

**Ministry of Education and Science of Ukraine  
Central Ukrainian National Technical University**

Compiled by: Huryevska O.M., Kovalov S.H.

**PHYSICS PROBLEM BOOK Part 1**  
**Mechanics. Electricity. Magnetism. Oscillations.**

Study Guide

Kropyvnytskyi 2026

**UDC 53 (075.8)**

Approved by the Academic Council of CNTU (Minutes No.10, Dated 25.05.2026 ) as a study guide for students of engineering specialities.

Huryevska, O. M., Kovalov, S. H. (2026). Physics Problem Book. Part 1. Mechanics. Electricity. Magnetism. Oscillations: study guide [Electronic edition]. Kropyvnytskyi: Central Ukrainian National Technical University. – 82 p.

This study guide contains a collection of physics problems covering mechanics, electricity, magnetism, and oscillations. Its aim is to help students master the fundamental physical concepts, laws, and develop skills for solving typical problems. The guide is recommended for students of higher educational institutions studying the general course of physics. Electronic edition.

## Contents

|   |     |
|---|-----|
| Kinematics                                      | 3   |
| Dynamics and Forces                             | 8   |
| Conservation Laws in Mechanics                  | 13  |
| Mechanics of a Rigid Body                       | 18  |
| Fluid and Gas Mechanics                         | 21  |
| Non-inertial Reference Frames. Inertial Forces. | 26  |
| Elements of Special Relativity                  | 29  |
| Elastic Properties of Solids                    | 32  |
| Law of Universal Gravitation                    | 36  |
| Electrostatics                                  | 39  |
| Electric Current                                | 48  |
| Magnetic Field                                  | 58  |
| Sound Oscillations                              | 71  |
| Electrical Oscillations. Alternating Current.   | 75  |
| References.                                     | 144 |

## KINEMATICS

### Key Formulae

**Average velocity** of a body or material point:

$$v = \Delta r / \Delta t$$

$\Delta r$  — change of position vector over time.

Instantaneous velocity:

$$v = \lim(\Delta t \rightarrow 0) \Delta r / \Delta t = dr/dt$$

Magnitude of the velocity vector:

$$v = |v| = \sqrt{[(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2]} = ds/dt$$

$ds$  — instantaneous value of the path element.

Average acceleration:

$$a = \Delta v / \Delta t$$

Instantaneous acceleration:

$$a = dv/dt = d^2r/dt^2$$

**Total acceleration** of a material point in curvilinear motion:

$$a = a_n + a_\tau = n(v^2/R) + \tau(dv/dt)$$

$a_n$  and  $a_\tau$  — normal and tangential accelerations;  $n$  — unit vector perpendicular to trajectory;  $R$  — radius of curvature.

Direction of the acceleration vector:

$$\operatorname{tg} \alpha = a_n / a_\tau$$

$\alpha$  — angle between total and tangential acceleration vectors.

Angular velocity and acceleration:

$$\omega = d\varphi/dt, \quad \varepsilon = d\omega/dt$$

Relationship between linear and angular velocity:

$$v = [\omega, R]$$

Period of revolution:

$$T = 2\pi/\omega = 1/\nu$$

$\nu$  — frequency of revolution.

Accelerations for uniform circular motion:

$$a_n = \omega^2 R, \quad a_\tau = \varepsilon R$$

## Problems

1. In what case does the distance travelled by a material point equal the magnitude of its displacement vector?
2. The position of a point in space is described by:  $x = 4t$ ,  $y = 3t$ ,  $z = 0$ . Determine the functional dependence of the distance travelled on time. Calculate the total distance covered in 5 seconds.
3. A motorcyclist covered the first quarter of the route at  $v_1 = 10$  m/s, the second at  $v_2 = 15$  m/s, the third at  $v_3 = 20$  m/s, and the last at  $v_4 = 5.0$  m/s. Find his average speed over the entire route.
4. A car travelled one quarter of the total time at 16 m/s and the rest at 8 m/s. Find the average speed.
5. A passenger on a train moving at 15 m/s observes an oncoming train of length 210 m passing him in 6.0 s. Find the speed of the oncoming train.
6. Find the speeds of a cyclist and a pedestrian if, when moving in the same direction, the pedestrian falls behind by  $s_1 = 210$  m every minute, and when moving toward each other, the distance decreases by  $s_2 = 780$  m in 2 minutes.
7. If the escalator is stationary a passenger covers its length in  $t_1 = 120$  s at his normal pace. With the escalator moving and the passenger walking at the same personal speed, the total ascent time decreases to  $t_2 = 30$  s. How long does the ascent take for a stationary passenger on the moving escalator?
8. A boat travels a river from one point to another and back. By how many times is the upstream travel time greater than downstream, if the current speed  $v_1 = 2.0$  m/s and the boat speed relative to water  $v_2 = 10$  m/s?
9. Find the speed of a motorboat relative to water if its speed downstream is 10 m/s and upstream is 6.0 m/s. What is the river current speed?
10. A plane flies between two cities 1000 km apart. Find the total flight time if the headwind speed is 25 m/s and the plane's speed relative to still air is 250 m/s. Also find the flight time with a tailwind.
11. A bus moves at  $v_1 = 15$  m/s. Raindrops leave traces on its windows at an angle  $\alpha = 30^\circ$ . The vertical component of the raindrops' velocity

relative to the ground is  $v_2 = 10$  m/s. Determine the actual speed of the rain.

12. A plane makes a flight between two points  $S = 500$  km apart. Plane speed relative to air  $v_1 = 100$  m/s. Wind blows against the plane at angle  $\alpha = 30^\circ$  to its motion at speed  $v_2 = 30$  m/s. Find the flight time.
13. What is the maximum walking speed of a person in rain so that drops do not hit his legs, if the umbrella is held at height 2.0 m and its edge extends 0.30 m forward? Drops fall vertically at 8.0 m/s.
14. A turboprop aircraft covered 700 km in 1.5 hours. Find the wind speed if its direction makes  $\alpha = 90^\circ$  with the plane's motion, and the plane's speed relative to air is 200 m/s.
15. Two submarines move toward each other, each at speed  $v$ . An ultrasonic signal sent from the first submarine returns after time  $t$ . Signal speed  $c$ . At what distance were the submarines when the signal was sent?
16. Two rowers must cross a river from point A to point B. The first rows directly toward B but arrives at point D; he then moves upstream along the bank to B. The second rows so as to arrive directly at B. Who gets to B faster and by how many times, if both row at  $v_1 = 2.0$  m/s relative to water and current speed is  $v_2 = 1.2$  m/s?
17. Motion of a material point:  $x = a + bt + ct^2$  where  $a = 5.0$  m,  $b = 0.20$  m/s,  $c = 0.01$  m/s<sup>2</sup>. Find the velocity at  $t_1 = 2.0$  s and  $t_2 = 5.0$  s, and the average velocity between them.
18. Find the trajectory of motion given by:  $x = 4t^2 - 3$ ;  $z = 0$ . Plot the motion in the  $(x, y, z)$  plane and indicate coordinates at  $t = 0$  and  $t = 1$  s.
19. Motion:  $x = 2t^2 - 2t + 2$ ;  $z = 4t^2 - t + 3$ . Find the distance at  $t = 2$  s if at  $t = 0$  the point was at the origin.
20. Two bodies A and B move according to given graphs. They move on a straight line from the same starting position. After what time do they have the same displacement again?
21. Straight-line motion:  $r = 3t^3 \mathbf{j} + 8t \mathbf{k}$ . Find the distance covered in the first 4 seconds.
22. An object moves in a straight line:  $v_x = 0.2 - 0.1t$ . If at  $t = 0$  the coordinate  $x = 1$ , what is its coordinate after 10 s?

23. A graph of the lift cabin's vertical velocity versus time is given. Find the acceleration and plot the acceleration vs time graph.
24. A plane moves uniformly at 100 m/s. Find the distance covered in 600 seconds.
25. A car moves at 20 m/s. Braking occurs over  $s = 40$  m reducing speed to 10 m/s. Find the braking time and the deceleration magnitude.
26. A car moves uniformly at 20 m/s, then decelerates uniformly over  $s = 10$  m. Find the deceleration and braking time.
27. Find the distance covered by a point in uniform circular motion with radius  $r = 10$  m over  $t = 10$  s if  $v = 2$  m/s.
28. A pile-driving weight is raised uniformly at  $v = 2$  m/s, then falls onto the pile. Find the lifting height  $h$ .
29. A body thrown vertically upward passes the 45 m mark twice with a 5.5 s interval. Find the initial speed. Neglect air resistance.
30. Find the bullet's speed if, when fired horizontally, the hole on a second paper sheet 20 m away is  $h = 5$  cm lower than on the first.
31. A projectile is fired at  $60^\circ$  above the horizontal at 90.4 m/s. The active phase ends at the highest point. How long was the active phase?
32. At what angle to the horizontal must a water jet be directed so that the maximum height equals the horizontal range?
33. Prove that maximum range is achieved at  $\alpha = 45^\circ$  (in the absence of air resistance).
34. At what angle was the body thrown if the maximum height is  $\frac{1}{4}$  of the range? Neglect air resistance.
35. A body is thrown horizontally at 10 m/s from a tower of height 19.6 m. Write the trajectory equation. What angle does the velocity make with the horizontal at the moment of landing?
36. Two bodies are thrown simultaneously from the same point with equal initial speeds of 10 m/s, at angles of  $30^\circ$  and  $60^\circ$ . Find the distance between them after 2.0 s.
37. A body is thrown at 20 m/s at  $45^\circ$ . At what height is the velocity directed at  $30^\circ$  to the horizontal? Neglect air resistance.
38. After what time does the velocity vector of a body thrown at  $\alpha = 60^\circ$  with  $v_0 = 20$  m/s make angle  $\beta = 30^\circ$  with the horizontal? Neglect air resistance.

39. A body is thrown horizontally at 19.6 m/s from a mountain top. Find the tangential and normal accelerations after 2.0 s. What angle does the total acceleration make with the velocity?
40. A material point:  $x = 3\sin(\omega t)$ ,  $y = 3\cos(\omega t)$ . Write the trajectory equation. Find path as a function of time ( $s = 0$  at  $t = 0$ ).
41. Material point:  $x = 3\sin(\omega t)$ ,  $y = 2\cos(\omega t)$ . Find the trajectory and displacement over a given time interval.
42. Find the average flight speed of a plane that left Kyiv at 6:00 local time and arrived in Paris at 6:00 local time. Assume both cities are at latitude  $40^\circ$ .
43. Two thin discs are mounted on a horizontal shaft rotating at 2000 rpm, spaced 20 cm apart. A bullet passing through them leaves holes at the same distance from the axis. Find the bullet speed if the angle between holes is  $18^\circ$ .
44. A tractor turning at 24 km/h traces an arc of radius  $R = 9.0$  m with its centre of mass. Find the wheelbase if the minimum turning radius is 1.5 m.
45. A cart moves along the X axis at  $v = 5.0$  m/s. A disc of diameter  $D = 20$  cm mounted on it rotates at 10 rev/s. The bottom of the disc moves in the same direction as the cart. In what direction does the top of the disc move relative to the ground?
46. At what speed was a cart moving during filming if wheels with 10 or 12 spokes appear stationary on film? Wheel diameter 1.0 m, filming rate 30 frames/s.
47. A car wheel stops after braking from  $v = 60$  km/h over  $t = 3.0$  s. Wheel diameter 0.6 m. Find the average angular deceleration.
48. A bullet makes one full turn inside the barrel. Find its average angular velocity and angular acceleration if muzzle velocity  $v = 860$  m/s and barrel length  $l = 1.0$  m.
49. A rigid body rotates about the fixed Z axis with  $\varphi(t) = \alpha t - bt^2$  where  $\alpha = 20$  s $^{-1}$  and  $b = 1$  s $^{-2}$ . Find angular velocity and angular acceleration as functions of time. Plot them.

## DYNAMICS AND FORCES

### Key Formulae

Momentum:

$$P = mv$$

Newton's second law:

$$F = ma = dP/dt$$

F — resultant force; m — mass; a — acceleration; P — momentum.

Centre of mass of a system of particles:

$$r_c = \Sigma(m_i r_i) / \Sigma m_i$$

Equation of motion with variable mass:

$$m dv/dt = u dm/dt$$

u — velocity of the mass being ejected.

Tsiolkovsky formula:

$$v = u \ln(M_0/M)$$

u — exhaust gas speed;  $M_0$  — initial rocket mass; M — current mass.

Sliding friction force:

$$F = \mu N$$

$\mu$  — friction coefficient; N — normal force.

**Problems**

1. A body of mass 3 kg moves under a force. Its position:  $x(t) = 2t^3 + 5t + 4$ . Find the force at  $t = 5$  s and plot force vs time.
2. Find the muzzle velocity of an air rifle bullet (mass 2.5 g, barrel length 0.70 m, calibre 5.0 mm, mean air pressure 9.8 MPa).
3. A wagon (1000 kg) detached from a moving train covered 70 m uniformly in 20 s then stopped. Find the friction force, friction coefficient, and initial speed.
4. Why do people take small steps on a slippery road? Find the maximum step length if leg length is 1 m and  $\mu = 0.1$ .
5. A car travels up an inclined road at  $v = 10$  m/s. Find the stopping distance and time if  $\mu = 0.5$  and inclination  $\alpha = 10^\circ$ .
6. Find the friction coefficient between an inclined plane and a body that started at  $v_0 = 5.0$  m/s and covered  $s = 2.0$  m. Inclination  $\alpha = 30^\circ$ .
7. A body rises on an inclined plane then slides back. Find the ratio of ascent to descent distances, compare times, and check if the final speed equals the initial one.  $\mu = 0.20$ ,  $\alpha = 45^\circ$ .
8. Find the friction coefficient between car wheels and road if the car stops in 8.0 m from 10 m/s.
9. A cart (10 kg) carries a load (5 kg). A force  $F$  causes acceleration of  $2$  m/s<sup>2</sup>. Neglect cart-ground friction. Friction between load and cart  $\mu = 0.20$ ; load does not slide on cart. Find  $F$ .
10. Body  $m_1 = 2$  kg and cart  $m_2 = 1$  kg are connected by an inextensible string on an incline ( $\alpha = 30^\circ$ ). Find cart acceleration and string tension if  $m_1$  accelerates at  $a = 2$  m/s<sup>2</sup> via a pulley.  $\mu = 0.15$  between cart and plane.
11. A load  $m = 10$  kg is lifted by a movable and fixed pulley system. Find its acceleration if force  $F = 60$  N is applied. Neglect masses of string and pulleys.
12. A uniform rod  $L = 5.0$  m is lifted vertically by force  $F = 500$  N at one end. Find the tensile force at cross-section  $l = 1.0$  m from the lower end.

13. A body (mass  $m$ ) on an incline ( $\alpha = 30^\circ$ ,  $\mu = 0.20$ ) is given downward vertical acceleration  $a = 3.8 \text{ m/s}^2$ . Find the distance covered along the incline in  $t = 10 \text{ s}$  from rest.
14. Two loads  $m_1 = 4.0 \text{ kg}$  and  $m_2 = 1.0 \text{ kg}$  are connected by a string over a pulley on a prism with angles  $\alpha = 60^\circ$  and  $\beta = 30^\circ$ .  $\mu = 0.20$  on both sides. Find accelerations and string tension.
15. What is the maximum acceleration of a car going up a road at  $\alpha = 20^\circ$  if  $\mu = 0.5$ ? What distance does it cover in  $t = 10 \text{ s}$  from  $v_0 = 10 \text{ m/s}$ ?
16. A body ( $m = 5 \text{ kg}$ ) on a horizontal surface is acted on by  $F = 50 \text{ N}$  at  $\alpha = 60^\circ$ .  $\mu = 0.20$ . Find the distance in  $t = 10 \text{ s}$  from rest.
17. A body ( $m = 0.7 \text{ kg}$ ) on a horizontal surface is acted on by  $F = 5.0 \text{ N}$  at  $\alpha = 45^\circ$ .  $\mu = 0.20$ . Find the horizontal acceleration.
18. A uniform flexible heavy rope of length  $l$  lies on a rough inclined surface ( $\alpha$  to horizontal) with one end hanging vertically. Find the rope's acceleration if friction coefficient is  $\mu$ .
19. A uniform chain lies on a horizontal table with one end hanging over the edge. Find  $\mu$  if the chain starts sliding when the hanging part is  $1/6$  of total length.
20. A  $5.0 \text{ kg}$  load hangs from a dynamometer in a lift. The lift moves up. Find the acceleration if during acceleration the dynamometer reads  $15 \text{ N}$  more than during deceleration.
21. In a lift, a dynamometer holds a pulley with loads  $m_1 = 1.0 \text{ kg}$  and  $m_2 = 2.0 \text{ kg}$  on a string. Find the dynamometer reading when the lift is stationary or accelerates upward at  $3.0 \text{ m/s}^2$ .
22. Three loads  $m_1 = 3.0 \text{ kg}$ ,  $m_2 = 1.0 \text{ kg}$ ,  $m_3 = 2.0 \text{ kg}$  hang via two ideal pulleys A (fixed) and B (movable). Find accelerations of all loads and of pulley B.
23. A load  $m_1 = 2.0 \text{ kg}$  on a moving cart is connected by a string over a fixed pulley to a hanging load  $m_2 = 1.0 \text{ kg}$ .  $\mu = 0.2$  between  $m_1$  and cart. Find system accelerations. Neglect rolling friction.
24. A  $1.0 \text{ kg}$  load on an incline ( $\alpha = 30^\circ$ ). Find its downward acceleration if  $\mu = 0.1$ .
25. A pulley is attached to the ceiling of a lift. A string with  $m_1 = 1.0 \text{ kg}$  and  $m_2 = 0.5 \text{ kg}$  hangs over it. Find their accelerations and string tension when lift is stationary or accelerates upward at  $2.0 \text{ m/s}^2$ .

26. A body  $m = 1.0$  kg slides on a wedge  $M = 2.0$  kg on a smooth horizontal surface. Wedge half-angle  $\alpha = 45^\circ$ . Find wedge acceleration; neglect all friction.
27. A block of length  $l$  and mass  $M$  lies on a smooth table with a small brick  $m$  on its end. What initial speed must be given to the block so it slides out from under the brick? Sliding friction coefficient  $\mu$ .
28. A string with loads  $m_1 = 1.0$  kg and  $m_2 = 2.0$  kg is thrown over a pulley. Both are initially at the same height. Find the displacement of the centre of mass after  $t = 1.0$  s. Find the centre of mass acceleration.
29. How far does a stationary boat shift if a load  $m_1 = 70$  kg moves from bow to stern? Boat length  $2.5$  m, boat mass  $m_2 = 100$  kg. Neglect water resistance.
30. Three blocks  $m_1 = 0.1$  kg,  $m_2 = 0.2$  kg,  $m_3 = 0.3$  kg are on a smooth surface. Block  $m_3$  is held at  $h = 30$  cm above the surface. If released, do the others move, and in what direction?
31. Two rockets  $m_1 = 1.0$  kg and  $m_2 = 2.0$  kg are connected by a light string on a smooth surface. One starts under constant force  $F = 15 \times 10^3$  N. Find the acceleration.  $g = 10$  m/s<sup>2</sup>, no friction.
32. A platform  $M_0 = 20 \times 10^3$  kg initially at rest. Sand flows out at  $q = 10$  kg/s. A constant horizontal force  $F = 1500$  N begins to act simultaneously. Find the platform's speed after  $t = 2$  min.
33. A rotating horizontal disc has a load at  $R = 10$  cm from the axis. Find the minimum static friction coefficient if the load starts sliding at  $n = 0.5$  rev/s.
34. Find the maximum safe speed for a car on a horizontal curve of radius  $R = 100$  m. Static friction coefficient  $\mu = 0.6$ .
35. At what speed should a car travel so that the pressure on a concave bridge is twice that on a convex bridge of the same radius  $R = 60$  m?
36. Find the period of a conical pendulum moving in a horizontal circle, if string length  $l = 65$  cm and the string makes angle  $\alpha = 30^\circ$  with the vertical.
37. A body ( $m = 300$  g) hangs on a string ( $l = 90$  cm) and is deflected to the level of the suspension point then released. The string breaks during motion. Find the height at which the string broke, if  $F_{\text{max}} = 4.9$  N.

- 38.** A vessel with water on a rope of length  $l = 2$  m rotates in a vertical plane. Find the maximum period of rotation for which water does not spill at the top.
- 39.** A body is on the inner surface of a vertical cone rotating at  $\omega = 6.0$  rad/s; half-angle  $\alpha = 90^\circ$ ;  $\mu = 0.23$ . Find the minimum and maximum distances from the apex for stable positioning.

## CONSERVATION LAWS IN MECHANICS

### Key Formulae

Work done by a force:

$$dA = F \cdot dr$$

Power:

$$dA/dt = F \cdot v$$

Kinetic energy:

$$E_k = mv^2/2$$

Potential energy:

$$E_p = mgh$$

Momentum:

$$P = mv$$

Conservation of energy (closed system):

$$E_{\text{total}} = \Sigma(m_i v_i^2/2 + m_i g h_i) = \text{const}$$

Conservation of momentum:

$$P_{\text{total}} = \Sigma m_i v_i = \text{const}$$

Torque:

$$M = r \times F = dL/dt$$

Angular momentum:

$$L = [r, p]$$

Conservation of angular momentum:

$$L_{\text{total}} = \Sigma L_i = \text{const}$$

### Problems

1. A load ( $m = 10 \text{ kg}$ ) is raised along an incline ( $\alpha = 60^\circ$ ) by  $s = 3 \text{ m}$  in  $t = 2.0 \text{ s}$ .  $\mu = 0.20$ . Find total work done on the load.
2. A parachutist ( $m = 65 \text{ kg}$ ) in a delayed jump reaches  $v = 50 \text{ m/s}$  in  $t = 15 \text{ s}$ . Assuming uniform acceleration, find work done against air resistance.
3. What minimum power must a tractor develop to move a trailer ( $m = 5 \times 10^3 \text{ kg}$ ) on a horizontal surface at constant speed?  $\mu = 0.20$ .
4. A body ( $m = 1.0 \text{ kg}$ ) is thrown from the ground at  $v_0 = 8.0 \text{ m/s}$  at  $\alpha = 30^\circ$ . Find instantaneous power of gravity at  $t = 5.0 \text{ s}$ , and work done by gravity in  $5.0 \text{ s}$ .
5. What work do the motors do over  $100 \text{ m}$  while uniformly accelerating at  $1.5 \text{ m/s}^2$  up a  $10^\circ$  slope? Train mass  $1.2 \times 10^3 \text{ kg}$ , friction coefficient  $0.05$ .
6. Find the power of a mine hoist lifting  $1.0 \times 10^4 \text{ kg}$  from  $200 \text{ m}$  in  $30 \text{ s}$  with efficiency  $75\%$ .
7. A train ( $M = 2.0 \times 10^6 \text{ kg}$ ) moves up a slope ( $\alpha = 15^\circ$ ) at constant  $v = 10 \text{ m/s}$  over  $S = 1.0 \text{ km}$ .  $\mu = 0.04$ . Find total work and mean power of the locomotive.
8. An electric train ( $n = 6$  wagons, each  $m = 4.0 \times 10^3 \text{ kg}$ ) accelerates from rest to  $v = 7.5 \text{ m/s}$  in  $t = 8 \text{ s}$ .  $\mu = 0.15$ . Find total power at the moment of reaching that speed.
9. Find the minimum work needed to tip a uniform cube ( $m = 4 \text{ kg}$ , edge  $a = 0.2 \text{ m}$ ) from one face to an adjacent one.
10. What work is needed to lift a uniform telegraph pole ( $M = 400 \text{ kg}$ ,  $L = 12.0 \text{ m}$ ) from horizontal to vertical?
11. A stone is thrown twice with equal initial kinetic energies: horizontally on ice and at  $45^\circ$  in air. Air resistance negligible, ice friction  $\mu = 0.2$ . In which case is the initial speed greater, and by how many times, if horizontal displacement is equal in both cases?
12. A body ( $m = 2.0 \text{ kg}$ ) is raised along a  $30^\circ$  incline by  $F = 50 \text{ N}$  to  $h = 1.0 \text{ m}$ .  $\mu = 0.20$ . Find work done by  $F$  and analyse how it is spent.

13. A spring pistol ( $M = 250$  g) hangs on a string ( $L = 0.60$  m) with barrel horizontal. After firing (bullet  $m = 15$  g at  $v = 13$  m/s horizontal), find the maximum angle of string deflection.
14. Find the mean hydroelectric power of a waterfall of height  $h = 50$  m and mean annual flow  $Q = 5900$  m<sup>3</sup>/s.
15. A body ( $m = 1.8$  kg) rises along a  $30^\circ$  incline to  $h = 2.0$  m at constant speed.  $\mu = 0.20$ . Find kinetic energy at that height.
16. A bullet (mass  $m$ ) undergoes perfectly inelastic collision with a ballistic pendulum (mass  $M$ ). What fraction of the initial kinetic energy converts to heat?
17. Two balls  $m_1 = 0.3$  kg and  $m_2 = 0.90$  kg hang on parallel strings of length  $L = 2.2$  m and touch in equilibrium.  $m_1$  is deflected  $90^\circ$  and released. Perfectly inelastic collision. Find velocities after collision and fraction of energy converted to heat.
18. Find the energy spent on deformation of two balls ( $m_1 = m_2 = 4.0$  kg) moving toward each other at  $v_1 = 3.0$  m/s and  $v_2 = 8.0$  m/s, perfectly inelastic collision.
19. Two balls on strings of equal length touch in equilibrium.  $M = 1.0$  kg. The smaller ball is deflected  $90^\circ$  and released. After perfectly inelastic collision both move as one. Find the smaller mass.
20. A hammer (0.80 kg) at 1.5 m/s drives a nail 5.0 mm deep. Find the mass to be placed on the nail head to drive it the same depth statically.
21. Find the instantaneous power of gravity on a 1.0 kg body at the end of its first second of free fall.
22. A small ball rests on a large sphere of radius 1.0 m. What initial speed must be given to the ball so it leaves the sphere at point M where  $\alpha = 60^\circ$ ? Friction coefficient  $\mu = 0.30$ .
23. A table-tennis ball ( $r = 15$  mm,  $m = 5.0$  g) is submerged to  $h = 30$  cm and released; it rises  $h_1 = 10$  cm above water. Find heat generated by friction.
24. A load (5.0 kg) starts from rest on an incline ( $\alpha = 30^\circ$ ). Force  $F = 40$  N at  $\beta = 30^\circ$  to incline acts on it.  $\mu = 0.10$ . Find displacement until speed reaches 1.0 m/s.
25. A ball is thrown vertically upward. Compare ascent time to highest point with descent time back to start. Neglect air resistance.

26. A large inflated balloon is thrown from a tower: first upward, then downward with equal initial speeds. Which has greater vertical speed at impact? Neglect air resistance.
27. Does the total work done by a hoist to lift a load depend on the lifting speed? Explain.
28. A uniform bar ( $m$ ,  $L = 0.60$  m) lies on a table ( $\mu = 0.15$ ) and rotates about its centre of mass at constant speed  $0.6$  m/s (speed of endpoint). Find work to rotate it by  $\varphi = 0.1$  rad.
29. Water from a hose ( $S = 5.0$  cm<sup>2</sup>,  $v = 10$  m/s) hits a vertical wall of a trolley ( $m = 200$  kg) and flows freely. Trolley resistance  $r = 0.01$  of its weight. Find trolley acceleration.
30. Find the initial speed of a hockey puck if it covered  $s_1 = 5.0$  m before hitting a board; after a perfectly elastic bounce it stopped in  $t = 2.0$  s. Ice friction  $0.10$ .
31. A person ( $m_2 = 60$  kg) jumps onto a moving trolley ( $m_1 = 240$  kg,  $v = 2.0$  m/s) with velocity perpendicular to the trolley. Find the speed of the system after the jump.
32. A cannon ( $M = 1.0 \times 10^3$  kg) fires horizontally. Shell mass  $m = 54$  kg, shell speed relative to Earth  $u = 900$  m/s. Find the free-recoil speed.
33. A recoilless gun on a platform ( $M = 2.0 \times 10^3$  kg) fires at  $45^\circ$  to the horizon. Platform rolls back  $S = 3.0$  m. Shell  $m = 10$  kg; rolling friction  $\mu = 0.002$ . Find the initial shell speed.
34. A grenade thrown at  $45^\circ$  with  $v_0 = 20$  m/s explodes at  $t_0 = 3.0$  s at height  $h$ . Fragment masses in ratio 1:2. Find the velocity of the larger fragment if the smaller one moved horizontally in the direction of throw.
35. Three boats (each  $M = 250$  kg) move in line at  $v = 5.0$  m/s. Two equal bodies ( $m = 20$  kg each) are thrown simultaneously from the middle boat at  $u = 20$  m/s relative to that boat: one toward the first boat, one toward the third. Find the speed of each boat.
36. A sledgehammer ( $20$  kg) falls from  $h = 1.2$  m onto an anvil. Impact is inelastic and lasts  $\Delta t = 0.005$  s. Find the mean impact force on the anvil.
37. A material point at  $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$  m has force  $\mathbf{F} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$  N acting on it. Find the torque vector  $\mathbf{M}$ , its magnitude, and component  $M_z$ .

- 38.** A body ( $m = 100 \text{ g}$ ) is thrown at  $\alpha = 45^\circ$  with  $v_0 = 20 \text{ m/s}$ . Find the angular momentum about the launch point when it is at the highest point.
- 39.** Prove that the angular momentum of a body moving under a central force is constant about the force centre.
- 40.** Show that the orbits of planets moving under central forces are planar.
- 41.** A uniform rod ( $L = 0.50 \text{ m}$ ,  $M = 1.0 \text{ kg}$ ) lies on a smooth surface. A ball ( $m = 0.30 \text{ kg}$ ,  $v = 10 \text{ m/s}$ ) hits the rod perpendicularly at  $l_1 = 20 \text{ cm}$  from its centre; perfectly inelastic. Find translational velocity of the rod's centre of mass and angular velocity.
- 42.** Prove that a person standing on a perfectly smooth horizontal surface can rotate about a vertical axis by swinging their arms above their head.
- 43.** Show that Kepler's second law (equal areas in equal times) is a direct consequence of conservation of angular momentum.
- 44.** A body ( $m = 0.5 \text{ kg}$ ) is thrown at  $\alpha = 30^\circ$  with initial speed  $v_0$ . Find the angular momentum magnitude about the launch point as a function of time.
- 45.** A body ( $m = 0.5 \text{ kg}$ ) is thrown vertically upward from  $(0, 3, 0) \text{ m}$  with  $v_0 = 5 \text{ m/s}$ . Y-axis points up. Find the change in angular momentum about the origin over the full ascent-and-return.

## MECHANICS OF A RIGID BODY

### Key Formulae

Equation of dynamics of a rigid body rotating about a fixed axis:

$$M_x = \varepsilon I$$

$M_x$  — algebraic sum of torques about axis X.

Moments of inertia:

Point mass  $m$  at distance  $R$  from axis:  $I = mR^2$

Uniform rod (length  $l$ , axis through centre, perpendicular to rod):  $I = ml^2/12$

Uniform rod (axis through end, perpendicular to rod):  $I = ml^2/3$

Uniform disc or cylinder (axis of symmetry):  $I = mR^2/2$

Thin-walled circle or tube (axis through centre, along tube):  $I = mR^2$

Uniform sphere (axis through diameter):  $I = 2mR^2/5$

Uniform disc (axis through diameter):  $I = mR^2/4$

Steiner (parallel axis) theorem:

$$I = I_0 + ma^2$$

$I_0$  — moment of inertia about axis through centre of mass;  $a$  — distance between axes.

Kinetic energy in plane motion:

$$E_k = mv^2/2 + I\omega^2/2$$

Equilibrium conditions:

$$\Sigma F_i = 0, \Sigma M_i = 0$$

## Problems

1. Find the moment of inertia of a sphere about an axis tangent to its surface. Radius 0.1 m, mass 5 kg.
2. Find the moment of inertia of a thin rod ( $l = 0.5$  m,  $m = 0.2$  kg) about an axis perpendicular to its length at 0.15 m from one end.
3. Find the moment of inertia of the Earth, treating it as a sphere ( $R = 6.4$  Mm,  $M = 6 \times 10^{21}$  kg).
4. A string wound on a drum ( $R = 10$  cm) supports a load ( $m = 0.50$  kg) that descends at  $a = 1.0$  m/s<sup>2</sup>. Find the drum's moment of inertia.
5. A flywheel disk ( $m = 10$  kg,  $R = 10$  cm) rotating at 6 rad/s stops in  $t = 5$  s. Find the braking torque.
6. A uniform disc ( $R = 6$  cm,  $m = 10$  kg) acts as a pulley. Two loads  $m_1 = 200$  g and  $m_2 = 300$  g hang on a string. Initially held; find acceleration of  $m_1$  when released.
7. A bucket ( $m = 10$  kg) is drawn from a well. At  $h = 5.0$  m above water, the handle is released. Find the linear speed at impact. Handle radius  $R = 30$  cm, axle radius  $r = 10$  cm, axle mass  $m_1 = 20$  kg. Neglect friction and string mass.
8. A flywheel ( $m_1 = 1.0$  kg,  $J = 500$  g·cm<sup>2</sup>) on a shaft has a string with load  $m_2 = 200$  g. How long to reach  $\omega = 10$  rad/s after release?
9. A load ( $m = 10$  kg) moves on an incline ( $\alpha = 45^\circ$ ). String over pulley; held stationary. Find sliding friction if load accelerates at  $a = 2$  m/s<sup>2</sup>.
10. A load ( $m = 10$  kg) moves along an incline ( $\alpha = 30^\circ$ ). Find sliding friction if  $a = 2$  m/s<sup>2</sup>.
11. A ball rolls down an incline ( $\alpha = 30^\circ$ ) from rest. What is the speed of the ball's centre after 1.5 s?
12. What power must a motor develop to accelerate a gyroscope disk ( $R = 1.0$  m,  $m = 1000$  kg) to  $\omega = 31$  rad/s in 1 minute? Neglect friction and air resistance.
13. Find the kinetic energy of a 2 kg disk rolling without slipping on a horizontal surface at 2 m/s.
14. What work is needed to bring a flywheel disk ( $m = 100$  kg,  $R = 0.4$  m) to  $n = 10$  rev/s from rest?

15. Find the braking torque applied to a flywheel ( $m = 50 \text{ kg}$ ,  $R = 0.30 \text{ m}$ ,  $n = 20 \text{ rev/s}$ ) to stop it in  $t = 20 \text{ s}$ . Mass uniformly distributed on rim. Find work done by braking torque.
16. Find the useful power of a motor driving a platform disk ( $m = 280 \text{ kg}$ ,  $R = 1 \text{ m}$ ) with a person ( $m = 60 \text{ kg}$ ) on its edge, reaching  $\omega = 1.2 \text{ rad/s}$  in  $t = 30 \text{ s}$ .
17. A rotating disk ( $m = 5 \text{ kg}$ ,  $R = 5 \text{ cm}$ ,  $n = 10 \text{ rev/s}$ ) contacts a stationary disk ( $m = 10 \text{ kg}$ ,  $R = 5 \text{ cm}$ ). Find the energy converted to heat when no slipping remains after contact.
18. In two cases a body slides down an incline (first case) or rolls (second case). In which case is the final speed greater and by how much?
19. A ball rolling without slipping on an incline ( $\alpha = 30^\circ$ ) flies off a horizontal table ( $h = 1 \text{ m}$ ). Find the horizontal displacement before it reaches a height of  $h = 12.5 \text{ cm}$  below the table.
20. A disk (mass  $m$ , radius  $r$ , angular velocity  $\omega$ ) falls onto a platform disk (mass  $M$ , radius  $R$ , angular velocity  $\Omega$ ) rotating about a vertical axis. Find the new angular velocity of the platform.
21. A uniform rod ( $l = 1.0 \text{ m}$ ,  $m = 5.0 \text{ kg}$ ) is suspended horizontally on two parallel strings. A load ( $10 \text{ kg}$ ) is attached  $0.25 \text{ m}$  from one end. Find string tensions.
22. A pulley attached to a dynamometer has a rope over it. One end is fixed at angle  $\alpha = 60^\circ$  from vertical; the other has load  $m = 5.0 \text{ kg}$ . Find the dynamometer reading.
23. A load ( $m = 10 \text{ kg}$ ) on a horizontal surface has ropes over pulleys to loads  $m_1$  and  $m_2$ . Find the maximum value of  $m_1$  for equilibrium if  $m_2 = 5.0 \text{ kg}$ ,  $\alpha_1 = 45^\circ$ ,  $\alpha_2 = 30^\circ$ ,  $\mu = 0.50$ .
24. A uniform rod ( $m = 2.0 \text{ kg}$ ) is suspended on two strings ( $a = 50 \text{ cm}$ ). Find the rod length if string tension  $T = 30 \text{ N}$ .
25. Rod AB ( $m = 5 \text{ kg}$ ) is hinged at its lower end to a vertical wall. A load ( $m = 3.0 \text{ kg}$ ) hangs from the upper end; rope SV length is half rod AB. Find tension in SV.

## FLUID AND GAS MECHANICS

### Key Formulae

For steady (stationary) flow the continuity equation holds:

$$vS = \text{const}$$

where  $v$  — fluid velocity;  $S$  — cross-sectional area of the stream tube.

Volume flow rate (discharge) — volume of fluid passing through any cross-section per unit time:

$$Q = vS$$

Speed of efflux of an ideal fluid through a small orifice in a wide vessel (Torricelli's theorem):

$$v = \sqrt{(2gh)},$$

where  $h$  — depth of the orifice below the free surface.

Bernoulli's equation:

$$p + \frac{1}{2}\rho v^2 + \rho gh = \text{const},$$

where  $\rho$  — fluid density;  $p$  — static pressure;  $v$  — flow velocity;  $h$  — height of the cross-section above a reference level.

Work done by external pressure on volume  $V$  of fluid moving from a zone with pressure  $p_1$  to a zone with pressure  $p_2$ :

$$A = (p_2 - p_1)V$$

Drag on a body in a fluid during laminar flow:

$$F = \gamma\eta v,$$

where  $\gamma$  — coefficient depending on body shape and size;  $\eta$  — dynamic viscosity;  $v$  — flow velocity.

Drag on a sphere in a viscous medium (Stokes' law):

$$F = 6\pi\eta Rv,$$

where  $R$  — radius of the sphere.

Volume  $V$  of fluid flowing through a pipe of length  $l$  and radius  $R$  during laminar flow in time  $t$  (Poiseuille's law):

$$V = (\pi R^4 / 8\eta l) \cdot \Delta p \cdot t,$$

where  $\Delta p$  — pressure difference between the pipe ends.

Drag during turbulent flow at moderate velocities:

$$F = C_\chi S \rho v^2,$$

where  $C_\chi$  — drag coefficient (depends on body shape and Reynolds number);  $S$  — frontal area of the body;  $\rho$  — fluid density.

Reynolds number:

$$Re = l\rho v / \eta,$$

where  $l$  — characteristic linear dimension of the vessel.

## Problems

1. Find the depth of submersion in water at which a pneumatic pistol (calibre  $d = 7.0$  mm, barrel length  $l = 22$  cm) will not fire when the trigger is pulled. Bullet mass  $m = 7.0$  g; muzzle velocity in air  $v = 27$  m/s.

2. Determine the cable tension during the lifting of a reinforced-concrete slab (volume  $2.4$  m<sup>3</sup>) out of water with acceleration  $0.50$  m/s<sup>2</sup>. Neglect all resistance. Density of reinforced concrete  $\gamma = 2.2 \cdot 10^3$  kg/m<sup>3</sup>.

3. What is the efficiency of the motor driving a hydraulic press whose piston areas are in the ratio 1:100, if, when lifting a load of mass  $m = 5 \cdot 10^3$  kg, the small piston makes  $n = 100$  strokes in  $t = 1.5$  min? Stroke of the small piston  $h_1 = 20$  cm; motor power  $N_1 = 2.0$  kW.

4. Two balls — one aluminium and one copper, one hollow and one solid — are hung from the ends of an unequal-arm balance and are in equilibrium in air. Determine which ball is hollow if immersing both in water does not disturb the equilibrium. What is the volume of the cavity if the mass of the copper ball is  $0.4$  kg?

5. A steel ball is dropped into a vessel containing mercury and water. What fraction of its volume will be in the water layer?

6. Find the mass of a cork life-belt capable of keeping a person of mass  $m_1 = 60$  kg afloat so that the head and shoulders ( $1/8$  of body volume) remain above water. Human body density  $\rho_1 = 1007$  kg/m<sup>3</sup>.

7. A vessel with water moves vertically with acceleration  $a_1 = 1.2$  m/s<sup>2</sup> directed upward. Find the pressure at depth  $h_1 = 0.20$  m.

8. An iron weight is tied by a string to a cork life-belt (volume  $V_1 = 50$  dm<sup>3</sup>) floating in water. Find the mass of the weight and the string tension if half the belt volume is submerged.

9. Two vertically placed tubes with cross-sections  $S_1$  and  $S_2$  are connected to each other and closed by massless pistons joined by a thin massless string of length  $l$ . The space between the pistons is filled with water of density  $\rho$ . Find the string tension if both tube ends are open to the atmosphere.

10. A petrol tanker (fully loaded) moving at  $v = 36$  km/h stops in  $t = 10$  s due to braking. By how many times does the pressure on the front wall of the tank exceed that on the rear wall during braking, if the tank can be

treated as a rectangular parallelepiped of length  $l = 3.0$  m? Height of petrol filling  $h = 0.50$  m.

**11.** A wide cylindrical reservoir (water level 75 cm) has two holes through which jets flow. The lower hole is at height 25 cm. At what height is the upper hole located if both jets intersect on the horizontal plane coinciding with the reservoir bottom?

**12.** A water tank on a locomotive tender (length  $l = 4.0$  m) is filled with water. A hole forms in the rear wall at height  $h = 1.0$  m below the top water level. Find the efflux speed of the jet if the locomotive accelerates at  $a = 0.50$  m/s<sup>2</sup>. What is the jet speed when braking at the same acceleration?

**13.** A wide reservoir filled with water has a hole of area  $S = 2$  cm<sup>2</sup> at height  $h = 0.1$  m. Water is continuously supplied from above so that the level remains constant. How much water must be poured into the reservoir each second so that the efflux jet reaches the ground at horizontal distance  $l = 0.5$  m from the reservoir edge?

**14.** A bucket of water is suspended from a spring and performs vertical harmonic oscillations with period  $T = 2.0$  s and amplitude  $A = 10$  cm. Water flows out through a small hole in the bucket. Find the maximum and minimum efflux speeds if the water level in the bucket is 10 cm.

**15.** Water flows through a pipe of cross-section  $S = 4.0$  cm<sup>2</sup> bent at a right angle. Find the force exerted by the water on the bend if the volume flow rate  $Q = 2.0$  L/s.

**16.** What force must be applied to the piston of a horizontal cylindrical pipe so that the efflux jet has speed  $v = 1.0$  m/s? Piston radius  $R = 2.0$  cm, pipe length  $l = 1.0$  m.

**17.** What drag force does the air exert on a car moving at 144 km/h head-on into the airflow? Frontal area 3.0 m<sup>2</sup>; drag coefficient 0.27.

**19.** Determine the power of the airflow impinging on an electric train moving at 100 km/h if its frontal area is 10 m<sup>2</sup>.

**20.** Water flows at 2 m<sup>3</sup>/min through a horizontal pipe with varying cross-section. Find the difference in water levels in manometric tubes at points with pipe diameters 0.3 m and 0.1 m.

**21.** Find the water velocity in the wide section of a horizontal pipe with varying cross-section if the radius of the narrow section is 3 times

smaller than that of the wide section and the pressure difference between them is  $\Delta p = 10 \text{ kPa}$ .

**22.** Determine the power developed by the left ventricle of the heart if it ejects 70 g of blood into the aorta at a pressure of 26 kPa during each contraction, with approximately 75 contractions per minute.

**23.** Find the work done in moving  $2 \text{ m}^3$  of water through a horizontal pipe with varying cross-section when pressure changes from 50 to 20 kPa.

**24.** A glass sphere of radius 0.50 mm falls at constant velocity 5.0 cm/s in a large container of glycerine. Find the viscosity of glycerine. Glass density  $2.7 \cdot 10^3 \text{ kg/m}^3$ ; glycerine density  $1.2 \cdot 10^3 \text{ kg/m}^3$ .

**25.** A basin of volume  $2.0 \text{ m}^3$  must be filled with water in 5.0 min. Find the minimum pipe radius through which this is possible if the head provides water velocity  $v = 1.50 \text{ m/s}$ .

**26.** How much heat is required to warm  $2 \text{ m}^3$  of water from  $10^\circ\text{C}$  to  $20^\circ\text{C}$ ? Specific heat capacity of water  $c = 4.2 \cdot 10^3 \text{ J/(kg}\cdot^\circ\text{C)}$ .

**27.** Find the water flow velocity in a pipe of radius  $r = 0.50 \text{ m}$  if the pressure difference between two cross-sections is  $\Delta p = 0.20 \text{ kPa}$  and water viscosity is  $\eta = 1.0 \cdot 10^{-3} \text{ Pa}\cdot\text{s}$ .

**28.** Water flows through a pipe of radius  $r = 0.50 \text{ m}$  at velocity  $v = 0.50 \text{ m/s}$ . Find the pressure difference between two cross-sections of the pipe. Water viscosity  $\eta = 1.0 \cdot 10^{-3} \text{ Pa}\cdot\text{s}$ .

## NON-INERTIAL REFERENCE FRAMES. INERTIAL FORCES.

### Key Formulae

If a body is in a reference frame that moves translationally and rectilinearly with acceleration  $a$  relative to an inertial frame, an inertial force acts on the body:

$$F^N = -ma$$

In a reference frame rotating at constant angular velocity  $\omega$  relative to an inertial frame, a centrifugal inertial force acts on the body:

$$F^L = m\omega^2 R,$$

where  $R$  — position vector from the rotation axis to the body.

In addition, the Coriolis force acts in the rotating frame:

$$F^L = 2m[v \times \omega],$$

where  $v$  — velocity of the body in the rotating frame;  $\times$  denotes the vector (cross) product.

## Problems

1. Find the initial speed of an electric train before braking, given that the motion was uniformly decelerated, the train stopped after covering  $s = 200$  m, and a pendulum suspended in a carriage deflected by angle  $\alpha = 5^\circ$  from the vertical during braking.

2. With what acceleration must a horizontal inclined plane (inclination angle  $\alpha = 30^\circ$ ) move so that a body on it remains stationary relative to the plane in the absence of friction?

3. What minimum horizontal acceleration must an inclined plane (angle  $\alpha$ ) have so that a body on it starts to move up its surface? Friction coefficient between body and plane is  $\mu$ .

4. A body rests on an inclined plane with angle  $\alpha = 10^\circ$ . The inclined plane is accelerated horizontally at  $a = 3$  m/s<sup>2</sup> (as shown in Figure 6.1). Find the friction coefficient  $\mu$  between the body and the plane for which the body remains stationary relative to the plane.

5. A ball moves inside a horizontal tube (Fig. 6.2). Each ball has mass  $m = 1.0$  g. The ball can move horizontally with acceleration  $a = 2$  m/s<sup>2</sup> directed at angle  $\alpha = 45^\circ$  to the tube. Find the interaction force between the ball and the tube.

6. A body lies on a horizontal rotating disc at distance  $R = 8$  cm from the rotation axis. Find the friction coefficient between the disc and the body if the body does not slide on the disc rotating at angular velocity  $\omega$ .

7. A cyclist rides uniformly on a horizontal road at speed  $v = 6$  m/s. Friction coefficient between tyres and road  $\mu = 0.2$ . Find the work done by the cyclist on a road section of length  $l = 100$  m.

8. What minimum speed must a cyclist have on a horizontal road to be able to climb a rise of height  $h = 1$  m, if friction coefficient  $\mu = 0.2$ ?

9. Analyse the motion of a body falling freely from height  $h = 10$  m. Find the speed of the body just before impact with the ground.

10. A ball moves along an inclined plane at angle  $\alpha = 60^\circ$ . At distance  $l = 1$  m from the base the speed is  $v = 5$  m/s. Find the acceleration of the ball.

11. A target is at the centre of a horizontal disc of radius  $R = 10$  cm and a pneumatic pistol is at its edge. The bullet flies along the radius toward the target. If the disc rotates about a vertical axis through its centre at

constant angular velocity  $\omega = 60 \text{ rad/s}$ , find the bullet speed for it to hit the target, if the distance from the centre to the target is  $r = 10 \text{ cm}$ .

**12.** An electric train of mass  $1 \cdot 10^3 \text{ kg}$  moves from north to south at angle  $60^\circ$  to the horizontal. Find the Coriolis force acting on the train at latitude  $\varphi = 60^\circ$ .

**13.** Find the change in force acting on the surface of the Earth if a body of mass  $m$ , moving along the equator from east to west at speed  $v$ , reverses its direction of motion.

**14.** A body falls freely from height  $h = 200 \text{ m}$  at geographical latitude  $\varphi = 60^\circ$ . Find its eastward deflection due to the Coriolis force caused by Earth's rotation.

**15.** A rifle is fired vertically upward at latitude  $\varphi = 60^\circ$ . The bullet returns to the ground after some time. Find the displacement of the landing point from the firing point if the initial bullet speed  $v_0 = 200 \text{ m/s}$ . Neglect air resistance.

**16.** A Foucault pendulum of length  $98 \text{ m}$  is installed in a cathedral. Through what angle does its plane of oscillation rotate in 10 minutes? Geographic latitude  $\varphi = 60^\circ$ . Solve the problem in two reference frames: the non-inertial (Earth) frame and the heliocentric (inertial) frame.

## ELEMENTS OF SPECIAL RELATIVITY

### Key Formulae

Lorentz transformations:

$$x' = (x - vt)/\sqrt{(1-v^2/c^2)}, \quad y' = y, \quad z' = z, \quad t' = (t - vx/c^2)/\sqrt{(1-v^2/c^2)}$$

where  $c$  — speed of light in vacuum.

Length contraction of a moving body:

$$l' = l\sqrt{(1 - v^2/c^2)},$$

where  $l'$  — length of the moving body;  $l$  — proper (rest) length.

Time dilation of a moving clock:

$$\Delta t' = \Delta t / \sqrt{(1 - v^2/c^2)},$$

where  $\Delta t'$  — time interval in the moving frame;  $\Delta t$  — proper time interval.

Relativistic velocity addition law:

$$u_x = (u_x' + v)/(1 + v \cdot u_x'/c^2), \quad u_y = u_y' \sqrt{(1-v^2/c^2)}/(1 + v \cdot u_x'/c^2)$$

where  $u_x, u_y$  — velocity components in the rest frame;  $u_x', u_y'$  — components in the frame moving at velocity  $v$ .

Relativistic mass and momentum:

$$m = m_0/\sqrt{(1-v^2/c^2)}, \quad p = m_0 v/\sqrt{(1-v^2/c^2)},$$

where  $m_0$  — rest mass.

Total energy of the body:

$$E = mc^2 = m_0 c^2 / \sqrt{(1-v^2/c^2)},$$

where  $E_0 = m_0 c^2$  — rest energy.

Kinetic energy of a moving body:

$$E_k = E - m_0 c^2 = m_0 c^2 (1/\sqrt{(1-v^2/c^2)} - 1)$$

**Problems**

1. Derive the inverse Lorentz transformations.
2. A rod moves at constant velocity  $v$ . Its length in the rest frame is  $l_1 = 3.0$  m, and in the frame attached to the rod  $l_2 = 6.0$  m. Find the proper length of the rod and its speed relative to the rest frame.
3. The orbital speed of Earth around the Sun is  $v = 30$  km/s. Find the contraction of Earth's diameter in the coordinate system attached to the Sun.
4. A jet aircraft flies at 1000 m/s. By how many hours does the onboard clock lag behind Earth clocks after a 24-hour flight?
5. One of a pair of twins leaves Earth at age 20 on a cosmic voyage to the star Arcturus at  $v = 0.99c$ . For observers on Earth, the distance to Arcturus is 40 light-years. How many years younger will the space traveller be than the twin who stayed on Earth?
6. Using the Lorentz transformations, derive the relativistic velocity addition law.
7. Using the relativistic velocity addition formulae, prove that the resulting speed of an object never exceeds the speed of light.
8. Show that a photon emitted toward Earth by a star approaching at speed  $v$  has speed  $c$ , not  $c + v$ .
9. Two rockets move away from Earth in opposite directions, each at  $0.8c$  relative to Earth. Find the speed of one rocket in the reference frame of the other.
10. An accelerator gave a radioactive nucleus speed  $v = 0.4c$ . At the moment of leaving the accelerator the nucleus emitted a  $\beta$ -particle in the direction of its motion at speed  $0.75c$  relative to the accelerator. Find the particle's speed relative to the nucleus.
11. What is the astronaut's age in Earth time if he left at age 30 and flew to a destination 20 light-years away, his own clock showing 35 years?
12. By how many times does the relativistic mass of an electron moving at  $v = 0.999c$  exceed its rest mass?
13. The relativistic mass of a moving body increased by 200% compared to its rest mass. By how many times did its length contract?

**14.** The relativistic mass of a moving proton is  $10^2$  times its rest mass. Find the proton's speed.

**15.** A body moves at 200.0 Mm/s. By how many times has its density increased compared to its rest density? Assume dimensions perpendicular to the motion are unchanged.

**16.** An electron moves at 200.0 Mm/s. Find its kinetic energy using classical and relativistic formulae. Compare the results.

**17.** Find the ratio of kinetic energy to rest energy for an electron moving at 150 Mm/s. What is the relativistic momentum of the electron?

**18.** The total energy of a meson is 8 times its rest energy. What is the meson's speed?

**19.** By how many times does the change in mass correspond to a change in energy of 10 J?

## ELASTIC PROPERTIES OF SOLIDS

### Key Formulae

Relative longitudinal strain:

$$\varepsilon_l = \Delta l / l_0,$$

where  $\Delta l$  — elongation under tension or compression;  $l_0$  — initial length.

Relative torsional strain:

$$\varepsilon_\varphi = \varphi / l,$$

where  $\varphi$  — twist angle;  $l$  — length of the rod.

Relative volume change under longitudinal deformation:

$$\Delta V/V = \varepsilon_l(1 - 2\mu),$$

where  $\mu$  — Poisson's ratio (ratio of transverse to longitudinal relative strain).

Stress at elastic deformation:

$$\sigma = dF / dS.$$

Hooke's law (force-deformation relation):

$$\varepsilon_l = \alpha F/S = (1/E)(F/S),$$

where  $\alpha$  — elasticity coefficient;  $E$  — Young's modulus.

Breaking (ultimate) force:

$$F_m = \sigma_m S,$$

where  $\sigma_m$  — ultimate stress.

Shear deformation (shear angle):

$$\psi = F_t / (GS) = \sigma_t / G,$$

where  $F_t$  — shear force;  $\sigma_t$  — shear stress;  $G$  — shear modulus.

Relation between Young's modulus  $E$ , shear modulus  $G$ , and Poisson's ratio  $\mu$ :

$$G = E / (2(1 + \mu))$$

Twist angle of a rod:

$$\varphi = 2Ml / (\pi GR^4),$$

where  $M$  — torque;  $l$  — rod length;  $R$  — rod radius.

Potential energy of an elastically deformed rod:

$$E_p = E\varepsilon_1^2 V / 2,$$

where  $V$  — volume of the rod.

Elastic strain energy density:

$$\omega = E\varepsilon_1^2 / 2$$

## Problems

1. A load hung on a rubber cord of length 50 cm rotates in a horizontal plane at constant speed so that the cord forms a cone with apex angle  $120^\circ$ . Find the relative elongation of the cord during rotation if under static loading its elongation is 1 cm. Assume elongation is proportional to the applied force.

2. What compressive load must be applied to an aluminium rod to keep its length at  $10^\circ\text{C}$  equal to its length at  $0^\circ\text{C}$ ? Cross-sectional area  $S = 1.5 \text{ cm}^2$ ; Young's modulus  $E = 70 \text{ GPa}$ .

3. A rubber cord is stretched so that its length doubles. What is the diameter of the stretched cord if before stretching it had diameter 1 cm and Poisson's ratio for rubber is 0.5?

4. Find the relative volume change of a steel wire of diameter 2 mm when stretched by a force of 1 kN. Poisson's ratio  $\mu = 0.3$ .

5. At what length will a vertically suspended steel wire begin to break under its own weight? Ultimate strength of steel  $\sigma_m = 0.69 \text{ GPa}$ .

6. The relative volume change under longitudinal deformation of a rod is zero. Find Poisson's ratio of the rod material.

7. A straight wire of length  $l$  is lifted vertically upward by a force applied to its upper end. At what force will the wire begin to break?

10. Find the diameter of the suspension wire of a mirror galvanometer frame if under a torque  $M = 0.3 \text{ N}\cdot\text{m}$  it rotates by angle  $\varphi = 2^\circ$ . Wire length  $l = 10 \text{ cm}$ ; shear modulus  $G = 6.5 \text{ GPa}$ .

11. Find the relative elongation of a copper rod if during stretching work of 0.12 J is done. Rod length 2 m, cross-sectional area  $1 \text{ mm}^2$ .

12. What is the elastic energy density of a stretched steel rod if the relative elongation is 0.001?

13. Two wagons, each of mass  $m = 2.0 \cdot 10^4 \text{ kg}$ , moving toward each other at  $v = 2 \text{ m/s}$ , collide. Find the compression of the buffer springs if a force  $F = 40 \text{ kN}$  compresses the spring by  $x_0 = 1 \text{ cm}$ . Assume compression is proportional to force.

14. Find the force exerted by a gymnast of mass  $m = 60 \text{ kg}$  on an elastic net when jumping from height  $h = 8.0 \text{ m}$ , if under the gymnast's weight the net sags by  $x_0 = 16 \text{ cm}$ .

**15.** What force must be developed when drawing a bow by  $x = 0.20$  m if all the work done is transferred to the arrow as kinetic energy and the maximum range of the arrow is  $s = 36$  m? Arrow mass  $m = 50$  g.

**16.** To what height does a stone of mass  $m = 30$  g rise when shot vertically upward from a slingshot whose rubber band has cross-section  $S = 0.20$  cm<sup>2</sup>, length  $l = 30$  cm, and was stretched by  $\Delta l = 20$  cm? Neglect air resistance. Young's modulus for rubber  $E = 7.8$  MPa.

**17.** An aircraft lands on an aircraft carrier at 100 km/h. Catching a braking cable, the plane travels 50 m to a full stop. Find the aircraft mass if the overload factor is 3g.

## LAW OF UNIVERSAL GRAVITATION

### Key Formulae

Two bodies of masses  $m_1$  and  $m_2$  separated by distance  $R$  attract each other with force:

$$F = G m_1 m_2 / R^2$$

Kepler's third law (for planets orbiting the Sun or satellites orbiting a planet):

$$a_1^3 / a_2^3 = T_1^2 / T_2^2,$$

where  $a_1, a_2$  — semi-major axes of ellipses;  $T_1, T_2$  — orbital periods.

## Problems

1. The first artificial Earth satellite had an orbital period of 1 h 36 min. Treating the orbit as circular and the motion as uniform, find the satellite's altitude above Earth's surface.

2. At what altitude must an artificial Earth satellite be launched so that it appears stationary from Earth and its circular orbit lies in the equatorial plane? Earth radius  $R = 6.37$  Mm; period of daily rotation  $T = 24$  h.

3. What speed must be given to a body to allow it to escape from the Moon's surface to infinity? Mass of Moon  $M_1 = 7.3 \cdot 10^{22}$  kg; Moon radius  $R_1 = 1.74$  Mm.

4. Find the orbital speed of the Moon, assuming a circular orbit. Earth mass  $M_n = 5.96 \cdot 10^{24}$  kg; distance between Earth and Moon centres  $R = 384.4$  Mm.

5. At what distance from Earth's centre will a rocket travelling to the Moon be attracted equally by Earth and Moon? The Moon's mass is 81 times less than Earth's; the distance between their centres is 60 Earth radii.

6. At what speed does Earth orbit the Sun if the Earth–Sun distance is approximately 150 Mm and the Sun's mass is  $1.97 \cdot 10^{30}$  kg?

7. Find the mean density of Earth if its radius is 6.37 Mm and gravitational acceleration is  $9.8$  m/s<sup>2</sup>.

8. Find the mass of Earth given that gravitational acceleration decreases by 10% at an altitude of 1 Mm.

9. Find the first cosmic velocity for an orbit whose radius equals the sum of Earth's radius and 384 Mm. Earth mass  $5.96 \cdot 10^{24}$  kg.

10. Find the second cosmic velocity, i.e. the minimum speed needed for a body to escape from Earth's surface to infinity.

11. By how much does the weight of a body decrease at the summit of Mount Elbrus ( $h = 6$  km) compared to sea level?

12. At what altitude is gravitational acceleration half its surface value?

13. What acceleration does Earth experience due to the Sun's gravitational attraction?

**14.** A rocket that has become an artificial planet orbits the Sun with period 450 days. Find its mean distance from the Sun, given that Earth's mean distance from the Sun is 149.5 million km and its period is 365 days. Assume a circular orbit.

**15.** What is Earth's orbital speed at perihelion ( $r_1 = 147$  million km) and aphelion ( $r_2 = 152$  million km) if the mean orbital speed is  $\langle v \rangle = 29.8$  km/s?

**16.** Find the work required to send a body of mass  $5 \times 10^3$  kg from Earth's surface into interplanetary space. Neglect air resistance.

**17.** It is known that as the orbital radius increases, the speed of an artificial Earth satellite decreases. Does this mean that the rocket engine must do less work to launch the satellite into a larger orbit?

## ELECTROSTATICS

### Key Formulae

Coulomb's law:

$$F = q_1 q_2 / (4\pi\epsilon\epsilon_0 r^2),$$

where  $q_1, q_2$  — electric charges;  $\epsilon$  — relative permittivity of the medium;  $\epsilon_0 \approx 8.85 \cdot 10^{-12}$  F/m — electric constant;  $r$  — distance between charges.

Electric field intensity:

$$E = F / q,$$

where  $F$  — force acting on charge  $q$ .

Field intensity of a point charge:

$$E = q / (4\pi\epsilon\epsilon_0 r^2)$$

Gauss's theorem: The flux of the electric field vector through a closed surface is directly proportional to the algebraic sum of the electric charges enclosed by that surface.

Field intensity of an infinite charged plane with surface charge density  $\sigma$ :

$$E = \sigma / (2\epsilon\epsilon_0)$$

Field intensity of an infinitely long charged wire with linear charge density  $\tau$ :

$$E = \tau / (2\pi\epsilon\epsilon_0 a)$$

## Problems

1. Calculate the attractive force between the nucleus of a hydrogen atom and the electron, given the hydrogen atom radius  $r = 0.5 \times 10^{-10}$  m. The nuclear charge equals the electron charge in magnitude.

2. Find the electrostatic interaction force between two point charges  $q_1 = 2 \times 10^{-9}$  C and  $q_2 = -3 \times 10^{-9}$  C separated by  $r = 5$  cm in vacuum.

3. Two point charges  $q_1 = 4 \times 10^{-8}$  C and  $q_2 = 6 \times 10^{-8}$  C are separated by  $r = 10$  cm. What force acts on a third charge  $q_3 = -2 \times 10^{-8}$  C placed midway between them?

4. Find the attraction force between a helium nucleus (charge equals two elementary charges) and an electron orbiting at  $r = 0.3 \times 10^{-10}$  m.

5. Find the distance between two point charges  $q_1 = 1 \times 10^{-7}$  C and  $q_2 = -4 \times 10^{-7}$  C if their interaction force is  $F = 0.036$  N.

6. What charge must a point object have so that at  $r = 1$  m from it the force on another point charge  $q_2 = 5 \times 10^{-9}$  C equals  $F = 4.5 \times 10^{-5}$  N?

7. An electron and a proton are separated by distance  $r$ . By how many times does their electrostatic interaction force change if the distance is halved?

8. Find the electric field intensity at the point midway between two point charges  $q_1 = 8$  nC and  $q_2 = -6$  nC separated by  $r = 10$  cm. Medium: vacuum ( $\epsilon = 1$ ).

9. Calculate the electric field intensity at a point 3 cm from a point charge  $q = 5$  nC. Medium: air ( $\epsilon \approx 1$ ).

10. Two point charges  $q_1 = 10$  nC and  $q_2 = 15$  nC are 20 cm apart. Find the field intensity at a point on the line connecting the charges, 5 cm from  $q_1$  (between the charges).

11. Find the electric field intensity at one vertex of a square with side  $a = 10$  cm if point charges  $q_1 = 2$  nC,  $q_2 = -2$  nC,  $q_3 = 4$  nC are placed at the other three vertices.

12. Charge  $q = 4$  nC creates an electric field. At what distance does the field intensity equal  $E = 3600$  V/m?

13. A point charge  $q = 10$  nC is at the centre of a square with side  $a = 20$  cm. Find the field intensity at one vertex.

**14.** Two point charges  $q_1 = 7 \text{ nC}$  and  $q_2 = -7 \text{ nC}$  are 14 cm apart. Find the field intensity at a point on the perpendicular bisector of the line joining the charges, 7 cm from the midpoint.

**15.** A negative charge  $q_0$  is placed at the centre of a square, each vertex of which carries charge  $q = 2.33 \text{ nC}$ . Find  $q_0$  if the net force on each charge  $q$  is zero.

**16.** Equal positive charges  $q$  are placed at the vertices of an equilateral triangle with side  $a$ . What charge  $q_0$  must be placed at the centroid so that the net force on each charge  $q$  is zero?

**17.** Four equal positive charges  $q$  are at the vertices of a square with side  $a$ . What is the net force on a charge placed at the centre?

**18.** Three charges of equal magnitude but alternating sign  $q_1 = q$ ,  $q_2 = -q$ ,  $q_3 = q$  are at the vertices of an isosceles right triangle. Find the field intensity at the midpoint of the hypotenuse.

**19.** Charges  $q_1 = q$ ,  $q_2 = -q$ ,  $q_3 = q$ ,  $q_4 = -q$  are at the vertices of a square with side  $a$ . Find the net force on charge  $q_1$ .

**20.** Three charges lie on a straight line:  $q_1 = 5 \text{ nC}$ ,  $q_2 = -3 \text{ nC}$ ,  $q_3 = 8 \text{ nC}$ . Distance between  $q_1$  and  $q_2$  is 10 cm; between  $q_2$  and  $q_3$  is 15 cm. Find the force on charge  $q_2$ .

**21.** Charges  $q_A = 2 \text{ nC}$ ,  $q_B = -2 \text{ nC}$ ,  $q_C = 2 \text{ nC}$ ,  $q_D = -2 \text{ nC}$  are at the vertices of a square with side  $a = 1 \text{ m}$ . Find the electric field intensity at the centre.

**22.** Two point charges  $q_1 = 7.5 \text{ nC}$  and  $q_2 = -14.7 \text{ nC}$  are 5 cm apart. Find the field intensity  $E$  at a point located 3 cm from  $q_1$  and 4 cm from  $q_2$ .

**23.** Two point charges  $q_1 = 6 \text{ nC}$  and  $q_2 = -8 \text{ nC}$  are 10 cm apart. Find the field intensity at a point on the line connecting the charges, 4 cm from  $q_1$  on the far side of  $q_1$ .

**24.** The field intensity at a point 5 cm from a point charge is 1000 V/m. If another point charge of equal magnitude but opposite sign is placed 5 cm from that point, what is the resultant field intensity there?

**25.** Point charges  $q_1 = 1 \text{ nC}$ ,  $q_2 = 2 \text{ nC}$ ,  $q_3 = -3 \text{ nC}$  are at the vertices of an equilateral triangle with side 10 cm. Find the field intensity at the centroid.

**26.** Two point charges  $q_1 = 5 \text{ nC}$  and  $q_2 = 12 \text{ nC}$  are 13 cm apart. Find the field intensity at the point that forms a right triangle with the charges, with legs of 5 cm and 12 cm.

**27.** Find the field intensity at the centre of a regular hexagon with side  $a = 20 \text{ cm}$  if equal charges  $q = 2 \text{ nC}$  are placed at four of its vertices and the other two vertices are empty.

**28.** At what distance from point charge  $q = 9 \text{ nC}$  is the field intensity equal to  $8100 \text{ V/m}$ ?

**29.** Two balls of equal radii and masses are suspended on strings of equal length so that their surfaces touch. After being given charge  $q_0 = 0.4 \text{ }\mu\text{C}$  they repel and diverge to angle  $2\alpha = 60^\circ$ . Find the mass  $m$  of each ball if the string length (centre to suspension point)  $l = 20 \text{ cm}$ .

**30.** Two identical small balls, each of mass  $m = 10 \text{ g}$ , are suspended on strings of length  $l = 50 \text{ cm}$  and carry equal charges. Find the charge on each ball if in equilibrium the strings diverge to angle  $2\alpha = 90^\circ$ .

**31.** Two identical balls, each of mass  $m = 20 \text{ g}$ , are suspended on strings of length  $l = 30 \text{ cm}$ . After being given equal charges they diverge so that the distance between them is  $d = 10 \text{ cm}$ . Find the charge on each ball.

**32.** A small ball of mass  $m = 5 \text{ g}$  and charge  $q = 1 \text{ }\mu\text{C}$  is hung on a string. What field intensity must be created for the string to deflect  $30^\circ$  from vertical?

**33.** Two identical charged balls are suspended on strings of length  $l = 40 \text{ cm}$ . In equilibrium the strings make angle  $2\alpha = 70^\circ$ . Find the mass of each ball if the charge is  $q = 0.5 \text{ }\mu\text{C}$ .

**34.** Two small balls of equal mass  $m = 15 \text{ g}$  and charges  $q_1 = 0.3 \text{ }\mu\text{C}$  and  $q_2 = 0.6 \text{ }\mu\text{C}$  are suspended on strings of equal length. Find the angle between the strings if each string has length  $l = 25 \text{ cm}$ .

**35.** One of two identical charged balls suspended on strings has charge  $q_1$ ; the other is uncharged. After touching they diverge to  $2\alpha = 45^\circ$ . Find  $q_1$  if each ball has mass  $m = 12 \text{ g}$  and string length  $l = 35 \text{ cm}$ .

**36.** Find the force per unit length with which the field of a charged infinite plane acts on an infinitely long charged wire placed in that field. Linear charge density on wire  $\tau = 3 \text{ }\mu\text{C/m}$ ; surface charge density on plane  $\sigma = 20 \text{ }\mu\text{C/m}^2$ .

**37.** An infinite plane is charged with surface density  $\sigma = 15 \text{ nC/m}^2$ . Find the electric field intensity it creates.

**38.** An infinitely long wire has linear charge density  $\tau = 5 \text{ nC/m}$ . What force does its field exert on a point charge  $q = 2 \text{ nC}$  at distance  $r = 2 \text{ cm}$  from the wire?

**39.** Find the force per unit area on a large charged plane with surface charge density  $\sigma = 10 \text{ }\mu\text{C/m}^2$ .

**40.** A point charge  $q = 0.5 \text{ nC}$  is placed in the field of an infinite charged plane with  $\sigma = 25 \text{ nC/m}^2$ . What force acts on the charge?

**41.** An infinitely long wire with linear charge density  $\tau = 4 \text{ }\mu\text{C/m}$  is parallel to an infinite charged plane with  $\sigma = 30 \text{ }\mu\text{C/m}^2$ . Find the force per unit length on the wire.

**42.** Two parallel infinite planes have surface charge densities  $\sigma_1 = 10 \text{ nC/m}^2$  and  $\sigma_2 = -5 \text{ nC/m}^2$ . Find the electric field intensity between the planes and outside them.

**43.** Two balls with charges  $q_1 = 6.66 \text{ nC}$  and  $q_2 = -13.33 \text{ nC}$  are  $r_1 = 40 \text{ cm}$  apart. What work  $A$  must be done to bring them to distance  $r_2 = 25 \text{ cm}$ ?

**44.** Calculate the work required to move a point charge  $q = 5 \text{ nC}$  in the field of another point charge  $Q = 10 \text{ nC}$  from distance  $r_1 = 20 \text{ cm}$  to  $r_2 = 5 \text{ cm}$ .

**45.** Two point charges  $q_1 = 8 \text{ nC}$  and  $q_2 = -4 \text{ nC}$  start infinitely far apart. What work does the electric field do if they are brought to distance  $r = 10 \text{ cm}$ ?

**46.** A point charge  $q = 2 \text{ nC}$  moves in a uniform electric field  $E = 500 \text{ V/m}$  through distance  $d = 10 \text{ cm}$  along the field lines. What work does the field do?

**47.** What potential does the field of a point charge  $q = 12 \text{ nC}$  have at distance  $r = 30 \text{ cm}$ ? What work must be done to move a charge  $q_0 = 1 \text{ nC}$  from that point to a point with potential  $\varphi = 0$ ?

**48.** Two point charges  $q_1 = 15 \text{ nC}$  and  $q_2 = 20 \text{ nC}$  are  $r = 50 \text{ cm}$  apart. What work must be done to separate them to  $r' = 1 \text{ m}$ ?

**49.** An electron moves in an electric field from a point with potential  $\varphi_1 = 100 \text{ V}$  to a point with  $\varphi_2 = 250 \text{ V}$ . What work does the field do? Electron charge  $e = -1.6 \times 10^{-19} \text{ C}$ .

**50.** A ball of mass  $m = 1 \text{ g}$  and charge  $q = 10 \text{ nC}$  moves from point 1 ( $\varphi_1 = 600 \text{ V}$ ) to point 2 ( $\varphi_2 = 0$ ). Find its speed at point 1 if at point 2 it is  $v_2 = 20 \text{ cm/s}$ .

**51.** An electron moves from  $\varphi_1 = 300 \text{ V}$  to  $\varphi_2 = 100 \text{ V}$ , starting from rest. Find its speed at point 2.  $m_e = 9.11 \times 10^{-31} \text{ kg}$ ,  $e = -1.6 \times 10^{-19} \text{ C}$ .

**52.** A proton starts from rest at  $\varphi_1 = 500 \text{ V}$ . Find its speed at  $\varphi_2 = 0$ .  $m_p = 1.67 \times 10^{-27} \text{ kg}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ .

**53.** A ball of mass  $m = 5 \text{ g}$  and charge  $q = 20 \text{ nC}$  moves at  $v_1 = 50 \text{ cm/s}$  at potential  $\varphi_1 = 200 \text{ V}$ . What is the ball's speed at potential  $\varphi_2 = 100 \text{ V}$ ?

**54.** A particle of mass  $m = 0.1 \text{ mg}$  and charge  $q = -5 \text{ nC}$  enters a uniform electric field  $E = 100 \text{ V/m}$  perpendicularly to the field lines at  $v_0 = 10 \text{ m/s}$ . Find its speed after travelling  $s = 20 \text{ cm}$  along the field direction.

**55.** A charged particle of mass  $m = 2 \text{ g}$  and charge  $q = 15 \text{ nC}$  moves in an electric field with speed  $v_1 = 1 \text{ m/s}$  at potential  $\varphi_1 = 400 \text{ V}$ . What is its speed at  $\varphi_2 = 600 \text{ V}$ ?

**56.** What minimum speed must an electron be given at potential  $\varphi_1 = 0$  to reach a point with potential  $\varphi_2 = -50 \text{ V}$ ?

**57.** A horizontal flat capacitor with plate separation  $d = 1 \text{ cm}$  contains a charged droplet of mass  $m = 5 \times 10^{-11} \text{ g}$ . Without a field the droplet falls at constant speed due to air resistance. When voltage  $U = 600 \text{ V}$  is applied, the droplet falls twice as slowly. Find the droplet charge.

**58.** A charged droplet of mass  $m = 2 \times 10^{-10} \text{ g}$  is suspended between horizontal capacitor plates separated by  $d = 5 \text{ mm}$ . What voltage is applied if the droplet charge is  $q = 3.2 \times 10^{-19} \text{ C}$ ?

**59.** A charged particle of mass  $m = 10^{-14} \text{ kg}$  enters a uniform field  $E = 200 \text{ V/m}$  perpendicularly to the field lines. Find its charge if after  $10 \text{ ms}$  its vertical deflection is  $2 \text{ cm}$ . Neglect gravity.

**60.** A charged droplet of mass  $m = 10^{-11} \text{ kg}$  falls in a viscous medium at constant speed  $v_0$ . When a vertical field  $E = 10^5 \text{ V/m}$  is switched on, the droplet moves upward at the same speed  $v_0$ . Find the droplet charge.

**61.** In a flat capacitor with voltage  $U = 400 \text{ V}$  and plate separation  $d = 2 \text{ cm}$ , a dust particle of mass  $m = 10^{-9} \text{ kg}$  moves upward with acceleration  $a = 2 \text{ m/s}^2$ . What is the particle's charge? (Include gravity.)

**62.** Two horizontal capacitor plates are separated by  $d = 1.5$  cm. Field intensity between plates  $E = 500$  V/m. A charged droplet is suspended. What charge does it carry if its mass is  $m = 1.6 \times 10^{-10}$  g?

**63.** A charged ball is between horizontal capacitor plates ( $d = 8$  mm) at zero voltage and falls at constant speed  $v_0$ . At voltage  $U = 800$  V the ball is suspended. Find  $v_0$  if air resistance coefficient  $k = 3 \times 10^{-8}$  N·s/m and ball mass  $m = 4 \times 10^{-10}$  kg. (Resistance force is proportional to speed:  $F_{\text{res}} = kv$ .)

**64.** An electron enters a horizontal flat capacitor parallel to the plates at  $v_0 = 9 \times 10^6$  m/s. Voltage  $U = 100$  V, plate separation  $d = 1$  cm. Find the total  $a$ , normal  $a_n$  and tangential  $a_\tau$  accelerations at time  $t = 10$  ns after entering.

**65.** A proton enters a uniform field  $E = 5 \times 10^4$  V/m perpendicularly to field lines at  $v_0 = 10^6$  m/s. Find its speed and deflection angle from the initial direction at  $t = 5$  ns.

**66.** An electron enters a horizontal flat capacitor parallel to the plates at  $v_0 = 5 \times 10^6$  m/s. Plate length  $L = 5$  cm; field intensity  $E = 1000$  V/m. Find the electron's transverse displacement at the exit.

**67.** A charged particle of mass  $m = 10^{-12}$  kg and charge  $q = 2 \times 10^{-10}$  C enters a uniform field  $E = 500$  V/m at  $v_0 = 2 \times 10^5$  m/s at  $30^\circ$  to the field. Find the particle's speed at  $t = 20$  ns.

**68.** Between flat capacitor plates ( $d = 2$  cm,  $U = 200$  V) an electron enters parallel to the plates along the axis of symmetry. Find the initial speed if at exit it has deflected  $y = 0.5$  cm from the axis. Plate length  $L = 4$  cm.

**69.** An alpha particle (charge  $2e$ , mass  $4m_p$ ) enters a uniform field  $E = 10^3$  V/m parallel to field lines at  $v_0 = 10^4$  m/s, moving against the field direction. Find the time for its speed to halve.

**70.** An electron enters a flat capacitor perpendicularly to field lines at speed  $v_0$ . Plate length  $L$ , plate separation  $d$ . What minimum voltage must be applied for the electron not to exit the capacitor?

**71.** Find the capacitance of the Earth, treated as a sphere. Earth radius  $R = 6400$  km. By how much does Earth's potential change if it is given a charge of  $1$  C?

**72.** Calculate the capacitance of a metal sphere of radius  $R = 10$  cm in vacuum. What charge must be given to it to raise its potential to  $1000$  V?

**73.** By how many times does the capacitance of an isolated conductor change when it is immersed in a liquid dielectric with permittivity  $\epsilon = 4$ ?

**74.** A sphere of radius  $R = 20$  cm is charged to potential  $\phi = 500$  V. Find the charge.

**75.** Two concentric conducting spheres have radii  $R_1 = 5$  cm and  $R_2 = 10$  cm. Find the capacitance of this system.

**76.** What charge must be given to a metal sphere of radius  $R = 1$  m to raise its potential relative to infinity to  $9 \times 10^5$  V?

**77.** Find the potential of Earth's surface if it is treated as an isolated conducting sphere given charge  $q = -5 \times 10^5$  C. Earth radius  $R = 6400$  km.

**78.** A flat air capacitor has plate area  $S = 1$  m<sup>2</sup> and plate separation  $d = 1.5$  mm. Find its capacitance  $C$ .

**79.** A flat capacitor has plates of area  $S = 0.5$  m<sup>2</sup> separated by  $d = 2$  mm and filled with dielectric  $\epsilon = 5$ . Find its capacitance.

**80.** What plate area must a flat air capacitor have for its capacitance to equal  $1$   $\mu$ F with plate separation  $d = 0.1$  mm?

**81.** A flat capacitor has capacitance  $C = 200$  pF and plate separation  $d = 1$  mm. Find the plate area if the dielectric is glass ( $\epsilon = 7$ ).

**82.** A flat capacitor ( $C = 100$  pF) is charged to  $U = 100$  V. What charge is accumulated on its plates?

**83.** What plate separation must a flat capacitor have (plate area  $S = 0.2$  m<sup>2</sup>) for its capacitance in air to be  $C = 1$  nF?

**84.** A flat capacitor consists of two plates each of area  $S = 0.8$  m<sup>2</sup>. The space ( $d = 3$  mm) between them is filled with paraffin ( $\epsilon = 2.1$ ). Find its capacitance.

**85.** Find the total capacitance  $C$  of the capacitor network shown in Figure 17. Each capacitor has capacitance  $C_1 = 0.5$   $\mu$ F.

**86.** Three capacitors  $C_1 = 2$   $\mu$ F,  $C_2 = 3$   $\mu$ F,  $C_3 = 6$   $\mu$ F are connected in series. Find the total capacitance.

**87.** Two capacitors  $C_1 = 4$   $\mu$ F and  $C_2 = 6$   $\mu$ F are connected in parallel. Find the total capacitance.

**88.** Two capacitors  $C_1 = 1$   $\mu$ F and  $C_2 = 2$   $\mu$ F are in series; a third  $C_3 = 3$   $\mu$ F is connected in parallel to them. Find the total capacitance.

**89.** Four identical capacitors  $C_0 = 1 \mu\text{F}$  are connected as follows: two in parallel, then this pair in series with two others also in parallel. Find the total capacitance.

**90.** Find the total capacitance of a circuit with  $C_1 = 0.2 \mu\text{F}$ ,  $C_2 = 0.3 \mu\text{F}$  in series and  $C_3 = 0.5 \mu\text{F}$  connected in parallel to them.

**91.** How many identical capacitors of  $C_0 = 0.1 \mu\text{F}$  must be connected in series to obtain total capacitance  $C = 0.02 \mu\text{F}$ ?

## ELECTRIC CURRENT

### Key Formulae

Ohm's law for an arbitrary circuit section (generalised):  $I = (E_{12} + U_1 - U_2) / R$

Ohm's law for a closed circuit:  $I = E / (R + r)$

Resistance of a uniform conductor with constant cross-section:  $R = \rho l / S$

Total resistance in series:  $R = R_1 + R_2 + \dots$

Total resistance in parallel:  $1/R = 1/R_1 + 1/R_2 + \dots$

Kirchhoff's laws for branched circuits:

- Junction rule:  $\Sigma I = 0$  (algebraic sum of currents at a node equals zero)

- Loop rule:  $\Sigma E = \Sigma IR$  (algebraic sum of EMFs equals algebraic sum of voltage drops in a closed loop)

Total power dissipated in a circuit section:  $P = UI = I^2R = U^2/R$

Joule–Lenz law:  $Q = I^2Rt$

## Problems

1. How many turns of nichrome wire of diameter  $d = 1 \text{ mm}$  must be wound on a porcelain cylinder of radius  $a = 2.5 \text{ cm}$  to make a furnace of resistance  $R = 40 \text{ }\Omega$ ? ( $\rho_{\text{nichrome}} \approx 1.1 \times 10^{-6} \text{ }\Omega \cdot \text{m}$ )
2. What length of nichrome wire of diameter  $d = 0.5 \text{ mm}$  is needed to make a heating element of resistance  $R = 25 \text{ }\Omega$ ?
3. Find the resistance of a copper wire of length  $L = 100 \text{ m}$  and diameter  $d = 0.8 \text{ mm}$ . ( $\rho_{\text{copper}} = 1.7 \times 10^{-8} \text{ }\Omega \cdot \text{m}$ )
4. To what length must an aluminium wire of diameter  $d = 0.3 \text{ mm}$  be drawn to have resistance  $R = 10 \text{ }\Omega$ ? ( $\rho_{\text{aluminium}} = 2.8 \times 10^{-8} \text{ }\Omega \cdot \text{m}$ )
5. What diameter must a nichrome wire of length  $L = 5 \text{ m}$  have for its resistance to be  $R = 50 \text{ }\Omega$ ?
6. Calculate the resistance of a coil consisting of 100 turns of nichrome wire of diameter  $d = 0.6 \text{ mm}$  wound on a cylindrical former of radius  $a = 3 \text{ cm}$ .
7. 200 turns of nichrome wire are wound on a porcelain cylinder of radius  $a = 2 \text{ cm}$ . Find the wire diameter if the heater resistance is  $R = 30 \text{ }\Omega$ .
8. A copper-wire coil has resistance  $R = 108 \text{ }\Omega$ . Mass of the copper wire  $m = 3.41 \text{ kg}$ . Find the length  $l$  and diameter  $d$  of the wire. ( $\rho_{\text{copper}} = 1.7 \times 10^{-8} \text{ }\Omega \cdot \text{m}$ ; copper density  $\rho = 8900 \text{ kg/m}^3$ )
9. How many metres of copper wire of diameter  $d = 0.5 \text{ mm}$  are in a coil of resistance  $R = 4.2 \text{ }\Omega$ ?
10. An aluminium wire of length  $L = 200 \text{ m}$  has resistance  $R = 5.6 \text{ }\Omega$ . Find its diameter and mass. (Aluminium density  $\rho = 2700 \text{ kg/m}^3$ )
11. A nichrome wire of length  $L = 10 \text{ m}$  has resistance  $R = 11 \text{ }\Omega$ . Find its cross-sectional area and diameter.
12. Find the mass of a copper wire of length  $L = 50 \text{ m}$  and diameter  $d = 0.3 \text{ mm}$ . What is its resistance?
13. What resistance does an iron coil of mass  $m = 1 \text{ kg}$  have if the iron wire diameter is  $d = 0.2 \text{ mm}$ ? ( $\rho_{\text{iron}} = 9.8 \times 10^{-8} \text{ }\Omega \cdot \text{m}$ ; iron density =  $7800 \text{ kg/m}^3$ )

**14.** A heating element uses nichrome wire of length  $L = 8 \text{ m}$  and diameter  $d = 0.7 \text{ mm}$ . Find the mass and resistance of the wire.

**15.** Find the potential drop  $U$  across a copper conductor of length  $l = 500 \text{ m}$  and diameter  $d = 2 \text{ mm}$  carrying current  $I = 2 \text{ A}$ .

**16.** What current must pass through a nichrome wire of length  $l = 20 \text{ m}$  and diameter  $d = 0.5 \text{ mm}$  for the voltage drop to be  $U = 12 \text{ V}$ ?

**17.** A copper conductor of length  $L = 150 \text{ m}$  and cross-section  $S = 0.785 \text{ mm}^2$  is connected to a voltage source  $U = 5 \text{ V}$ . Find the current.

**18.** An aluminium wire of length  $l = 100 \text{ m}$  has resistance  $R = 2.8 \Omega$ . What voltage drop appears when current  $I = 1.5 \text{ A}$  flows through it?

**19.** A heating element has resistance  $R = 20 \Omega$ . When connected to  $U = 220 \text{ V}$ , what current flows and what is the potential drop?

**20.** What length of copper wire of diameter  $d = 0.4 \text{ mm}$  is needed so that a current  $I = 0.5 \text{ A}$  produces a voltage drop  $U = 0.1 \text{ V}$ ?

**21.** How much electrical energy is released in a nichrome heater of resistance  $R = 30 \Omega$  in 5 minutes if current  $I = 3 \text{ A}$  flows through it?

**22.** Find the potential drop  $U$  across resistors  $R_1 = 4 \Omega$ ,  $R_2 = 2 \Omega$  and  $R_3 = 4 \Omega$  (Fig. 19) if the ammeter reads  $I_1 = 3 \text{ A}$ . Also find currents  $I_2$  and  $I_3$  through  $R_2$  and  $R_3$ .

**23.** Three resistors  $R_1 = 5 \Omega$ ,  $R_2 = 10 \Omega$ ,  $R_3 = 15 \Omega$  are in series. An ammeter reads  $I = 2 \text{ A}$ . Find the voltage drop across each resistor and the total voltage.

**24.** Two resistors  $R_1 = 6 \Omega$  and  $R_2 = 12 \Omega$  are in parallel. The total current in the main line is  $I = 3 \text{ A}$ . Find the current through each resistor and the total voltage.

**25.**  $R_1 = 3 \Omega$  is in series with parallel combination of  $R_2 = 6 \Omega$  and  $R_3 = 4 \Omega$ . Total voltage  $U = 30 \text{ V}$ . Find the current through each resistor and voltage drop on  $R_1$ .

**26.** Three resistors  $R_1 = 2 \Omega$ ,  $R_2 = 3 \Omega$ ,  $R_3 = 5 \Omega$  in series. Voltage drop on  $R_2$  is  $U_2 = 6 \text{ V}$ . Find total current and voltage drops on  $R_1$  and  $R_3$ .

**27.**  $R_1 = 8 \Omega$  and  $R_2 = 4 \Omega$  are in parallel;  $R_3 = 6 \Omega$  is in series with them. Current through  $R_3$  is  $I_3 = 2 \text{ A}$ . Find currents through  $R_1$  and  $R_2$  and total voltage.

**28.** Three resistors  $R_1 = 12 \Omega$ ,  $R_2 = 6 \Omega$ ,  $R_3 = 4 \Omega$  are in parallel. Total current  $I = 6 \text{ A}$ . Find the current through each and total voltage.

**29.** Two identical cells, each with EMF  $E = 2 \text{ V}$  and internal resistance  $r = 0.3 \Omega$ , can be connected in series or parallel. Which gives greater current if: (a) external resistance  $R = 0.2 \Omega$ ; (b)  $R = 16 \Omega$ ? Find the current in each case.

**30.** Two identical current sources ( $E = 1.5 \text{ V}$ ,  $r = 0.4 \Omega$ ) are in series, connected to  $R = 1.2 \Omega$ . Find the current.

**31.** Three identical cells ( $E = 1.1 \text{ V}$ ,  $r = 0.6 \Omega$ ) in parallel. Find the current in external circuit  $R = 0.5 \Omega$ .

**32.** Four identical batteries ( $E = 12 \text{ V}$ ,  $r = 0.1 \Omega$ ): (a) in series; (b) in parallel. Find the current in both cases. External resistance  $R = 2 \Omega$ .

**33.** Two cells  $E_1 = 2 \text{ V}$ ,  $r_1 = 0.5 \Omega$  and  $E_2 = 1.5 \text{ V}$ ,  $r_2 = 0.3 \Omega$  in series with  $R = 1.2 \Omega$ . Find the current.

**34.** Sources ( $E = 4 \text{ V}$ ,  $r = 1 \Omega$ ) are combined into a battery. Which connection (series or parallel) is more efficient for external resistance  $R = 0.8 \Omega$ ? Justify.

**35.** Six identical cells ( $E_0 = 1.2 \text{ V}$ ,  $r_0 = 0.2 \Omega$  each) must be combined to give maximum current in  $R = 0.1 \Omega$ . How should they be connected? Find the maximum current.

**36.** Two cells ( $E_1 = E_2 = 2 \text{ V}$ ,  $r_1 = 1 \Omega$ ,  $r_2 = 1.5 \Omega$ ) in parallel, connected to  $R = 1.4 \Omega$  (Fig. 22). Find the current through each cell and the whole circuit.

**37.** Two sources ( $E_1 = 6 \text{ V}$ ,  $r_1 = 0.5 \Omega$  and  $E_2 = 4 \text{ V}$ ,  $r_2 = 0.3 \Omega$ ) in parallel, connected to  $R = 2 \Omega$ . Find the external circuit current and the current through each source.

**38.** Three identical cells ( $E = 1.5 \text{ V}$ ,  $r = 0.2 \Omega$ ) in parallel, connected to  $R = 0.8 \Omega$ . Find the current.

**39.** Sources  $E_1 = 12 \text{ V}$ ,  $r_1 = 1 \Omega$  and  $E_2 = 8 \text{ V}$ ,  $r_2 = 0.5 \Omega$  in series with  $R = 3 \Omega$ . Find the circuit current and voltage across  $R$ .

**40.** Two cells ( $E_1 = E_2 = 2 \text{ V}$ ,  $r_1 = r_2 = 0.5 \Omega$ ) in parallel with  $R = 0.2 \Omega$ . Find the current through  $R$ .

**41.** Two cells ( $E_1 = 3 \text{ V}$ ,  $r_1 = 0.6 \text{ } \Omega$  and  $E_2 = 2 \text{ V}$ ,  $r_2 = 0.4 \text{ } \Omega$ ) in parallel with an external resistor. Current through the first cell  $I_1 = 0.5 \text{ A}$ . Find the current through the second and the external resistance.

**42.** Three cells ( $E_1 = 4 \text{ V}$ ,  $r_1 = 0.2 \text{ } \Omega$ ;  $E_2 = 3 \text{ V}$ ,  $r_2 = 0.3 \text{ } \Omega$ ;  $E_3 = 5 \text{ V}$ ,  $r_3 = 0.1 \text{ } \Omega$ ) in parallel with  $R = 1 \text{ } \Omega$ . Find the currents through each source and through  $R$ .

**43.** Battery EMF  $E = 100 \text{ V}$ ; resistances  $R_1 = R_3 = 40 \text{ } \Omega$ ,  $R_2 = 80 \text{ } \Omega$ ,  $R_4 = 34 \text{ } \Omega$  (Fig. 27). Find current  $I_2$  through  $R_2$  and voltage drop  $U_2$ .

**44.** Battery  $E = 60 \text{ V}$ ,  $r = 2 \text{ } \Omega$ , with  $R_1 = 10 \text{ } \Omega$  in parallel with  $R_2 = 15 \text{ } \Omega$ , and  $R_3 = 20 \text{ } \Omega$  in series. Find current through  $R_1$  and voltage across  $R_3$ .

**45.** Source  $E = 50 \text{ V}$  (no internal resistance),  $R_1 = 20 \text{ } \Omega$  in series, then  $R_2 = 30 \text{ } \Omega$  in parallel with  $R_3 = 60 \text{ } \Omega$ . Find current in  $R_2$  and voltage across  $R_3$ .

**46.** Source  $E = 24 \text{ V}$ ,  $r = 1 \text{ } \Omega$ ;  $R_1 = 5 \text{ } \Omega$  in series with parallel  $R_2 = R_3 = 8 \text{ } \Omega$ . Find total current and voltage drop on  $R_1$ .

**47.** Source  $E = 80 \text{ V}$ ,  $r = 1 \text{ } \Omega$ ;  $R_1 = 10 \text{ } \Omega \parallel R_2 = 20 \text{ } \Omega$ , then  $R_3 = 40 \text{ } \Omega$  and  $R_4 = 5 \text{ } \Omega$  in series. Find current through  $R_1$  and total power in external circuit.

**48.** Source  $E = 120 \text{ V}$ ,  $r = 5 \text{ } \Omega$ ;  $R_1 = 25 \text{ } \Omega$  in series with parallel  $R_2 = R_3 = 50 \text{ } \Omega$  and  $R_4 = 100 \text{ } \Omega$ . Find current through  $R_3$  and voltage across  $R_4$ .

**49.** Battery  $E = 36 \text{ V}$ ,  $r = 0.5 \text{ } \Omega$ ;  $R_1 = 10 \text{ } \Omega$  in series with parallel  $R_2 = 12 \text{ } \Omega$  and  $R_3 = 24 \text{ } \Omega$ . Find current through  $R_2$  and power dissipated in  $R_1$ .

**50.**  $R_1 = R_2 = R_3 = 200 \text{ } \Omega$ , voltmeter resistance  $R_V = 1 \text{ k}\Omega$  (Fig. 29). Voltmeter reads  $U = 100 \text{ V}$ . Find battery EMF  $E$ .

**51.** Bridge circuit (Fig. 29):  $R_1 = 100 \text{ } \Omega$ ,  $R_2 = 150 \text{ } \Omega$ ,  $R_3 = 50 \text{ } \Omega$ . Voltmeter between nodes reads  $U_V = 10 \text{ V}$ . Source EMF  $E = 200 \text{ V}$  (no internal resistance). Find voltmeter resistance  $R_V$ .

**52.** Source  $E = 12 \text{ V}$  connected to  $R_1 = 4 \text{ } \Omega$  and  $R_2 = 6 \text{ } \Omega$  in series. Voltmeter ( $R_V = 24 \text{ } \Omega$ ) is connected in parallel to  $R_2$ . What does the voltmeter read?

**53.** Source  $E = 50 \text{ V}$ ,  $r = 2 \text{ } \Omega$ ;  $R_1 = 8 \text{ } \Omega$  and  $R_2 = 12 \text{ } \Omega$  in series. Voltmeter ( $R_V = 60 \text{ } \Omega$ ) in parallel to  $R_1$ . Find the voltmeter reading.

**54.** All resistors  $R_1 = R_2 = R_3 = 100 \text{ } \Omega$  (Fig. 29). Voltmeter with very high resistance ( $R_V \rightarrow \infty$ ). Source EMF  $E = 150 \text{ V}$ . What does the voltmeter read?

**55.**  $R_1 = 300 \Omega$  and  $R_2 = 600 \Omega$  in series; source  $U = 90 \text{ V}$ . Voltmeter in parallel to  $R_1$  reads  $U_V = 20 \text{ V}$ . Find the voltmeter's internal resistance.

**56.**  $R_1 = 150 \Omega$ ,  $R_2 = 250 \Omega$ ,  $R_3 = 100 \Omega$ ; voltmeter  $R_V = 500 \Omega$  reads  $U_V = 75 \text{ V}$  (Fig. 29). Source internal resistance  $\approx 0$ . Find EMF  $E$ .

**57.** Ammeter (max current  $I = 10 \text{ A}$ ,  $R_A = 0.18 \Omega$ , 100 divisions). Find the shunt resistance  $R$  to extend range to  $I_0 = 100 \text{ A}$ . How does the scale division value change?

**58.** Milliammeter ( $I_{\max} = 100 \text{ mA}$ ,  $R_A = 10 \Omega$ ). Find shunt to extend range to  $I_0 = 1 \text{ A}$ . How and where to connect it?

**59.** Ammeter (max  $I = 5 \text{ A}$ ,  $R_A = 0.05 \Omega$ ). Find shunt to extend range to  $I_0 = 25 \text{ A}$ .

**60.** Galvanometer ( $R_G = 50 \Omega$ , full deflection at  $I_G = 2 \text{ mA}$ ). Find shunt to measure up to  $I = 100 \text{ mA}$ .

**61.** Ammeter ( $R_A = 0.2 \Omega$ , max  $I = 5 \text{ A}$ , 50 divisions). What is the scale division value? Find shunt to double the division value.

**62.** Ammeter ( $R_A = 0.1 \Omega$ , max  $I = 20 \text{ A}$ ). Find shunt to extend range 10 times.

**63.** Ammeter ( $R_A = 0.02 \Omega$ ) with shunt  $R_{\text{sh}} = 0.005 \Omega$ . Current through shunt  $I_{\text{sh}} = 1 \text{ A}$ . What does the ammeter read?

**64.** Voltmeter (max  $U = 30 \text{ V}$ ,  $R_V = 2 \text{ k}\Omega$ , 150 divisions). Find series resistor  $R$  to extend range to  $U_0 = 75 \text{ V}$ . How does scale division value change?

**65.** Millivoltmeter ( $U_{\max} = 100 \text{ mV}$ ,  $R_V = 100 \Omega$ ). Find series resistor to extend range to  $U_0 = 10 \text{ V}$ .

**66.** Voltmeter (max  $U = 100 \text{ V}$ ,  $R_V = 5 \text{ k}\Omega$ ). Find series resistor to extend range to  $U_0 = 500 \text{ V}$ .

**67.** Galvanometer ( $R_G = 50 \Omega$ , full deflection at  $U_G = 100 \text{ mV}$ ). Find series resistor to measure up to  $U = 50 \text{ V}$ .

**68.** Voltmeter ( $R_V = 1 \text{ k}\Omega$ , max  $U = 15 \text{ V}$ , 75 divisions). What is scale division value? Find series resistor to halve the division value.

**69.** Voltmeter ( $R_V = 2.5 \text{ k}\Omega$ , max  $U = 25 \text{ V}$ ). Find series resistor to extend range 4 times.

**70.** Voltmeter in parallel with  $R = 300 \Omega$ . Reading  $U_V = 12 \text{ V}$ . Voltmeter internal resistance  $R_V = 1.5 \text{ k}\Omega$ . What current flows through  $R$ ?

**71.** A cell (EMF  $E = 6 \text{ V}$ ) gives maximum current  $I = 3 \text{ A}$ . Find the maximum heat  $Q_T$  that can be released in the external resistance per unit time.

**72.** Source:  $E = 12 \text{ V}$ ,  $r = 1 \ \Omega$ . What external resistance must be connected for maximum power? What is that maximum power?

**73.** Accumulator:  $E = 2.2 \text{ V}$ ,  $r = 0.1 \ \Omega$ . What power is dissipated in  $R = 0.4 \ \Omega$ ? What power is lost inside the accumulator?

**74.** Source: short-circuit current  $I_{sc} = 10 \text{ A}$ ; with  $R = 4 \ \Omega$  current is  $I = 2 \text{ A}$ . Find the EMF and internal resistance.

**75.** Heater  $R = 25 \ \Omega$  connected to  $U = 220 \text{ V}$ . How much heat is released in 10 minutes?

**76.** Cell EMF  $E = 9 \text{ V}$ . With external  $R = 2 \ \Omega$  the power on  $R$  is  $P = 8 \text{ W}$ . Find the internal resistance.

**77.** Maximum power from battery  $E = 4.5 \text{ V}$ ,  $r = 0.5 \ \Omega$ . At what external resistance is it achieved?

**78.** Battery  $E = 120 \text{ V}$ ;  $R_2 = 30 \ \Omega$ ,  $R_1 = 60 \ \Omega$  (Fig. 35). Ammeter reads  $I = 2 \text{ A}$ . Find power  $P$  dissipated in  $R_1$ .

**79.** Source  $E = 90 \text{ V}$ ;  $R_1 = 15 \ \Omega$  and  $R_2 = 25 \ \Omega$  in series; ammeter reads  $I = 2 \text{ A}$ . Find power dissipated on  $R_2$ .

**80.** Battery  $E = 12 \text{ V}$ ,  $r = 0.5 \ \Omega$ ,  $R = 3.5 \ \Omega$ . Find power dissipated in  $R$ .

**81.** Source  $E = 40 \text{ V}$  (no internal resistance);  $R_1 = 20 \ \Omega$  in parallel with  $R_2 = 30 \ \Omega$ . Find power dissipated in  $R_1$ .

**82.** Source  $E = 60 \text{ V}$ ,  $r = 1 \ \Omega$ ; external  $R_{ext} = 9 \ \Omega$ . What power does the external circuit consume?

**83.** Source  $E = 220 \text{ V}$ ; heater  $R = 50 \ \Omega$ . What power does the heater consume?

**84.** Source  $E = 10 \text{ V}$ ,  $r = 1 \ \Omega$ ,  $R = 4 \ \Omega$ . Find power dissipated in  $R$  and power lost inside the source.

**85.** Battery  $E = 100 \text{ V}$ ,  $r = 2 \ \Omega$ ;  $R_1 = 25 \ \Omega$  and  $R_2 = 78 \ \Omega$  (Fig. 35). Power dissipated in  $R_1$  is  $P_1 = 16 \text{ W}$ . What current does the ammeter show?

**86.** Source  $E = 50 \text{ V}$ ,  $r = 1 \ \Omega$ ; power in external resistance  $P = 150 \text{ W}$ . Find the resistance and current.

**87.** Accumulator  $E = 12 \text{ V}$ ; lamp  $R = 10 \ \Omega$  dissipates  $P = 12.1 \text{ W}$ . Find internal resistance.

**88.** Source  $E = 75 \text{ V}$ ,  $r = 1.5 \ \Omega$ ;  $R_1 = 20 \ \Omega \parallel R_2 = 30 \ \Omega$ ,  $R_3 = 10 \ \Omega$  in series. Find power on  $R_2$  and ammeter reading.

**89.** Source  $E = 24 \text{ V}$ ,  $R = 6 \ \Omega$ ; power in  $R$  is  $P = 72 \text{ W}$ . Find internal resistance.

**90.** What resistance must a resistor have ( $E = 40 \text{ V}$ ,  $r = 3 \ \Omega$ ) for it to dissipate  $P = 120 \text{ W}$ ?

**91.** With  $R_1 = 4 \ \Omega$ , power  $P_1 = 9 \text{ W}$ . With  $R_2 = 9 \ \Omega$ , power  $P_2 = 9 \text{ W}$ . Find EMF and internal resistance.

**92.** Electric kettle: volume  $V = 1 \text{ L}$  boils in  $\tau = 5 \text{ min}$ . Find heater power  $P$  and resistance  $R$  at  $U = 120 \text{ V}$ . Initial temperature  $t_0 = 13.5^\circ\text{C}$ . ( $c_{\text{water}} = 4200 \text{ J}/(\text{kg}\cdot^\circ\text{C})$ ,  $\rho_{\text{water}} = 1000 \text{ kg}/\text{m}^3$ ; assume 100% efficiency.)

**93.** Electric kettle power  $P = 2 \text{ kW}$ . How long to heat 1.5 L of water from  $20^\circ\text{C}$  to boiling? (100% efficiency)

**94.** Heating element  $R = 40 \ \Omega$  on  $U = 220 \text{ V}$ . How much heat is released in 10 minutes?

**95.** What current flows through a kettle heater of power  $P = 1.5 \text{ kW}$  on  $U = 220 \text{ V}$ ?

**96.** In 7 minutes a kettle (volume 0.8 L) heats water from  $15^\circ\text{C}$  to  $100^\circ\text{C}$ . Find heater power if efficiency is 85%.

**97.** A kettle coil ( $R = 30 \ \Omega$ ,  $U = 120 \text{ V}$ ): what mass of water can boil in 6 minutes starting at  $25^\circ\text{C}$ ? (100% efficiency)

**98.** What resistance must a 1.8 kW heating element have for  $U = 220 \text{ V}$ ? What current flows through it?

**99.** Batteries  $E_1 = 2 \text{ V}$ ,  $E_2 = 4 \text{ V}$ ,  $E_3 = 6 \text{ V}$ ; resistors  $R_1 = 4 \ \Omega$ ,  $R_2 = 6 \ \Omega$ ,  $R_3 = 8 \ \Omega$  (Fig. 46). Find currents in all branches.

**100.** Source  $E_1 = 10 \text{ V}$ ,  $r_1 = 1 \ \Omega$  with  $R_1 = 5 \ \Omega$ ; a second source  $E_2 = 5 \text{ V}$ ,  $r_2 = 0.5 \ \Omega$  with  $R_2 = 2 \ \Omega$  is connected in parallel. Find all branch currents.

**101.** Two batteries ( $E_1 = 6 \text{ V}$ ,  $r_1 = 0.5 \ \Omega$  and  $E_2 = 4 \text{ V}$ ,  $r_2 = 0.3 \ \Omega$ ) in parallel with  $R = 1 \ \Omega$ . Use Kirchhoff's laws to find all branch currents.

**102.** Circuit:  $R_1 = 2 \ \Omega$ ,  $R_2 = 3 \ \Omega$ ,  $R_3 = 4 \ \Omega$ ; sources  $E_1 = 8 \text{ V}$  (in branch  $R_1$ ) and  $E_2 = 12 \text{ V}$  (in branch  $R_2$ ).  $R_3$  is in parallel to  $R_1 + R_2$ . Find all branch currents.

**103.** Three parallel branches: (1)  $E_1 = 5 \text{ V}$ ,  $R_1 = 2 \ \Omega$ ; (2)  $E_2 = 7 \text{ V}$ ,  $R_2 = 3 \ \Omega$ ; (3)  $E_3 = 4 \text{ V}$ ,  $R_3 = 1 \ \Omega$ . Find currents in each branch.

**104.** Sources  $E_1 = 20 \text{ V}$  and  $E_2 = 10 \text{ V}$  in series;  $R_1 = 5 \ \Omega$  in parallel with  $R_2 = 10 \ \Omega$ . Find all currents.

**105.**  $E_1 = 1.5 \text{ V}$  in series with  $R_1 = 10 \ \Omega$ ;  $E_2 = 3 \text{ V}$  in series with  $R_2 = 20 \ \Omega$  — these two branches in parallel;  $R_3 = 30 \ \Omega$  in parallel to the whole. Find all branch currents.

**106.** Batteries  $E_1 = E_2 = E_3 = 6 \text{ V}$ ;  $R_1 = 20 \ \Omega$ ,  $R_2 = 12 \ \Omega$  (Fig. 46). Short-circuit current through the closing wire  $I = 1.6 \text{ A}$ . Find all branch currents and  $R_3$ .

**107.** Circuit similar to Fig. 46:  $E_1 = E_2 = E_3 = 4 \text{ V}$ ;  $R_1 = 8 \ \Omega$ ,  $R_2 = 10 \ \Omega$ ,  $R_3 = 12 \ \Omega$ . Find all branch currents and voltage on each branch.

**108.** Three identical sources ( $E = 1.5 \text{ V}$ ,  $r = 0.1 \ \Omega$ ) in parallel with  $R = 0.3 \ \Omega$ . Find the external circuit current and current through each source.

**109.** Source  $E = 9 \text{ V}$ ;  $R_1 = 18 \ \Omega$  and  $R_2 = 9 \ \Omega$  in parallel. Short-circuit current  $I_{sc} = 4.5 \text{ A}$ . Find internal resistance.

**110.** Battery  $E = 24 \text{ V}$ ,  $r = 1 \ \Omega$ ;  $R_1 = 5 \ \Omega$  and  $R_2 = 8 \ \Omega$  in series. A wire of zero resistance connects the midpoint between  $R_1$  and  $R_2$  to the negative terminal. Find the current through this wire.

**111.** Three parallel branches:  $E_1 = E_2 = E_3 = 10 \text{ V}$ ;  $R_1 = R_2 = R_3 = 5 \ \Omega$  (internal resistance included in  $R$ ). Find current in each branch.

**112.** Circuit (Fig. 46):  $E_1 = 3 \text{ V}$ ,  $E_2 = 6 \text{ V}$ ,  $E_3 = 9 \text{ V}$ ;  $R_1 = 15 \ \Omega$ ,  $R_2 = 10 \ \Omega$ . With  $R_{ext} = 5 \ \Omega$  connected at upper node, current through it  $I = 0.5 \text{ A}$ . Find  $R_3$ .

**113.** Batteries  $E_1 = E_2 = 100 \text{ V}$ ;  $R_1 = 20 \ \Omega$ ,  $R_2 = 10 \ \Omega$ ,  $R_3 = 40 \ \Omega$ ,  $R_4 = 30 \ \Omega$  (Fig. 47). Find ammeter reading.

**114.** Fig. 47:  $E_1 = 50 \text{ V}$ ,  $E_2 = 25 \text{ V}$ ;  $R_1 = 15 \ \Omega$ ,  $R_2 = 5 \ \Omega$ ,  $R_3 = 20 \ \Omega$ ,  $R_4 = 10 \ \Omega$ . Find ammeter reading.

**115.** Sources  $E_1 = 12 \text{ V}$  and  $E_2 = 8 \text{ V}$  connected in opposition in series with  $R_1 = 10 \ \Omega$ ;  $R_2 = 20 \ \Omega$  in parallel. Find ammeter current in main branch.

**116.** Fig. 47:  $E_1 = 70 \text{ V}$ ,  $E_2 = 30 \text{ V}$ ;  $R_1 = 10 \ \Omega$ ,  $R_2 = 5 \ \Omega$ ,  $R_3 = 25 \ \Omega$ ,  $R_4 = 15 \ \Omega$ . Find current through  $R_3$ .

**117.** Source  $E_1 = 40 \text{ V}$ ;  $R_1 = 8 \ \Omega$  in one branch;  $R_2 = 12 \ \Omega$  in other branch;  $E_2 = 20 \text{ V}$  in series with  $R_3 = 4 \ \Omega$  in the first branch. Ammeter in main branch. Find its reading.

**118.** Fig. 47:  $E_1 = E_2 = 150 \text{ V}$ ;  $R_1 = 25 \ \Omega$ ,  $R_2 = 15 \ \Omega$ ,  $R_3 = 50 \ \Omega$ ,  $R_4 = 35 \ \Omega$ . Find voltage across  $R_1$ .

**119.** Source  $E = 60 \text{ V}$  with  $R_1 = 10 \ \Omega$ ; in parallel: source  $E_2 = 30 \text{ V}$  in series with  $R_2 = 5 \ \Omega$ . Ammeter in  $R_1$  branch. Find its reading.

**120.** Fig. 47:  $E_1 = 2E_2$ ;  $R_1 = R_3 = 20 \ \Omega$ ,  $R_2 = 15 \ \Omega$ ,  $R_4 = 30 \ \Omega$ . Ammeter (upward direction) reads  $I = 1.5 \text{ A}$ . Find  $E_1$ ,  $E_2$  and currents  $I_2$ ,  $I_3$ .

**121.** Fig. 47:  $R_1 = 10 \ \Omega$ ,  $R_2 = 5 \ \Omega$ ,  $R_3 = 20 \ \Omega$ ,  $R_4 = 10 \ \Omega$ ;  $E_1 = 25 \text{ V}$ ,  $E_2 = 15 \text{ V}$ . Find ammeter current and its direction.

**122.** Fig. 47:  $R_1 = 12 \ \Omega$ ,  $R_2 = 6 \ \Omega$ ,  $R_3 = 18 \ \Omega$ ,  $R_4 = 9 \ \Omega$ ; ammeter reads  $I = 2 \text{ A}$ ;  $E_1 = E_2 = E$ . Find  $E$  and currents through  $R_1$  and  $R_2$ .

**123.** Circuit:  $E_1 = 6 \text{ V}$ ,  $E_2 = 4 \text{ V}$ ;  $R_1 = 3 \ \Omega$ ,  $R_2 = 2 \ \Omega$ ,  $R_3 = 5 \ \Omega$ . Ammeter reads  $I = 1 \text{ A}$ . Find internal resistances of both sources if they are equal.

**124.** Fig. 47:  $R_1 = 25 \ \Omega$ ,  $R_2 = 10 \ \Omega$ ,  $R_3 = 40 \ \Omega$ ,  $R_4 = 30 \ \Omega$ ;  $E_1 = 3E_2$ ; ammeter reads  $I = 0.8 \text{ A}$  (upward). Find EMF values.

**125.** Using mesh-current method, find all branch currents (Fig. 47):  $E_1 = 80 \text{ V}$ ,  $E_2 = 60 \text{ V}$ ;  $R_1 = 10 \ \Omega$ ,  $R_2 = 10 \ \Omega$ ,  $R_3 = 20 \ \Omega$ ,  $R_4 = 15 \ \Omega$ .

**126.** Circuit:  $E_1 = E_2$ ;  $R_1 = 5 \ \Omega$ ,  $R_2 = 10 \ \Omega$ ,  $R_3 = 15 \ \Omega$ ,  $R_4 = 20 \ \Omega$ . Ammeter reads  $I = 0.5 \text{ A}$ . Find EMF and currents through  $R_1$  and  $R_4$ .

## MAGNETIC FIELD

### Key Formulae

Superposition principle:  $H = \Sigma H_i$

Biot–Savart–Laplace law:  $dH = (I dl \times r) / (4\pi r^3)$

Magnetic field of an infinite straight conductor:  $H = I / (2\pi a)$

Magnetic field at the centre of a circular loop:  $H = I / (2R)$

Field on the axis of a circular loop at distance  $x$ :  $H = IR^2 / (2(R^2 + x^2)^{3/2})$

Ampère's circulation theorem (vacuum):  $\oint H dl = \Sigma I$

Gauss's theorem for magnetic field:  $\oint B dS = 0$

Lorentz force:  $F = q(v \times B)$

Ampère force law:  $F = I(l \times B)$

Work done when moving a current-carrying conductor:  $A = I\Delta\Phi$

Faraday's law of electromagnetic induction:  $\epsilon = -d\Phi/dt$

Self-induction EMF ( $L = \text{const}$ ):  $\epsilon_L = -L dI/dt$

Magnetic field energy ( $L = \text{const}$ ):  $W = LI^2/2$

## Problems

1. Find the magnetic field intensity  $H$  at a point located at distance  $a = 2$  m from an infinitely long conductor carrying current  $I = 5$  A.
2. Find the magnetic field intensity at distance  $r = 10$  cm from a long straight conductor carrying current  $I = 20$  A.
3. What current must flow through an infinitely long straight conductor for the magnetic field intensity at distance  $a = 5$  cm to equal  $H = 80$  A/m?
4. Find the magnetic flux density  $B$  at a point located at  $r = 0.2$  m from a straight conductor carrying current  $I = 15$  A. ( $\mu_0 = 4\pi \times 10^{-7}$  H/A<sup>2</sup>)
5. At what distance from a long straight conductor carrying  $I = 10$  A is the magnetic field intensity  $H = 25$  A/m?
6. A straight conductor carries current  $I = 3$  A in vacuum. Find the magnetic field intensity at distance  $r = 1$  m from the conductor.
7. Find the magnetic flux density at a point 5 cm from the axis of an infinitely long cylindrical conductor of radius 2 cm carrying uniformly distributed current  $I = 4$  A.
8. Find the magnetic field intensity  $H$  at the centre of a circular loop of radius  $R = 1$  cm carrying current  $I = 1$  A.
9. A circular loop of radius  $R = 5$  cm carries current  $I = 0.5$  A. Find the magnetic field intensity at its centre.
10. What current must flow through a circular loop of radius  $R = 2$  cm for the magnetic field intensity at its centre to equal  $H = 100$  A/m?
11. Find the magnetic flux density at the centre of a circular loop of radius  $R = 3$  cm carrying current  $I = 2$  A.
12. A solenoid has 500 turns per unit length. If current  $I = 0.2$  A flows through it, what is the magnetic field intensity inside?
13. A circular loop has magnetic field intensity  $H = 20$  A/m at its centre when carrying current  $I = 0.8$  A. Find the loop radius.
14. Find the magnetic flux density at the centre of a coil of diameter 10 cm consisting of 100 turns carrying current  $I = 0.1$  A.

**15.** Two long parallel straight conductors are  $d = 10$  cm apart, carrying equal and opposite currents  $I_1 = I_2 = 5$  A. Find the magnitude and direction of the magnetic field intensity  $H$  at a point located  $a = 10$  cm from each conductor.

**16.** Two parallel long conductors are  $d = 20$  cm apart, carrying currents  $I_1 = 10$  A and  $I_2 = 15$  A in the same direction. Find the magnetic field intensity at the midpoint between them.

**17.** Two parallel long conductors are  $d = 5$  cm apart, carrying currents  $I_1 = 3$  A and  $I_2 = 7$  A in opposite directions. Find the magnetic flux density at a point  $r = 5$  cm from the first and  $r = 10$  cm from the second (outside both conductors).

**18.** At what distance from the first of two parallel conductors ( $I_1 = 6$  A and  $I_2 = 8$  A in the same direction,  $d = 12$  cm apart) is the magnetic flux density equal to zero?

**19.** Two parallel conductors ( $I_1 = I_2 = 4$  A in opposite directions,  $d = 8$  cm apart). Find the magnetic field intensity at a point on the line connecting them, 4 cm from the first conductor.

**20.** Find the interaction force per unit length between two parallel conductors carrying  $I_1 = 2$  A and  $I_2 = 3$  A in the same direction, separated by  $d = 1$  cm.

**21.** Two long straight conductors are perpendicular to each other in the same plane, carrying  $I_1 = 5$  A and  $I_2 = 10$  A. Find the magnetic flux density at a point 3 cm from the first and 4 cm from the second.

**22.** Find the magnetic field intensity  $H$  on the axis of a circular loop at distance  $a = 3$  cm from its plane. Loop radius  $R = 4$  cm, current  $I = 2$  A.

**23.** A circular loop ( $R = 5$  cm,  $I = 1$  A) lies in the  $XY$  plane. Find the magnetic field intensity on the loop's axis ( $Z$ -axis) at distance  $a = 12$  cm from the centre.

**24.** A coil ( $D = 10$  cm,  $N = 20$  turns,  $I = 0.5$  A). Find the magnetic flux density on the coil's axis at distance  $a = 6$  cm from its centre.

**25.** At what distance from the centre of a circular loop ( $R = 6$  cm,  $I = 3$  A) is the on-axis magnetic field intensity  $H = 10$  A/m?

**26.** A circular loop has radius  $R = 8$  cm. What current must flow for the on-axis field intensity at distance  $a = 6$  cm from the loop plane to equal  $H = 15$  A/m?

**27.** Two circular loops ( $R = 10$  cm each, same current  $I = 4$  A in same direction) are parallel and  $d = 20$  cm apart. Find the magnetic field intensity at the midpoint on the common axis.

**28.** Find the magnetic field intensity at the centre of a circular loop ( $R = 7$  cm) if the on-axis field intensity at  $a = 24$  cm from the plane is  $H = 0.5$  A/m.

**29.** A square frame (side  $a = 4$  cm) is placed in a uniform magnetic field  $H = 79.6$  kA/m with its plane at angle  $\alpha = 45^\circ$  to the field. Find the magnetic flux  $\Phi$  through the frame.

**30.** A square frame (side  $a = 5$  cm) is in a uniform field  $B = 0.2$  T with the flux density vector perpendicular to the frame. Find the magnetic flux.

**31.** A coil ( $N = 100$  turns, cross-section  $S = 20$  cm<sup>2</sup>) is in a uniform field  $B = 0.1$  T with the coil axis at  $\alpha = 30^\circ$  to the field. Find the total magnetic flux.

**32.** The magnetic flux through a rectangular loop ( $S = 0.03$  m<sup>2</sup>) is  $\Phi = 0.006$  Wb. The loop plane is perpendicular to the field lines. Find the field induction.

**33.** By how many times does the magnetic flux through a circular loop ( $R = 10$  cm) change in a uniform field ( $B = 0.5$  T) when the angle between the normal and the field changes from  $0^\circ$  to  $90^\circ$ ?

**34.** A circular loop ( $R = 2$  cm) is in a uniform field  $H = 100$  A/m. Find the flux if the loop plane is parallel to the field.

**35.** A coil has  $N = 200$  turns, each of area  $S = 15$  cm<sup>2</sup>. What is the field induction if the total flux through the coil is  $\Phi = 0.003$  Wb when the field makes angle  $60^\circ$  with the normal to the turns?

**36.** How many ampere-turns are needed to create a magnetic flux  $\Phi = 0.42$  mWb in a solenoid with iron core of length  $l = 120$  cm and cross-section  $S = 3$  cm<sup>2</sup>? ( $\mu_{\text{iron}} \approx 5000$ )

**37.** A solenoid ( $l = 50$  cm,  $S = 5$  cm<sup>2</sup>,  $N = 200$  turns, no core). What current is needed to create  $B = 0.8$  T inside?

**38.** What magnetic flux appears in a solenoid with iron core ( $l = 80$  cm,  $S = 2$  cm<sup>2</sup>,  $N = 400$  turns,  $I = 0.5$  A)? ( $\mu \approx 2000$ )

- 39.** A solenoid has  $N = 300$  turns and requires 500 ampere-turns to create  $\Phi = 0.2$  mWb. Find the mean relative permeability of the core.  $l = 60$  cm,  $S = 4$  cm<sup>2</sup>.
- 40.** A toroidal iron core ( $l = 1.5$  m,  $S = 10$  cm<sup>2</sup>) has  $N = 1000$  turns wound on it. What current creates  $\Phi = 1$  mWb? ( $\mu \approx 3000$ )
- 41.** How many turns must be wound on a toroidal iron core ( $l = 100$  cm,  $S = 5$  cm<sup>2</sup>) so that current  $I = 0.2$  A produces  $B = 1.2$  T? ( $\mu \approx 4000$ )
- 42.** What force does a magnetic field  $B = 10$  mT exert on a conductor carrying  $I = 50$  A if the active conductor length is 0.1 m and the field lines are perpendicular to the current?
- 43.** A conductor (length 0.2 m, current 20 A) experiences force 0.05 N in a uniform field. Find the field induction if the conductor is perpendicular to the field lines.
- 44.** What force acts on a conductor (length 15 cm, current 10 A) in a uniform field  $B = 0.08$  T if the angle between the current and field is 30°?
- 45.** A conductor (length 0.5 m) is in a field  $B = 0.02$  T. What current must flow for a force of 0.1 N if the conductor is perpendicular to the field?
- 46.** What force does a uniform field  $B = 25$  mT exert on a straight conductor (length 0.3 m, current 15 A) if it is parallel to the field lines?
- 47.** A straight conductor (length 10 cm, current 5 A) is in a uniform field. The force on it is 0.002 N; angle between current and induction vector is 90°. Find  $B$ .
- 48.** Find the force on an electron moving at  $v = 10^6$  m/s perpendicular to a uniform field  $B = 0.5$  T. ( $e = 1.6 \times 10^{-19}$  C)
- 49.** A horizontal conductor (length 20 cm, mass 4 g) carries current  $I = 10$  A. Find the magnetic induction (magnitude and direction) needed for the Ampère force to balance gravity.
- 50.** A horizontal conductor (length 15 cm, mass 3 g). What current must flow for equilibrium in a uniform horizontal field  $B = 0.03$  T perpendicular to the conductor?
- 51.** A horizontal conductor (length 0.25 m, mass 5 g) is in a vertical uniform field  $B = 0.1$  T. What current is needed for the Ampère force to balance gravity?

**52.** A conductor (mass 10 g, current 5 A) is in a uniform field  $B = 0.04$  T. What length must it be if it lies horizontally and the Ampère force balances gravity? The field is perpendicular to the conductor.

**53.** A horizontal conductor (length 0.1 m, mass 2 g) carries current 8 A. Find the direction and minimum magnetic induction for the conductor to be in equilibrium.

**54.** A straight conductor (length 30 cm, mass 6 g) is suspended on two light strings in a horizontal field  $B = 0.05$  T. If current  $I = 12$  A is passed, by what angle does the conductor deflect from the vertical?

**55.** What mass per unit length of a horizontal conductor carries when current 2 A keeps it in equilibrium in a uniform vertical field  $B = 0.06$  T?

**56.** An electron enters a uniform field perpendicular to its motion. Speed  $v = 4 \times 10^7$  m/s,  $B = 1$  mT. Find the tangential  $a_\tau$  and normal  $a_n$  accelerations.

**57.** A proton enters a uniform field perpendicular to the field lines at  $v = 10^6$  m/s.  $B = 0.5$  T. Find the orbital radius. ( $m_p = 1.67 \times 10^{-27}$  kg,  $e = 1.6 \times 10^{-19}$  C)

**58.** An electron moves in a circle of radius  $R = 2$  cm in a field  $B = 0.01$  T. Find the electron's speed.

**59.** A particle ( $q = 3.2 \times 10^{-19}$  C,  $m = 6.64 \times 10^{-27}$  kg) moves in a field  $B = 0.2$  T with velocity perpendicular to  $B$ . Find the period of revolution.

**60.** An electron enters a uniform field at  $30^\circ$  to the field lines at  $v = 2 \times 10^7$  m/s,  $B = 0.1$  T. Find the radius of the helical trajectory.

**61.** What field induction is needed for an electron to move in a circle of radius  $R = 5$  cm at  $v = 10^7$  m/s?

**62.** A proton enters a field  $B = 0.05$  T. If the normal acceleration is  $a_n = 5 \times 10^{13}$  m/s<sup>2</sup>, find its speed (initial velocity perpendicular to field).

**63.** A charged particle moves in a circle at  $v = 10^6$  m/s in a field  $B = 0.3$  T with radius  $R = 4$  cm. Find its charge if its energy  $W = 12$  keV.

**64.** An electron moves in a circle of radius  $R = 5$  cm in a field  $B = 0.1$  T. Find its kinetic energy.

**65.** A proton moves in a field  $B = 0.5$  T in a circle. If its kinetic energy is  $W_k = 10$  keV, find the radius.

**66.** An alpha particle (charge  $2e$ , mass  $4m_p$ ) moves in a field  $B = 0.02$  T in a circle of radius  $R = 10$  cm. Find its speed.

**67.** A charged particle ( $m = 1.67 \times 10^{-27}$  kg) moves in a field  $B = 0.05$  T in a circle of radius  $R = 20$  cm at  $v = 10^5$  m/s. Find its charge.

**68.** What field induction is needed for an electron with kinetic energy  $W_k = 1$  keV to move in a circle of radius  $R = 1$  cm?

**69.** A charged particle ( $W_k = 5$  keV) moves in a circle in a uniform field.  $R = 3$  cm,  $B = 0.4$  T. Find the charge-to-mass ratio  $q/m$ .

**70.** A proton and an  $\alpha$ -particle enter a uniform field perpendicular to the lines. Find the ratio of the proton's revolution period  $T_1$  to the  $\alpha$ -particle's  $T_2$ .

**71.** An electron and a proton enter a uniform field perpendicularly with equal speeds. Compare their orbital radii.

**72.** Two charged particles with equal charges but different masses  $m_1$  and  $m_2$  enter a uniform field perpendicularly. Find the ratio of their revolution periods.

**73.** An alpha particle and a proton move in circles of equal radius in the same uniform field. Find the ratio of their kinetic energies.

**74.** An electron and a positron (same mass, opposite charge) enter a uniform field perpendicularly. Compare their revolution periods.

**75.** What speed must a proton have so that its period in field  $B_1 = 0.1$  T equals the period of an electron in field  $B_2 = 0.5$  T?

**76.** Prove that the period of revolution of a charged particle in a uniform magnetic field (perpendicular to its velocity) is independent of the particle's speed.

**77.** An electron accelerated through  $U = 6$  kV enters a uniform field at  $\alpha = 30^\circ$  to the field and moves in a helix.  $B = 13$  mT. Find the radius  $R$  and pitch  $h$  of the helix.

**78.** A proton accelerated through  $U = 5$  kV enters a uniform field perpendicular to the field lines.  $B = 0.2$  T. Find the orbital radius.

**79.** An electron enters a field  $B = 10$  mT at  $60^\circ$  to the field at  $v = 10^7$  m/s. Find the radius and pitch of the helical trajectory.

**80.** An ion (charge  $+e$ , mass  $m = 1.6 \times 10^{-27}$  kg) is accelerated through  $U = 1$  kV and enters a field  $B = 0.05$  T at  $45^\circ$ . Find the radius and pitch of the helix.

**81.** A particle accelerated through  $U = 2$  kV moves in a helix ( $R = 3$  cm,  $h = 8$  cm) in a field  $B = 0.1$  T. Find  $q/m$ .

**82.** A proton ( $v = 5 \times 10^6$  m/s) moves in a helix in a field  $B = 20$  mT. Find the angle between the velocity and field if the helix radius  $R = 10$  cm.

**83.** What field induction is needed for an electron accelerated through  $U = 3$  kV to move in a helix with pitch  $h = 5$  cm if the angle between velocity and field is  $\alpha = 30^\circ$ ?

**84.** An electron enters a horizontal flat capacitor parallel to the plates at  $v = 10^7$  m/s. Capacitor length  $l = 5$  cm, field  $E = 10$  kV/m. On exit it enters a magnetic field perpendicular to the electric field,  $B = 10$  mT. Find  $R$  and  $h$  of the helical trajectory.

**85.** An electron accelerated through  $U = 2$  kV enters a uniform field perpendicular to the field lines.  $B = 5$  mT. Find the orbital radius.

**86.** A proton enters a flat capacitor parallel to the plates at  $v_0 = 10^6$  m/s. Plate length  $L = 10$  cm,  $E = 500$  V/m. On exit it enters a uniform field  $B = 0.01$  T perpendicular to its velocity. Find the orbital radius in the field.

**87.** An electron enters a field  $B = 15$  mT at  $45^\circ$  at  $v = 5 \times 10^6$  m/s. Find the radius and pitch of the helical trajectory.

**88.** A charged particle ( $m = 3.34 \times 10^{-27}$  kg,  $q = 1.6 \times 10^{-19}$  C) enters a flat capacitor ( $L = 8$  cm,  $E = 2000$  V/m) at  $v_0 = 2 \times 10^5$  m/s parallel to the plates. After the capacitor it enters a field  $B = 0.02$  T perpendicular to its velocity. Find the radius of curvature.

**89.** An electron leaves an electron gun with kinetic energy  $W_k = 1.6 \times 10^{-16}$  J and enters a field  $B = 20$  mT at  $30^\circ$  to the field. Find the pitch of the helix.

**90.** A proton enters a flat capacitor ( $d = 2$  cm,  $U = 400$  V) parallel to the plates at  $v_0 = 10^5$  m/s. After exit it enters a field  $B = 0.05$  T perpendicular to the plane of motion. Find the orbital radius.

**91.** A conductor ( $l = 10$  cm) moves at  $v = 15$  m/s perpendicular to a field  $B = 0.1$  T. Find the induced EMF.

**92.** A conductor (length  $0.5$  m) moves at  $20$  m/s perpendicular to field lines. What is  $B$  if the induced EMF is  $1.5$  V?

**93.** A conductor (length 25 cm) moves in a field  $B = 0.05$  T; the induced EMF is 0.8 V. Find the speed if it is perpendicular to the field.

**94.** A straight conductor (length 1.2 m) moves in a field  $B = 0.02$  T at  $30^\circ$  to B. Find the speed for an induced EMF of 0.12 V.

**95.** A frame ( $S = 100$  cm<sup>2</sup>) rotates at  $\omega = 10$  rad/s in a field  $B = 0.3$  T. Find the peak induced EMF.

**96.** What length must a conductor have to induce  $EMF = 2.5$  V when moving at  $v = 5$  m/s perpendicularly to a field  $B = 0.4$  T?

**97.** A conductor (length 0.8 m) moves at  $v = 10$  m/s in a uniform field; induced  $EMF = 0.4$  V. Find B (motion perpendicular to field).

**98.** A coil ( $D = 10$  cm,  $N = 500$  turns) is in a magnetic field that increases from 0 to 2 T in  $t = 0.1$  s. Find the average induced EMF.

**99.** A solenoid ( $N = 300$  turns,  $S = 50$  cm<sup>2</sup>). The field inside decreases uniformly from 0.4 T to 0.1 T in 0.05 s. Find the average induced EMF.

**100.** An inductance coil has  $N = 1000$  turns. The flux through each turn changes by 0.002 Wb in 0.2 s. What is the average induced EMF?

**101.** A frame ( $S = 200$  cm<sup>2</sup>,  $N = 50$  turns) is in a uniform field perpendicular to its plane. B increases uniformly from 0.1 T to 0.6 T in 0.15 s. Find the average induced EMF.

**102.** Find the number of turns in a coil if a flux change of 0.001 Wb in 0.01 s induces an average EMF of 20 V.

**103.** A coil has  $N = 200$  turns and diameter 12 cm. What rate of change of B is needed to produce an average induced EMF of 0.5 V (field perpendicular to coil plane)?

**104.** A solenoid ( $N = 600$  turns,  $l = 40$  cm,  $S = 30$  cm<sup>2</sup>, no core). What average EMF is induced when the current decreases from 5 A to 0 in 0.08 s?

**105.** How many turns must a coil ( $S = 50$  cm<sup>2</sup>) have for B changing from 0.2 T to 0.3 T in 4 ms to induce  $EMF = 10$  V?

**106.** A coil ( $N = 200$  turns,  $A = 30$  cm<sup>2</sup>). The field perpendicular to the turns increases from 0.1 T to 0.5 T in 0.02 s. Find the average induced EMF.

**107.** A coil has  $N = 150$  turns and  $S = 25$  cm<sup>2</sup>. Find the change in B if in 0.01 s an average EMF of 5 V is induced.

**108.** What cross-section must a coil of  $N = 400$  turns have so that  $B$  changing from  $0.05\text{ T}$  to  $0.25\text{ T}$  in  $0.005\text{ s}$  induces  $\text{EMF} = 12\text{ V}$ ?

**109.** A coil ( $N = 300$  turns,  $S = 40\text{ cm}^2$ ). In what time must  $B$  change from  $0.3\text{ T}$  to  $0.7\text{ T}$  for an average induced EMF of  $8\text{ V}$ ?

**110.** A coil ( $N = 100$  turns,  $A = 20\text{ cm}^2$ ,  $\Delta t = 0.01\text{ s}$ ,  $E = 4\text{ V}$ ,  $B_1 = 0.1\text{ T}$ ). Find  $B_2$ .

**111.** By how many times does the average induced EMF increase if the number of turns is doubled and the rate of change of  $B$  is increased 1.5 times?

**112.** The magnetic flux inside a loop of radius  $5\text{ cm}$  changed by  $18.6\text{ mWb}$  in  $5.9\text{ ms}$ . Find the eddy electric field intensity in the loop.

**113.** The magnetic flux in a loop of radius  $10\text{ cm}$  changed by  $25\text{ mWb}$  in  $8\text{ ms}$ . Find the average eddy electric field intensity.

**114.** The flux through a circular loop ( $R = 3\text{ cm}$ ) changes at  $d\Phi/dt = 0.5\text{ Wb/s}$ . Find the eddy electric field intensity.

**115.** A loop of radius  $2\text{ cm}$  is in a magnetic field. What rate of flux change is needed for the eddy field intensity along the loop to be  $1\text{ V/m}$ ?

**116.** Find the loop radius if a flux change of  $0.01\text{ Wb}$  in  $0.002\text{ s}$  produces eddy field intensity of  $2\text{ V/m}$ .

**117.** The induced EMF in a conducting ring of radius  $R = 7\text{ cm}$  is  $0.4\text{ V}$ . Find the average eddy electric field along the ring.

**118.** A circular loop ( $R = 4\text{ cm}$ ) is perpendicular to a uniform field that increases uniformly from  $0.1\text{ T}$  to  $0.6\text{ T}$  in  $0.01\text{ s}$ . Find the average eddy electric field.

**119.** What charge passes through a loop ( $R = 0.03\ \Omega$ ) when the flux inside decreases by  $\Delta\Phi = 12\text{ mWb}$ ?

**120.** A coil ( $R = 0.1\ \Omega$ ,  $N = 500$  turns). What charge passes when the flux through each turn changes by  $0.005\text{ Wb}$ ?

**121.** Charge  $2\text{ C}$  passes through a conducting ring ( $R = 0.05\ \Omega$ ). By how much did the magnetic flux change?

**122.** The flux through a loop changed by  $8\text{ mWb}$ . Find the loop resistance if charge  $0.2\text{ C}$  passed through it.

**123.** A solenoid ( $N = 100$  turns,  $R = 0.5\ \Omega$ ). What charge passes if the flux through each turn changes by  $10\text{ mWb}$ ?

**124.** A ring ( $A = 20 \text{ cm}^2$ ,  $R = 0.01 \ \Omega$ ) is in a field perpendicular to its plane. By how much did  $B$  change if charge  $0.5 \text{ C}$  passed?

**125.** A field is switched off; charge  $0.1 \text{ C}$  passes through a loop ( $R = 0.02 \ \Omega$ ). Find the initial magnetic flux if the final flux is zero.

**126.** A circular loop ( $R = 34 \text{ cm}$ ) is made of copper wire ( $S = 1 \text{ mm}^2$ ) in a field  $B = 0.1 \text{ T}$  perpendicular to it. What charge passes when the field disappears?

**127.** A circular copper loop ( $R = 5 \text{ cm}$ , wire cross-section  $S = 0.5 \text{ mm}^2$ ) is in a field  $B = 0.2 \text{ T}$  perpendicular to the loop. What charge passes when the loop is rotated  $90^\circ$ ? ( $\rho_{\text{Cu}} = 1.7 \times 10^{-8} \ \Omega \cdot \text{m}$ )

**128.** An aluminium ring ( $R = 10 \text{ cm}$ , wire  $S = 2 \text{ mm}^2$ ) is in a field  $B = 0.05 \text{ T}$  perpendicular to it. What charge passes when it is removed from the field? ( $\rho_{\text{Al}} = 2.8 \times 10^{-8} \ \Omega \cdot \text{m}$ )

**129.** A loop ( $R = 0.01 \ \Omega$ ,  $A = 100 \text{ cm}^2$ ) is in a field. Find  $B$  if charge  $5 \times 10^{-3} \text{ C}$  passes when the field disappears.

**130.** A conducting ring (diameter  $20 \text{ cm}$ ,  $\rho = 10^{-7} \ \Omega \cdot \text{m}$ , wire  $S = 1.5 \text{ mm}^2$ ) is in field  $B = 0.3 \text{ T}$ . Find the charge that passes when the field drops to zero.

**131.** A solenoid ( $N = 500$  turns,  $A = 20 \text{ cm}^2$ ,  $R = 2 \ \Omega$ ) is in a uniform field parallel to its axis.  $B$  decreases from  $0.8 \text{ T}$  to  $0$  in  $0.01 \text{ s}$ . What charge passes?

**132.** What radius must a circular copper loop ( $S = 0.8 \text{ mm}^2$ ) have so that removing it from field  $B = 0.15 \text{ T}$  causes charge  $0.01 \text{ C}$  to pass? (Normal to plane is parallel to field)

**133.** A loop made of aluminium wire (length  $10 \text{ cm}$ ,  $S = 1.4 \text{ mm}^2$ ) has flux changing at  $10 \text{ mWb/s}$ . Find the induction current.

**134.** A conducting loop ( $R = 0.02 \ \Omega$ ). Find the rate of flux change for induction current  $0.5 \text{ A}$ .

**135.** A coil ( $N = 200$  turns,  $R = 10 \ \Omega$ ) has induction current  $0.1 \text{ A}$ . What is the rate of flux change through one turn?

**136.** Flux changes at  $0.03 \text{ Wb/s}$  through a loop. What resistance is needed for induction current  $0.6 \text{ A}$ ?

**137.** Copper wire (length 20 cm,  $S = 1 \text{ mm}^2$ ) forms a closed loop. Flux changes at 5 mWb/s. Find the induction current. ( $\rho_{\text{Cu}} = 1.7 \times 10^{-8} \Omega \cdot \text{m}$ )

**138.** Aluminium wire ( $L = 1 \text{ m}$ ,  $S = 0.8 \text{ mm}^2$ ) is bent into a circle. What flux change rate produces induction current 0.2 A?

**139.** A loop ( $R = 0.15 \Omega$ ,  $S = 50 \text{ cm}^2$ ).  $B$  increases uniformly from 0.1 T to 0.4 T in 0.003 s. Find the induction current.

**140.** How many turns does a coil have ( $L = 1 \text{ mH}$ ) if at  $I = 1 \text{ A}$  the flux through it is  $\Phi = 2 \mu\text{Wb}$ ?

**141.** A coil has  $N = 500$  turns. At  $I = 0.5 \text{ A}$  the flux is  $\Phi = 1.5 \mu\text{Wb}$ . Find the inductance.

**142.** Solenoid inductance  $L = 0.2 \text{ H}$ . How many turns does it have if at  $I = 2 \text{ A}$  the flux through each turn is  $10 \mu\text{Wb}$ ?

**143.** Find the inductance of a coil ( $N = 300$  turns) if at  $I = 0.8 \text{ A}$  the flux through each turn is  $3 \mu\text{Wb}$ .

**144.** A coil has inductance  $L = 5 \text{ mH}$ . What flux passes through it at  $I = 0.1 \text{ A}$ ?

**145.** Current  $I = 0.3 \text{ A}$  creates flux 0.6 mWb in a coil of  $N = 1000$  turns. Find the inductance.

**146.** By how many times does the inductance change if the number of turns is doubled (other parameters unchanged)?

**147.** A solenoid with iron core ( $S = 10 \text{ cm}^2$ ,  $l = 1 \text{ m}$ ). Find the core permeability  $\mu$  if the flux through the cross-section is  $\Phi = 1.4 \text{ mWb}$ . At what current  $I$  does this occur if  $L = 0.44 \text{ H}$ ?

**148.** Solenoid ( $l = 50 \text{ cm}$ ,  $N = 400$  turns,  $S = 5 \text{ cm}^2$ , core  $\mu = 1000$ ). Find the inductance.

**149.** Solenoid ( $N = 200$  turns,  $S = 20 \text{ cm}^2$ ,  $l = 20 \text{ cm}$ ) carries  $I = 0.5 \text{ A}$ ; flux = 0.5 mWb. Find the core permeability.

**150.** What current must pass through a solenoid ( $L = 0.1 \text{ H}$ ,  $N = 100$  turns) to create flux 2 mWb?

**151.** Solenoid ( $l = 0.8 \text{ m}$ ,  $S = 12 \text{ cm}^2$ ,  $N = 600$  turns,  $\mu = 800$ ,  $I = 1.5 \text{ A}$ ). Find the flux.

**152.** Solenoid with core:  $L = 0.3 \text{ H}$ ,  $I = 1 \text{ A}$ ,  $N = 250$  turns,  $l = 0.6 \text{ m}$ ,  $\mu = 500$ . Find the cross-sectional area.

**153.** What length must a solenoid ( $N = 700$  turns,  $S = 8 \text{ cm}^2$ ,  $I = 0.7 \text{ A}$ ,  $\mu = 600$ ) have to produce flux  $1.2 \text{ mWb}$ ?

## SOUND OSCILLATIONS

### Key Formulae

Wave propagation speed:  $v = \lambda\nu$  ( $\lambda$  — wavelength)

Speed of longitudinal waves in thin rods:  $v = \sqrt{E/\rho}$  ( $E$  — Young's modulus,  $\rho$  — density)

Speed of transverse waves:  $v = \sqrt{G/\rho}$  ( $G$  — shear modulus)

Speed of longitudinal waves in an unbounded elastic medium:  $v = \sqrt{k/\rho}$  ( $k$  — bulk modulus)

Sound level (dB):  $L_N = 10 \cdot \lg(I/I_0)$  ( $I$  — intensity;  $I_0$  — threshold intensity)

Acoustic Doppler effect:  $\nu' = [(c \pm v)/(c \pm u)] \nu$  ( $c$  — sound speed;  $v$  — listener speed;  $u$  — source speed)

Logarithmic decrement:  $\Theta = \ln(A_n/A_{n+1}) = \beta T$

Period of a simple pendulum:  $T = 2\pi\sqrt{l/g}$

Period of a spring oscillator:  $T = 2\pi\sqrt{m/k}$

Period of a physical pendulum:  $T = 2\pi\sqrt{L/g}$ ,  $L = I/(md)$

Period of torsional oscillations:  $T = 2\pi\sqrt{I/k}$

Period of damped oscillations:  $T = 2\pi/\sqrt{k/m - \beta^2}$ ,  $\beta = r/(2m)$

Amplitude of forced oscillations ( $F = F_0 \cos \omega t$ ):  $A = f_0 / \sqrt{[(\omega_0^2 - \omega^2)^2 + 4\beta^2 \omega^2]}$

Period of a uniform string:  $T = 2l\sqrt{m/F}$  ( $l$  — length;  $m$  — mass per unit length;  $F$  — tension)

Total energy of a harmonic oscillator:  $E = E_k + E_p = m\omega^2 A^2/2$

## Problems

1. Find the wavelength  $\lambda$  of the note A (frequency  $\nu = 435$  Hz). Speed of sound in air  $c = 340$  m/s.
2. A sound wave has frequency  $\nu = 262$  Hz (note C). What is its wavelength in air if  $c = 343$  m/s?
3. A sound wave in air has wavelength 0.8 m. If  $c = 340$  m/s, what is the frequency?
4. A sound source vibrates at 100 Hz and the wavelength in some medium is 3.3 m. What is the speed of sound in that medium?
5. Find the frequency of a sound wave whose wavelength in air is 0.5 m ( $c = 340$  m/s).
6. At what frequency does a source vibrate if it creates a wave of wavelength 1.7 m in air at  $c = 340$  m/s?
7. The speed of sound in water is approximately 1500 m/s. What is the wavelength of a 1000 Hz sound in water?
8. The human ear can detect frequencies from  $\nu_1 = 20$  Hz to  $\nu_2 = 20\,000$  Hz. What is the corresponding range of wavelengths?  $c = 340$  m/s.
9. Find the range of wavelengths audible to the human ear if  $c = 330$  m/s.
10. A bat emits ultrasound at 80 kHz. What is the wavelength in air ( $c = 340$  m/s)? Can the human ear detect it?
11. Infrasound has frequency 10 Hz. What is its wavelength in air ( $c = 340$  m/s)? Is it in the audible range?
12. If the audible wavelength range is from 1.7 cm to 17 m, what is the approximate speed of sound in the medium?
13. Marine animals may detect sounds from 100 Hz to 150 kHz;  $c_{\text{water}} = 1500$  m/s. What is their wavelength range?
14. What is the minimum audible frequency if the maximum detectable wavelength is 17 m?  $c = 340$  m/s.
15. Find the speed of sound in steel.
16. The speed of sound in water is approximately 1480 m/s. How long does it take for sound to travel 1.5 km?

17. What is the density of a medium if the speed of sound in it is 500 m/s and Young's modulus is  $2 \times 10^9$  Pa?
18. Find the speed of sound in copper:  $E = 11 \times 10^{10}$  Pa,  $\rho = 8960$  kg/m<sup>3</sup>.
19. Sound travels in aluminium at 6420 m/s. What is the frequency if the wavelength is 3.21 m?
20. How long does a sound signal take to travel 10 km in steel ( $c = 5100$  m/s)?
21. Compare the speed of sound in air (340 m/s) and in steel (5100 m/s). How many times faster is it in steel?
22. Two trains approach each other at  $v_1 = 72$  km/h and  $v_2 = 54$  km/h. The first train sounds a whistle at  $\nu = 600$  Hz. Find the frequency  $\nu'$  heard by a passenger on the second train: (a) before the trains meet; (b) after.  $c = 340$  m/s.
23. A car moves at 72 km/h toward a wall and sounds its horn at 500 Hz. What frequency does the driver hear in the reflection?  $c = 340$  m/s.
24. A 1000 Hz source moves away from a stationary observer at 30 m/s. What frequency does the observer hear?  $c = 340$  m/s.
25. An observer moves at 20 m/s toward a stationary 800 Hz source. What frequency is heard?  $c = 340$  m/s.
26. A plane flies at 360 km/h over a stationary observer who hears the plane's siren. What frequency does the observer hear as the plane approaches and then recedes, if the siren's natural frequency is 400 Hz?  $c = 340$  m/s.
27. Two ships move on parallel courses in the same direction. The first (50 km/h) sounds its horn at 200 Hz; the second moves at 30 km/h. What horn frequency does a passenger on the second ship hear?  $c = 340$  m/s.
28. A police car (siren frequency 700 Hz) moves at 90 km/h. What frequency does a stationary pedestrian hear: (a) as the car approaches; (b) as it recedes?  $c = 340$  m/s.
29. As a train passes a stationary observer the whistle frequency jumps. What percentage of the true frequency is the jump if the train moves at  $v = 60$  km/h?

**30.** A car moves at 72 km/h. What percentage of the true siren frequency is the frequency change heard by a stationary observer as the car passes?  $c = 340$  m/s.

**31.** A source moves at 40 m/s. Find the relative frequency jump (%) when the source passes a stationary observer.  $c = 340$  m/s.

**32.** An observer on a platform hears frequency  $\nu_1$  as the train approaches and  $\nu_2$  as it recedes. If  $\nu_1/\nu_2 = 1.1$ , what is the train's speed?  $c = 340$  m/s.

**33.** A stationary observer hears a 10% increase in frequency as a source approaches. What is the source speed?  $c = 340$  m/s.

**34.** A train moving at speed  $v$  approaches a tunnel. What is the relative change in the reflected signal frequency (%) compared with the original?  $c = 340$  m/s.

**35.** Find the relative frequency jump (%) when a source passes a stationary observer if the source speed is  $0.1c$ .  $c = 340$  m/s.

## ELECTRICAL OSCILLATIONS. ALTERNATING CURRENT.

### Key Formulae

Natural cyclic frequency of an LC circuit:  $\omega_0 = 1/\sqrt{LC}$

Damped oscillation cyclic frequency (series circuit):  $\omega = \sqrt{(\omega_0^2 - \beta^2)}$

Logarithmic decrement and Q-factor:  $\delta = \beta T$ ,  $Q = \pi/\delta$

Inductive, capacitive and total impedance:  $X_L = \omega L$ ;  $X_C = 1/(\omega C)$ ;  
 $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Ohm's law for AC:  $I = U/Z$

RMS values:  $I = I_{\max}/\sqrt{2}$ ;  $U = U_{\max}/\sqrt{2}$

Power in AC circuit:  $P = UI \cos\varphi$

Phase shift:  $\operatorname{tg}\varphi = (X_L - X_C)/R$

## Problems

1. An LC circuit has  $C = 888 \text{ pF}$  and  $L = 2 \text{ mH}$ . What wavelength  $\lambda$  is it tuned to?
2. An LC circuit has  $C = 100 \text{ pF}$  and  $L = 0.5 \text{ mH}$ . What is its natural oscillation frequency?
3. Find the inductance of an LC circuit tuned to  $\lambda = 300 \text{ m}$  if  $C = 150 \text{ pF}$ .
4. Find the capacitance for an LC circuit to receive  $\lambda = 600 \text{ m}$  if  $L = 3 \text{ mH}$ .
5. An LC circuit is tuned to  $\lambda = 100 \text{ m}$ . If  $L = 1 \text{ mH}$ , find  $C$ .
6. Natural frequency of an LC circuit is  $500 \text{ kHz}$ ;  $L = 4 \text{ mH}$ . Find  $C$ .
7. An LC circuit has  $C = 500 \text{ pF}$  and  $L = 0.2 \text{ mH}$ . What wavelength does it emit?
8. An LC circuit ( $L = 2 \text{ mH}$ ) can be tuned by varying  $C$  from  $69 \text{ pF}$  to  $533 \text{ pF}$ . Find the wavelength range.
9.  $L = 1.5 \text{ mH}$ ;  $C$  varies from  $50 \text{ pF}$  to  $400 \text{ pF}$ . Find the frequency range.
10. A variable capacitor ( $100 \text{ pF}$  to  $900 \text{ pF}$ ) and  $L = 0.8 \text{ mH}$ . What wavelength range can be received?
11. Frequency range  $1 \text{ MHz}$  to  $5 \text{ MHz}$ ;  $C$  varies from  $10 \text{ pF}$  to  $250 \text{ pF}$ . Find  $L$ .
12.  $L = 0.5 \text{ mH}$ ;  $C$  varies from  $20 \text{ pF}$  to  $320 \text{ pF}$ . Find the wavelength range.
13. The circuit must cover  $\lambda = 200 \text{ m}$  to  $600 \text{ m}$ .  $C_{\text{min}} = 20 \text{ pF}$ . Find  $C_{\text{max}}$  and  $L$ .
14. The capacitance increases 16 times. By how many times does the tunable wavelength range change ( $L$  constant)?
15. What inductance must be included in an LC circuit for a natural frequency of  $1000 \text{ Hz}$  with  $C = 2 \text{ }\mu\text{F}$ ?
16. What capacitance must be connected to  $L = 0.1 \text{ H}$  for natural frequency  $500 \text{ Hz}$ ?

17. An LC circuit is tuned to resonant frequency 10 kHz.  $L = 0.1$  H. Find C.
18. Find the natural frequency of an LC circuit with  $C = 100$  nF and  $L = 25$  mH.
19. An LC circuit ( $C = 300$  pF) must receive  $\lambda = 150$  m. Find L.
20. What inductance is needed with  $C = 5$  nF for a natural period  $T = 2 \times 10^{-4}$  s?
21. An LC circuit has  $T = 0.001$  s and  $L = 10$  mH. Find C.
22. An LC circuit has  $C = 25$  nF,  $L = 1.015$  H, initial charge  $q = 2.5$   $\mu$ C. Write equations (with numerical coefficients) for the voltage U across the capacitor and current I. Find U and I at  $T/8$ ,  $T/4$ , and  $T/2$ . Plot one full period.
23.  $C = 10$  nF,  $L = 0.4$  H,  $q_{\max} = 1$   $\mu$ C. Write equations for charge, voltage, and current as functions of time. Find values at  $T/6$  and  $T/3$ .
24.  $C = 50$  pF,  $L = 2$  mH,  $U_{\max} = 100$  V. Write equations for voltage and current.
25.  $I_{\max} = 0.5$  A,  $L = 0.1$  H,  $C = 20$  nF. Write equations for current and charge. Find current and charge when capacitor voltage is maximum.
26.  $C = 100$  nF,  $L = 0.2$  H,  $q_{\max} = 3$   $\mu$ C. Plot  $E_{\text{electric}}$  and  $E_{\text{magnetic}}$  vs time over one period.
27.  $C = 1$   $\mu$ F,  $L = 1$  H,  $\phi = 0$ ,  $I_{\max} = 0.1$  A. Write equations for charge and voltage.
28.  $U_{\max} = 200$  V,  $I_{\max} = 0.2$  A,  $f = 500$  Hz. Find C and L. Write equations for U and I.
29.  $U(t) = 50\cos(10^4\pi t)$  V,  $C = 0.1$   $\mu$ F. Find T, L,  $I(t)$ , and  $\lambda$ .
30.  $U(t) = 120\sin(2000\pi t)$  V,  $C = 0.5$   $\mu$ F. Find T, L,  $I(t)$ , and  $\lambda$ .
31.  $L = 0.2$  H,  $C = 0.05$   $\mu$ F,  $U_{\max} = 80$  V. Write equations for  $U(t)$  and  $I(t)$ .
32.  $I(t) = 0.4\cos(500\pi t)$  A,  $C = 2$   $\mu$ F. Find T, L,  $U_{\max}$ , and  $\lambda$ .
33.  $U(t) = 100\cos(10^5 t)$  V,  $L = 10$  mH. Find C,  $I_{\max}$ , and  $\lambda$ .
34. Circuit tuned to  $\lambda = 300$  m;  $U_{\max} = 60$  V;  $C = 200$  pF. Write equations for  $U(t)$  and  $I(t)$ .

**35.**  $T = 10$  ms,  $L = 0.1$  H,  $U_{\max} = 20$  V. Write full equations for  $U(t)$  and  $I(t)$ . Find  $\lambda$ .

**36.**  $I(t) = -0.02\sin(400\pi t)$  A,  $L = 1$  H. Find  $T$ ,  $C$ ,  $W_{\text{magnetic\_max}}$ ,  $W_{\text{electric\_max}}$ .

**37.**  $I(t) = 0.1\cos(200\pi t)$  A,  $C = 5$   $\mu\text{F}$ . Find  $T$ ,  $L$ ,  $W_{\text{magnetic\_max}}$ ,  $W_{\text{electric\_max}}$ .

**38.**  $U(t) = 40\sin(500\pi t)$  V,  $C = 10$  nF. Find  $L$ ,  $I_{\max}$ ,  $W_{\text{electric\_max}}$ ,  $W_{\text{magnetic\_max}}$ .

**39.**  $I_{\max} = 0.05$  A,  $L = 0.8$  H,  $f = 25$  Hz. Find  $C$  and total oscillation energy.

**40.**  $C = 2$   $\mu\text{F}$ ,  $L = 0.5$  H,  $W_{\text{electric}} = 0.01$  J. Write the equation for  $I(t)$ .

**41.**  $I(t) = I_{\max} \sin(\omega t)$ ,  $I_{\max} = 0.03$  A,  $\omega = 1000$  rad/s,  $C = 100$   $\mu\text{F}$ . Find  $L$  and total energy.

**42.**  $W_{\text{magnetic\_max}} = 0.02$  J,  $T = 0.01$  s,  $I_{\max} = 0.4$  A. Find  $L$  and  $C$ .

**43.**  $C = 405$  nF,  $L = 10$  mH,  $R = 2$   $\Omega$ . By how many times does the capacitor voltage decrease in one period?

**44.**  $C = 100$  nF,  $L = 5$  mH,  $R = 1$   $\Omega$ . By how many times does the current amplitude decrease per period?

**45.** Logarithmic decrement  $\delta = 0.02$ . By how many times does the capacitor voltage amplitude decrease in 5 periods?

**46.**  $C = 200$  nF,  $L = 8$  mH,  $R = 0.5$   $\Omega$ . Find the time for the capacitor voltage amplitude to halve.

**47.** Q-factor  $Q = 100$ . By how many times does the charge amplitude on the capacitor decrease per period?

**48.** The voltage amplitude halved in one period. Find the logarithmic decrement.

**49.**  $C = 500$  pF,  $L = 20$  mH,  $R = 5$   $\Omega$ . Find the number of oscillations for the circuit energy to decrease by a factor of  $e$ .

**50.**  $C = 2.22$  nF; coil: copper wire, diameter  $d = 0.5$  mm, length  $l = 20$  cm. Find the logarithmic decrement  $\kappa$ .

**51.**  $C = 10$  nF,  $L = 50$  mH; copper wire  $d = 0.2$  mm,  $l = 1$  m. Find the logarithmic decrement. ( $\rho_{\text{Cu}} = 1.7 \times 10^{-8}$   $\Omega \cdot \text{m}$ )

52.  $\delta = 0.01$ ,  $T = 10 \mu\text{s}$ . Find the damping coefficient  $\beta$ .
53.  $C = 1 \mu\text{F}$ ,  $L = 0.1 \text{ H}$ ,  $R = 0.1 \Omega$ . Find the logarithmic decrement.
54.  $\delta = 0.05$ ; the amplitude halves. How many periods have passed?
55.  $C = 4.44 \text{ nF}$ ; aluminium wire ( $d = 1 \text{ mm}$ ,  $l = 50 \text{ cm}$ ),  $L = 5 \text{ mH}$ . Find  $\delta$ . ( $\rho_{\text{Al}} = 2.8 \times 10^{-8} \Omega \cdot \text{m}$ )
56. In 100 oscillations the current amplitude decreased 2.72 times. Find the logarithmic decrement.
57. A coil ( $l = 25 \text{ cm}$ ,  $r = 2 \text{ cm}$ ,  $N = 1000$  turns, copper wire  $S = 1 \text{ mm}^2$ ) is connected to a 50 Hz AC supply. What fraction of the total impedance  $Z$  is the resistance  $R$  and the inductive reactance  $X_L$ ?
58. A coil ( $N = 500$  turns, copper wire  $d = 0.8 \text{ mm}$ ,  $l = 15 \text{ cm}$ ,  $r = 1.5 \text{ cm}$ ) is connected to a 100 Hz supply. Find  $R$  and  $X_L$ . ( $\rho_{\text{Cu}} = 1.7 \times 10^{-8} \Omega \cdot \text{m}$ )
59.  $L = 0.1 \text{ H}$ ,  $R = 5 \Omega$ ,  $f = 50 \text{ Hz}$ . Find the total impedance.
60.  $L = 20 \text{ mH}$ ,  $R = 10 \Omega$ . At what frequency is the inductive reactance equal to  $R$ ?
61.  $R = 2.5 \Omega$ ,  $X_L = 6 \Omega$  at  $f = 400 \text{ Hz}$ . Find  $L$  and total impedance  $Z$ .
62.  $f = 60 \text{ Hz}$ ,  $Z = 13 \Omega$ ,  $R = 5 \Omega$ . Find  $L$ .
63.  $f = 200 \text{ Hz}$ ,  $L = 0.08 \text{ H}$ ,  $R = 3 \Omega$ . By how many times is  $X_L$  greater than  $R$ ?
64.  $C = 20 \mu\text{F}$  and  $R = 150 \Omega$  in series,  $f = 50 \text{ Hz}$ . What fraction of the applied voltage  $U$  drops across  $C$  and across  $R$ ?
65.  $R = 100 \Omega$ ,  $C = 10 \mu\text{F}$  in series,  $f = 400 \text{ Hz}$ . Find the total impedance.
66.  $R = 50 \Omega$ ,  $C = 5 \mu\text{F}$ ,  $U = 100 \text{ V}$ ,  $f = 500/\pi \text{ Hz}$ . Find the voltage amplitudes across  $R$  and  $C$ .
67.  $C = 30 \mu\text{F}$ ,  $R = 80 \Omega$ ,  $U = 220 \text{ V}$ ,  $f = 50 \text{ Hz}$ . Find the current.
68. In a series RC circuit:  $U_R = 60 \text{ V}$ ,  $U_C = 80 \text{ V}$ . Find the total applied voltage.
69. Series RC circuit:  $C = 40 \mu\text{F}$ ,  $R = 200 \Omega$ ,  $I = 0.5 \text{ A}$ ,  $f = 50 \text{ Hz}$ . Find  $U_R$ ,  $U_C$ , and total  $U$ .

**70.** Capacitor and resistor in series,  $f = 100 \text{ Hz}$ ,  $Z = 100 \ \Omega$ ,  $R = 60 \ \Omega$ .  
Find C.

## References

1. Bondarenko, O. O., Hokov, O. M., Katrunov, K. O. (2009). Practicum on Physics. Electricity and Magnetism. Kharkiv: KhNEU.
2. Dem'yanyuk, B. P., Kolupaev, B. S. (2008). Modular Control in Mechanics. Rivne: RSHU.
3. Huryevska, O. M., Kovalov, S. H. (2025). Physics Problem Book. Part 1. Mechanics. Electricity. Magnetism. Oscillations: study guide [Electronic edition]. Kropyvnytskyi: Central Ukrainian National Technical University. – 144 p.
4. Kucheruk, I. M., Horbachuk, I. T., Lushchyk, P. P. (2007). General Course of Physics. Vol. 1. Mechanics. Molecular Physics and Thermodynamics. Kyiv: Tekhnika.
5. Kucheruk, I. M., Horbachuk, I. T., Lushchyk, P. P. (2007). General Course of Physics. Vol. 2. Electricity and Magnetism. Kyiv: Tekhnika.
6. Voroshylo, O. I., Nefedchenko, V. F., Yunda, A. M. (2004). Physics Problem Book. Part 1. Mechanics. Sumy: SumDU.