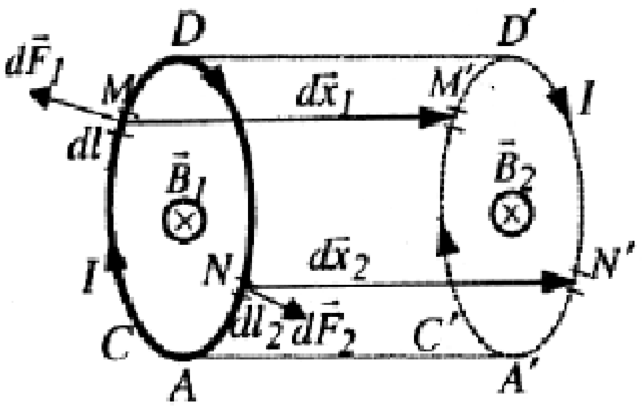
$$dA = IB_n dS = Id\Phi_B.$$

Assuming the current to be constant and integrating this expression, we obtain

$$A = I\Phi_B \, .$$

The work performed by an ampere force when a conductor with a constant current is crossed in a magnetic field is equal to the product of the current force and the magnetic flux through the surface described by the conductor during its movement.

Let's find the expression for the work done by the Ampere force when moving in magnetic field of a closed circuit through which a direct current *I* passes. Suppose that due to an infinitesimal displacement, the contour *C* has taken the position C^1(Fig. 13).



Let's imaginatively divide the circuit *C* into two connected by its the ends of the *AMD* and *DNA* conductors. The total work *dA* performed by the Ampere forces during the considered movement of the circuit is equal to the algebraic sum of the work of moving the *AMD* conductors *(dA1) and DNA*

(dA_2), i.e. $d A = dA_1 + dA_2$.

Suppose that the vector *B of* magnetic induction is directed perpendicular to the plane of the figure and at the initial position of the contour is equal to *B1* and at the final position - B_2, with $B_2 > B_1$ **.**

The ampere force $\overrightarrow{dF_2}$ acting on an arbitrary element dl_2 forms an acute angle with in the direction of its movement dx_2 and performs positive work.

The force dF_1 acting on the element *dl1 of the AMO* conductor forms with the direction of its displacement dx_1 is an obtuse angle and performs negative work, so the work dA_1 and dA_2

The displacements of the *AMD and DNA* conductors have different signs. To get the absolute the values of work *dA1* and *dA2* , it is necessary to differentiate the expression

$$A = I\bar{\Phi}_B .$$

Therefore.

$$dA = (dA_1) + (dA_2) = -Id\Phi_{B_1} + Id\Phi_{B_2} = I(d\Phi_{B_2} - d\Phi_{B_1}) ,$$

$d\Phi_B$- magnetic flux through the surface ***AMDD¹M¹A¹;*** $d\Phi_{B2}$ - through the surface *ANDD¹N¹A¹*

$d\Phi_{B_2} - d\Phi_{B_1} = d\Phi_B$ - change in the magnetic flux penetrating the surface, limited by the contour, when the contour is moved from position *C* to position C^1. *The* final expression for the elementary work *dA* is

$$A = Id\Phi_B$$

By integrating this expression, we find the work *A* performed by the Ampere force at any movement of the circuit in a magnetic field

$$A = I\Delta\Phi_B .$$

The work done by a force of Ampere when moving a closed circuit through which a direct current flows i n a magnetic field is equal to the product of the current strength and the change in magnetic flux through the surface bounded by the circuit.

Lecture 11. The phenomenon of electromagnetic induction.

Inductance. Energy of the magnetic field.

6.4.The phenomenon of electromagneticinduction.

6.4.1.Lenz's lawlaw of electromagnetic induction (Faraday's law)

After Ersted's discovery that a magnetic field exists around conductors with current, it was natural to raise the question of the possibility of generating electric current in conductors using a magnetic field. This question was solved by M. Faraday, who in 1831 showed that an electric current arises in a closed conductor at any change in the magnetic flux through the area covered by this conductor.

The phenomenon of electromotive force in a conductor when the magnetic field penetrating the conductor's contour area changes is called electromagnetic induction. If a conductor is closed, an electric current will flow in it. The current that occurs in a conductor during electromagnetic induction is called induction current.

The occurrence of induction current is always associated with changes in the magnetic flux of the conductor circuit. These changes can occur for a variety of reasons, including

- movement of a permanent magnet relative to a fixed conductor;
- movement of the contour relative to a stationary magnet;
- short-circuiting and opening current в winding of a stationary electromagnet placed near a conductor;
- relative movement of the circuit and the electromagnet;
- change in the magnetic field induction of an electromagnet (core removal at a constant current in the winding or changing the current by a rheostat);
- change of the current direction in the electromagnet winding by the commutator;
- constant movement of the contour in a heterogeneous magnetic field;
- rotational movement of the contour in a homogeneous magnetic field.

Consequently, an induction current in a closed loop only occurs when the magnetic flux passing through the area covered by the loop changes.

Faraday found that the direction of induction current in a conductor depends on the nature of the change (increase or decrease) in the magnetic flux ($\Delta\Phi > 0$ or $\Delta\Phi < 0$) through its contour. If the galvanometer arrow deflects to one side when a permanent magnet is inserted into the coil, it deflects to the opposite side when the magnet is removed.

A general rule that can be used to determine the direction of induction current in a closed conductor was formulated by E.H. Lenz:

The induction current in a closed conductor always has a direction such that the intrinsic magnetic flux generated by this current counteracts the changes in the external magnetic flux that excite the induction current.

Using Lenz's law to ***determine the direction of the induction current, we*** must:

1) find the cause of the induction current;

2) assume that the induction current opposes this cause, find the direction of its magnetic field;

3) determine the direction of the induction current by the direction of its magnetic field. From Lenz's law, it can be established that the energy of induction current in a conductor is formed by due to of theenergy which is spent to overcoming the magnetic field of the induction current.

Thanks to the phenomenon of electromagnetic induction, mechanical energy can be converted into electrical energy and electrical energy can be transferred from one circuit to another.

The induction current *I,* in a closed conductor with resistance *R,* arises under the action of ε_i , which can be expressed by Ohm's law

$$\varepsilon_i = I_i R \ .$$

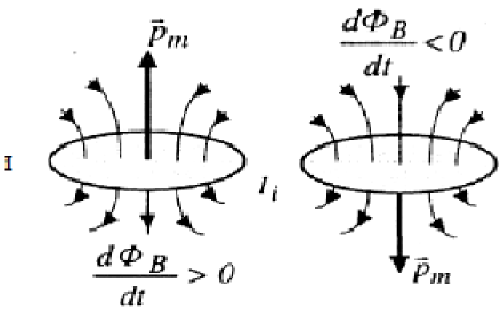
ISince the same EFS in conductors with different resistance creates unequal currents, it is more convenient to quantify the phenomenon of electromagnetic induction use the value of ε_i rather than t h e induction current *Ii* .

Studies of induction current in circuits of various shapes and sizes have shown that the electromagnetic induction ε_i , in the circuit is proportional to the rate of change magnetic flux Φ through a surface bounded by this circuit

(Faraday's law):

$$\varepsilon_i = k\frac{d\Phi_B}{dt}.$$

The electromagnetic induction EFS in a circuit is considered positive if the magnetic moment $\overrightarrow{pm}$ corresponding induction current forms an acute angle with the lines of the magnetic induction of the field that induces this current (Fig.).



Then for the case shown in the figure on the left, ε*i* < 0, and for the one on the right, ε*i* > 0. In the SI system, *k* = -1 and

$$\varepsilon_i = -\frac{d\Phi_B}{dt}.$$

The "-" sign is a mathematical expression of Lenz's Rule.

This formula, which combines Faraday's and Lenz's laws, is a mathematical expression of the basic law of electromagnetic induction:

the electromotive force of electromagnetic induction in a closed circuit is numerically equal and opposite in sign to the rate of change of the magnetic flux through the surface bounded by the circuit.

If the induction EFS occurs when the magnetic flux permeating a coil of *N* turns changes, then its value will be respectively *N* times greater than for one turn, i.e.

$$\varepsilon_i = -N\frac{d\Phi_B}{dt}.$$

Based on the law of electromagnetic induction, the unit of magnetic flux can be defined as a Weber flux: *1Bb* is a magnetic flux such that when it decreases to zero within *1 s, an* induction EFS occurs in the circuit it penetrates в *1B*.

6.4.2.The phenomenon of self-induction. Inductance

A magnetic field exists around any conductor with a current, magnetic induction *dB* which is at a point lying at a distance *r* from the element *dl of the* current loop *I*, is equal to

$$d\vec{B} = \frac{\mu\mu_o}{4\pi}\frac{I}{r^3}[d\vec{l},\vec{r}].$$

By integrating along the entire length of the circuit *l*, we find the magnetic induction $\vec{B}$ of the result field:

$$\vec{B} = \oint_l d\vec{B} = \frac{\mu\mu_o}{4\pi} I \oint_l \mu \frac{[d\vec{l},\vec{r}]}{r^3}.$$

If this contour is bounded by a surface *S*, then the magnetic flux generated by the contour's own magnetic field through the surface *S is* equal to

$$\Phi_B = \int_S B_n dS = \left\{\frac{\mu_o}{4\pi}\int_S dS \oint_l \frac{\mu}{r^3}[d\vec{l},\vec{r}]\right\} I = LI,$$

where *L* is a value called the ***inductance of the circuit***.

From this formula, it follows that the inductance of a circuit depends on the geometric shape of the circuit, its size, and the magnetic permeability of the medium in which it is located.

The inductance of a circuit is numerically equal to the magnetic flux generated by a unit current in the circuit.

In this case, it i s assumed that there are no other magnetic fields other than the magnetic field generated by the current in the circuit under consideration.

The unit of inductance is the henry (*H*): *1Gn is the* inductance of a circuit whose magnetic flux at a current of *1A is* equal to *1Vb.*

Let's calculate the inductance of the solenoid:

$$L = \frac{\Phi_{B_{con}}}{I} = \frac{N\Phi_B}{I},$$

where *N is the* total number of turns of the solenoid, *FB is the* magnetic flux through the area *S* bounded by one turn:

$$\Phi_B = BS = \mu\mu_o nIS = \mu\mu_o \frac{NI}{l} S$$

where *l* is the length of the solenoid.

Therefore, the inductance of the solenoid is equal to

$$L = \mu\mu_o \frac{N^2}{l} S = \mu\mu_o n^2 V,$$

V, where *V* = *Sl* is the volume of the solenoid.

When an alternating current passes through the circuit, the magnetic flux changes and an induction EFS is induced in the circuit.

The occurrence of an induction EFS due to a change in current in a circuit is called self-induction. In this case, the induction EFS is called the self-induction EFS*:*

$$\varepsilon_c = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(LI).$$

If the contour is not deformed and the magnetic permeability of the medium does not change, then *L=const* and

$$\varepsilon_c = -L\frac{dI}{dt}.$$

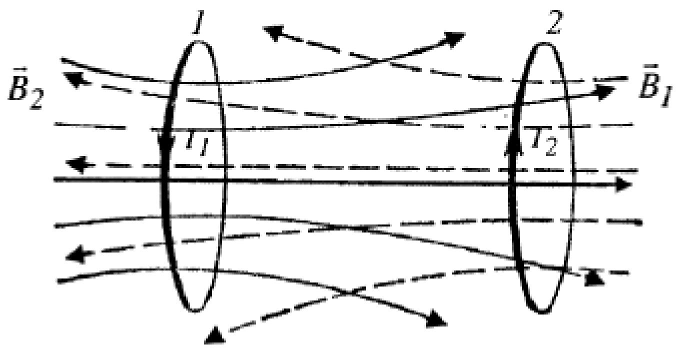
The "-" sign, caused by Lenz's rule, shows that the presence of inductance in a circuit slows down the change in current in it. In other words, the inductance of a circuit is a measure of its inertia with respect to current changes.

If the current increases over time, then $\frac{dl}{dt} > 0$ and $\varepsilon_i < 0$, i.e., the self-induction current is directed towards the current caused by an external source and inhibits it its growth. If the current decreases over time, then

$\frac{dl}{dt} < 0$ and $\varepsilon_i > 0$, *i*.e. the inductive current flows in the same direction as the downstream current in the circuit and slows its decline.

6.4.3. The phenomenon of mutual induction

Let us consider two fixed circuits with inductances *L1* and *L2* , which are located at a sufficiently are close to each other (Fig).



If a current *I1* flows in circuit *1,* the magnetic flux generated by this current is proportional to *I1* . Let Φ_{B21} denote the part of the flow that penetrates the contour 2. Then $\Phi_{21}=M_{21}I_1$, where *M*21 - proportionality factor. If the current *I1* changes, then in the office *1* induction of EFS ε_{i2} , which by law Faraday equals the rate of change magnetic flux Φ_{B21} :

$$\varepsilon_{i2} = -\frac{d\Phi_{21}}{dt} = -M_{21}\frac{dI_1}{dt}.$$

.

Similarly, when current I_2 flows in circuit 2, the magnetic flux penetrates the first circuit, If Φ_{B12} is a part of the flux that penetrates circuit *1*, then

$$\Phi_{B21} = M_{12}I_2.$$

If the current I_2 changes, the EFS εi1 is induced in circuit

$$\varepsilon_{i1} = -\frac{d\Phi_{B12}}{dt} = -M_{12}\frac{dI_2}{dt}.$$

Circuits *1* and *2* are called connected.

The phenomenon of EFS occurrence in one of the circuits when the current in the other is changed is called mutual induction.

The coefficients M_{21} and M_{12} are called the mutual inductance of the circuits. Calculations show that

$$M_{21} = M_{12}.$$

The coefficients M_{21} and M_{12} depend on the geometric shape, size, relative position of the circuits and on the magnetic permeability of the environment surrounding the circuits.

6.5. Magnetic field energy

A conductor through which an electric current flows is always surrounded by a magnetic field, and the magnetic field appears and disappears with the appearance and disappearance of the current. Therefore, part of the current energy is used to create the magnetic field.

Energy magnetic field is equal to the work, which isconsumed by the current to create this field.

Let's calculate the energy of the magnetic field of a current in the simplest case of an isotropic medium in which the relationship between induction and field strength is linear. To do this, consider a solenoid with *N* turns and an inductance *L*. If the current in the solenoid increases by *dI* in time *dt*, then its own magnetic flux changes by *dΦB* . If at time *t* the current in the solenoid was *I*, then when the magnetic flux changes by *dΦ, the* current source performs additional work *dA:*

$$dA = Id\Phi_B \, .$$

Since the solenoid remains stationary, this elementary work *dA is* associated with a change in the solenoid energy, which is caused by the presence of a magnetic field in it, by the value $dW_м$:

$$dW_м = dA \;\; ; \;\; dW_м = Id\Phi_B \, .$$

Since

$$d\Phi = LdI \text{ , то } dW_м = LIdl \, .$$

$$\int_0^{W_м} dW_м = \int_0^1 LIdI \; ; \qquad W_м = \frac{LI^2}{2} \, .$$

This is the energy consumed by the current source to generate a magnetic field in the solenoid. According to the law of conservation of energy, this energy is equal to the energy of the magnetic field *W,* which is associated with the current *I* flowing through a conductor with inductance *L.*

The study of t h e properties of alternating magnetic fields proved that the magnetic field energy is localised in space.

The energy of the current magnetic field can be determined through the characteristics of this field - the value of its strength *H* and induction *B.* To do this, consider a special case – a homogeneous magnetic field inside a long solenoid, induced by a whose

$$L = \mu\mu_o n^2 V$$

. Then.

$$W_м = \frac{LI^2}{2} = \frac{1}{2}\mu\mu_o n^2 I^2 V .$$

Magnetic field induction inside a long solenoid

$$B = \mu\mu_o In$$

. Hence

$$In = \frac{B}{\mu\mu_o} .$$

Then

$$W_м = \frac{1}{2}\frac{B^2}{\mu\mu_o}V = \frac{1}{2}BHV .$$

which takes into account that

$$B = \mu\mu_o H .$$

The magnetic field of the solenoid is homogeneous and concentrated inside the solenoid, and the field energy is distributed in it with a constant bulk density *wм* , which is equal to

$$w_м = \frac{W_м}{V} = \frac{BH}{2} = \frac{B^2}{2\mu\mu_o} = \frac{\mu\mu_o H^2}{2} .$$

The resulting expression for $w_м$ differs from the expression for the energy density of an electric field only in that the electric quantities are replaced by the corresponding magnetic ones. In the case of an inhomogeneous magnetic field, its energy in a certain volume *V* can be determined as follows. Let us divide the volume *V* into infinitesimal elements *dV* so that the field in each of them can be considered homogeneous. Then the energy of a volume element with local density $w_м$ in it is equal:

$$dW_м = w_м dV .$$

Integrating this expression over the entire volume of the field *V*, we obtain a formula for calculating the energy of an inhomogeneous field:

$$W_м = \int_V w_м dV .$$ [5,6]

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