АВТОМАТИЗАЦІЯ ТА КОМП'ЮТЕРНО-ІНТЕГРОВАНІ ТЕХНОЛОГІЇ

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Optimal control of nonlinear stationary systems at infinite control time

The article presents a solution to the problem of control synthesis for dynamical systems described by linear differential equations that function in accordance with the integral-quadratic quality criterion under uncertainty. External perturbations, errors and initial conditions belong to a certain set of uncertainties. Therefore, the problem of finding the optimal control in the form of feedback on the output of the object is presented in the form of a minimum problem of optimal control under uncertainty. The problem of finding the optimal control and initial state, which maximizes the quality criterion, is considered in the framework of the optimization problem, which is solved by the method of Lagrange multipliers after the introduction of the auxiliary scalar function - Hamiltonian. The case of a stationary system on an infinite period of time is considered. The formulas that can be used for calculations are given for the first and second variations.

It is proposed to solve the problem of control search in two stages: search of intermediate solution at fixed values of control and error vectors and subsequent search of final optimal control. The solution of -optimal control for infinite time taking into account the signal from the compensator output is also considered, as well as the solution of the corresponding matrix algebraic equations of Ricatti type.
minimax control, robustness, systems with uncertainties, optimization, matrix form

Formulation of the problem. Initially, the main results of research on linear automatic control systems were the concept of stability and its criteria based on characteristic polynomials. With the development of radio engineering and electronic automatic systems, frequency research methods became the main ones, which later spread to pulsed, discrete and nonlinear systems in connection with the development of computer technology. The progress of cosmonautics has led to the study of automatic systems in the space of states, the idea of optimizing control systems with the simultaneous optimization of their quality indicators.

Subsequent progress has made it possible to combine frequencies with methods of state space research, which, in addition to optimization, has made it possible to solve problems with any uncertainties - robust control. In this case, the uncertainty of the frequency response of control objects is limited in the H^{∞}-norm and can be specified in both parametric and matrix form when describing the state in space [1].

Analysis of recent and publication. The theory of robust control [4] has intrigued scientists since the 90s, although some fundamental ideas of robustness (for example, the allocation of areas of stability in the parameter space) come from Vyshnegradsky. The first results in this area concerned the analysis of systems with uncertainties – it was possible to construct robust analogs of the main criteria of stability of linear systems. Serious results were obtained in robust synthesis (design of regulators for robust systems).

The practical value of the use of robust control is due to the fact that synthesized by the criteria of stability, the optimal control system may be less sensitive to changes in parameters or greater. In the first case, talk about the roughness of the system or its robustness, in the second - the system is virtually inoperable, because at least the deviation of the parameters (their uncertainty) leads to a loss of stability [5]. Thus, the formulation of the problem of robust control is associated with the requirement to maintain the efficiency of the system in the presence of uncertainties in its description.

There are three types of uncertainties: parametric, when the parameters of the object are inaccurately known; structural, when the exact structure of the object is not known; mixed when both parameters and structure are incorrectly set.

The system will be stable robust if, according to the root criterion of stability, all the roots of its characteristic equation lie in the left complex root half-plane.

In the theory of robust control, the concept of in H_∞ and H^2 norm stability is used. For one-dimensional systems, the H_∞ norm is the maximum of the modulus of the frequency transfer function (amplitude-phase characteristic) when the frequency changes from zero to infinity. For example, the oscillation index is the H_∞ norm of a transfer function that relates a controlled variable to a setpoint [6].

The use of the H_∞ norm allowed us to use known methods of the theory of functions of a complex variable (Nehari's theorem, Nevanlin-Peak interpolation) to construct an optimal control that provides a minimum of this norm. Later, a construction method was proposed H_∞- suboptimal control, the so-called 2-Riccati approach, which develops the results obtained in the development of optimal stochastic systems, in the case where external perturbations and interferences are unknown damping functions with unknown statistical characteristics, or H^2 approach [7,8].

Statement of the objective. For uncertainty conditions, it is fruitful to use a minimax approach, when there is an optimal regulator for the state of the object, which operates in conditions of uncertainty so that it minimizes the maximum error (deviation of the current state of the system from the set or desired) from many possible values. perturbations that may affect an object or system. However, the way to solve this problem is not always obvious [2], and its search requires additional research [9].

Consider a dynamic object described by the following system of differential equations [3].

$$\begin{cases} \dfrac{dx(t)}{dt} = A(t)x(t) + B(t)u(t) + F_w(t)w(t), \qquad t_0 < t < T, \\ x(t_0) = F_0 x_0, \end{cases} \tag{1}$$

where $x(t) \in R^{n_x}$ – state vector, $u(t) \in R^{n_u}$ – control vector, $w(t) \in R^{n_w}$ – unknown vector of extremal perturbations acting on the object, $x_0 \in R^{n_0}$ – unknown vector of initial conditios, $A(t) \in R^{n_x \times n_x}$, $B(t) \in R^{n_x \times n_u}$, $F_w(t) \in R^{n_x \times n_w}$, $F_0 \in R^{n_x \times n_0}$ – given matrices of corresponding dimensions.

Consider and choose an integral-quadratic criterion for the quality of the object in the form

$$I(u) = \int_{t_0}^{T} \left(x^T(t) G_x(t) x(t) + u^T(t) G_u(t) u(t) \right) dt + x^T(T) G_f x(T), \tag{2}$$

where $y(t) \in R^{n_y}$ – the result of observation, $v(t) \in R^{n_v}$ – unknown errors (obstacles) of measurements, $C(t) \in R^{n_y \times n_x}$, $F_v(t) \in R^{n_y \times n_v}$ – known matrices.

Consider and choose an integral-quadratic criterion for the quality of the object in the form

$$I(u) = \int_{t_0}^{T} \left(x^T(t) G_x(t) x(t) + u^T(t) G_u(t) u(t) \right) dt + x^T(T) G_f x(T), \tag{3}$$

where $G_x(t) \in R^{n_x \times n_x}$, $G_u(t) \in R^{n_u \times n_u}$, $G_f \in R^{n_x \times n_x}$ – given symmetric weight matrices, and it is assumed that they satisfy the conditions $G_x(t) = G_x^T(t) \geq 0$, $G_u(t) = G_u^T(t) > 0$, $G_f = G_f^T \geq 0$.

Here " T " – means the operation of transposing the matrix, $G = G^T$ – means that the matrix G is symmetric, $G > 0$ ($G \geq 0$) – the condition of positive (non-negative) certainty of the matrix, i.e. the matrix G has positive or non-negative eigenvalues.

Regarding the unknown vector of external perturbations $w(t)$, the vector of measurement errors $v(t)$ and the vector of initial conditions x_0, it is assumed that they belong to the next set of permissible perturbations (unsertainties).

$$\Omega_\xi = \left\{ \xi : \ \xi = (w(t), v(t), x_0), \ w(t) \in L_2(t_0, T), \ v(t) \in L_2(t_0, T), \ x_0 \in R^{n_0}; \ \| \xi \|^2 \leq 1 \right\}, \tag{4}$$

where the norm $\| \xi \|$ of the vector-valued function ξ is determined by the following expression

$$\| \xi \|^2 = \int_{t_0}^{T} \left(w^T(t) R_w(t) w(t) + v^T(t) R_v(t) v(t) \right) dt + \left(x_0 - \hat{x}_0 \right)^T R_0 \left(x_0 - \hat{x}_0 \right), \tag{5}$$

in which $R_w(t) \in R^{n_w \times n_w}$, $R_v(t) \in R^{n_v \times n_v}$, $R_0 \in R^{n_0 \times n_0}$ – given weight matrices, and $R_w(t) = R_w^T(t) \geq 0$, $R_v(t) = R_v^T(t) > 0$, $R_0 = R_0^T \geq 0$, $\hat{x}_0 \in R^{n_0}$ – known vector, in the vicinity of which is an unknown vector of the initial condition x_0 [4].

In addition, in (4) through $L_2(t_0, T)$ the denoted set of vector-integrated square functions, ie

$$L_2(t_0, T) = \left\{ f(t) \in R^n : \ \int_{t_0}^{T} f^T(t) f(t) dt = \int_{t_0}^{T} \| f(t) \|^2 \, dt < \infty \right\}.$$

Main material. Consider the case of a stationary system (1) over an infinite period of time. H^∞ – optimal control of linear stationary systems at infinite control time.

Consider a stationary system:

$$\begin{cases} \dfrac{dx(t)}{dt} = Ax(t) + Bu(t) + F_w w(t), & t_0 < t < \infty, \\ x(t_0) = F_0 x_0, \end{cases} \tag{6}$$

in the equation of observation

$$y(t) = Cx(t) + F_v v(t), \tag{7}$$

and quality criteria

$$I(u) = \int_{t_0}^{\infty} \left(x^T(t) G_x x(t) + u^T(t) G_u u(t) \right) dt, \tag{8}$$

Regarding the unknown vector of external perturbations $w(t)$, the vector of measurement errors $v(t)$ and the vector of initial conditions x_0, it is assumed that they belong to the next set of permissible perturbations (uncertainties)

$$\Omega_\xi = \left\{ \xi : \ \xi = (w(t), v(t), x_0), \ w(t) \in L_2(t_0, \infty), \ v(t) \in L_2(t_0, \infty), \ x_0 \in R^{n_0}; \ \| \xi \|^2 \le 1 \right\}, \tag{9}$$

where the norm норма $\| \xi \|$ of the vector-valued function ξ is determined by the following expression

$$\| \xi \|^2 = \int_{t_0}^{\infty} \left(w^T(t) R_w w(t) + v^T(t) R_v v(t) \right) dt + \left(x_0 - \hat{x}_0 \right)^T R_0 \left(x_0 - \hat{x}_0 \right). \tag{10}$$

Then H^∞ - the optimal solution to the problem of minimax control

$$\inf_u \sup_{\xi \in \Omega_\xi} I(u) = \gamma_{\min}^2, \tag{11}$$

presentable in the form

$$u(t) = -G_u^{-1} B^T Q x_c(t), \tag{12}$$

where $x_c(t)$ – compensator output

$$\begin{cases} \dfrac{dx_c(t)}{dt} = A_c x_c(t) + B_c y(t), \\ x_c(t_0) = x_c^0, \end{cases} \tag{13}$$

in which it is marked

$$A_c = A - BG_u^{-1} B^T Q + \gamma^{-2} F_w R_w^{-1} F_w^T Q - \left(E - \gamma^{-2} PQ \right)^{-1} PC^T RC, \tag{14}$$

$$B_c = \left(E - \gamma^{-2} PQ \right)^{-1} PC^T R, \tag{15}$$

$$x_c^0 = \left(E - \gamma^{-2} F_0 R_0^{-1} F_0^T Q(t_0) \right)^{-1} F_0 \hat{x}_0, \qquad R = (F_v^{-1})^T R_v F_v^{-1}. \tag{16}$$

Matrices $P = P^T > 0$ і $Q = Q^T > 0$ are solutions of the following matrix algebraic equations of the Ricatti type

$$AP + PA^T - P\left(C^T RC - \gamma^{-2} G_x \right) P + F_w R_w^{-1} F_w^T = 0, \tag{17}$$

$$-A^T Q - QA + Q\left(BG_u^{-1} B^T - \gamma^{-2} F_w R_w^{-1} F_w^T \right) Q - G_x = 0, \tag{18}$$

in which the parameter γ^2 must satisfy the condition

$$E - \gamma^{-2} QP > 0. \tag{19}$$

The minimum value $\gamma^2_{\min}$ of the parameter γ^2, under which the condition is met (19), corresponds to optimal control.

The worst (most unfavorable) perturbations are given by formulas

$$w(t) = \gamma^{-2} R_w^{-1} F_w^T Q x_c(t), \quad v(t) = 0, \quad x_0 = \left(E - \gamma^{-2} R_0^{-1} F_0^T Q F_0\right)^{-1} \hat{x}_0. \qquad (20)$$

Estimation of the state vector $\hat{x}(t)$ can be found by the formula

$$\hat{x}(t) = \left(E - \gamma^{-2} P Q\right) x_c(t). \qquad (21)$$

Conclusions. Thus, the purpose of the article, declared at the beginning of the work, is achieved, the proposed solution of the problem of finding the optimal control as output feedback, which minimizes the integral-quadratic criterion of operation under uncertainty under the most adverse perturbations. The results of the research are presented in the form of practical formulas, according to which the corresponding calculations are allowed when modeling control processes in the considered linear dynamic non-stationary object with uncertainties. The theory of automatic control moves in the direction of complicating the studied phenomena, processes and reducing information about the control system, object, its features, properties, characteristics, operating conditions, uncertainties and external influences. Given all the above, the chosen area of research is promising and has a high level of relevance.

Список літератури

1. Баландин Д.В., Коган М.М. Синтез законов управления на основе линейных матричных неравенств . М.: Физматлит, 2007. 281 с.
2. Гантмахер Ф.Р. Теория матриц. М.: Физматлит, 2004. 560 с.
3. Поляк Б.Т., Хлебников М.В. Управление линейными системами при внешних возмущениях: Техника линейных матричных неравенств. М.: ЛЕНАНД, 2014. 560 с.
4. Якубович В.А. Решение некоторых матричных неравенств, встречающихся в теории автоматического регулирования . *ДАН СССР*. 1962. Т. 143, №6. С. 1304-1307.
5. Boyd S., El Ghaoui L., Feron E., Balakrishnan V. LinearMatrix Inequalities in System and Control Theory. Philadelphia: SIAM, 1994. 193 p.
6. Chilali M., Gahinet P. H^∞ design with pole placement constraints: An LMI approach. *IEEE Trans. Automat. Contr*., 1996. Vol.41. Pp. 358–367.
7. Ghaoui L.E., Niculescu S.I. Advances in linear matrix inequality methods in control. Advances in Design and Control. Philadelphia, PA: SIAM, 2000. 372 p.
8. Masubuchi I., Ohara A., Suda N. LMI-based controller synthesis: A unified formulation and solution. *Int. J. Robust Nonlinear Contr.* 1998. Vol. 8. Pp. 669–686.
9. Лобок О.П., Гончаренко Б.М., Віхрова Л.Г. Зведення задачі мінімаксного керування лінійними нестаціонарними системами до H^∞ робастного шляхом динамічної гри . *Центрально український науковий вісник. Технічні науки*. 2020. Вип..3(34). С.143-162.

References

1. Balandin, D.V. & Kogan, M.M. (2007). *Sintez zakonov upravlenija na osnove linejnyh matrichnyh neravenstv* [*Synthesis of control laws based on linear matrix inequalities*] . Moskow: Fizmatlit [in Russian].
2. Gantmaher, F.R. (2004). *Teorija matric* [*Matrix theory*]. Moskow: Fizmatlit [in Russian].
3. Poljak, B.T. & Hlebnikov, M.V. (2014). Upravlenie linejnymi sistemami pri vneshnih vozmushhenijah: Tehnika linejnyh matrichnyh neravenstv. Moskow: LENAND [in Russian].
4. Jakubovich, V.A. (1962). Reshenie nekotoryh matrichnyh neravenstv, vstrechajushhihsja v teorii avtomaticheskogo regulirovanija [Solution of some matrix inequalities found in automatic control theory]. *DAN SSSR.. Vol. 143, 6*, 1304-1307 [in Russian].
5. Boyd, S., El Ghaoui, L., Feron, E. & Balakrishnan, V. (1994). LinearMatrix Inequalities in System and Control Theory. Philadelphia: SIAM [in English].

6. Chilali, M. & Gahinet, P. (1996). H^{∞} design with pole placement constraints: An LMI approach. *IEEE Trans. Automat. Contr, Vol.41*, 358–367 [in English].
7. Ghaoui, L.E. & Niculescu, S.I. (2000). *Advances in linear matrix inequality methods in control*. Advances in Design and Control. Philadelphia, PA: SIAM [in English].
8. Masubuchi I., Ohara A., Suda N. (1998). LMI-based controller synthesis: A unified formulation and solution. *Int. J. Robust Nonlinear Contr.*, *Vol. 8*, 669–686 [in English].
9. Lobok, O.P. Honcharenko, B.M. & Vikhrova, L.H. (2020). Zvedennia zadachi minimaksnoho keruvannia linijnymy nestatsionarnymy systemamy do robastnoho shliakhom dynamichnoi hry [Reducing the Problem of Minimax Control of Linear non-Stationary Systems to a - Robust One by the Way of Dynamic Game]. *Tsentral'no ukrains'kyj naukovyj visnyk. Tekhnichni nauky – Central Ukrainian scientific Bulletin. Technical Sciences, Vol.3(34)*, 143 – 162 [in Ukrainian]

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Оптимальне керування нелінійними стаціонарними системами на нескінченому часі регулювання

В статті наведене вирішення проблеми синтезу керування для динамічних систем, які описуються лінійними диференційними рівняннями, що функціонують у відповідності з інтегрально-квадратичним критерієм якості в умовах невизначеності.

Зовнішні збурення, похибки та початкові умови при цьому належать певній множині невизначеностей. Тому проблема пошуку оптимального керування у вигляді зворотного зв’язку за виходом об’єкта представлена у вигляді мінімаксної задачі оптимального керування за умов невизначеностей. Завдання пошуку оптимального керування і початкового стану, які максимізують критерій якості, розглянуто в рамках оптимізаційної задачі, яку розв’язано методом множників Лагранжа після введення допоміжної скалярної функції – гамільтоніана. Розглянуто випадок стаціонарної системи на нескінченному відтинку часу. Приведені для перших та других варіацій формули, які можуть використовуватися для розрахунків.

Запропоновано задачу пошуку керування розв’язувати в два етапи: пошук проміжного розв’язку при фіксованих значеннях векторів керування та похибки і наступний пошук остаточного оптимального керування. Розглянуте також розв’язання H^{∞}-оптимального керування на нескінченому часі з врахуванням сигналу з виходу компенсатора, а також – розв’язання відповідних матричних алгебраїчних рівнянь типу Рікатті.

мінімаксне керування, робастність, системи з невизначеностями, оптимізація, матрична форма

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