

Optimal Control of Leader-Following Robots under Random Effects

Osadchy S.I.¹, Ladanyuk A.P.², Zozulya V.A.¹, Vikhrova L.G.¹, Kalich V.M.¹

¹ Kirovograd National Technical University, University Avenue 8, Kirovograd, Ukraine, 25006, srg2005@ukr.net, irish38@ukr.net, <http://www.kntu.kr.ua/avp/>

² National University of Food Technologies, Volodymyrska str. 68, Kyiv, Ukraine, 01601, ladanyuk@ukr.net

Abstract — the article focuses on the development of an on-board system design method for optimal control of an autonomous mobile group of objects. It is assumed that the group consists of a leader and some agents. A new method for the synthesis of an optimal multivariable control system, which is needed for preserving desired position of the agent relatively to the leader was substantiated in the article. The leader passes along a random trajectory and measurement of the agent position with respect to the leader is accompanied with random noise. All group members experience the action of random disturbances.

Keywords — micro-mechanic gyroscope, angular motion, hexapod, identification, factorization, separation, noise

I. INTRODUCTION

Many monographs and articles, for example, [1-4] prove that for the implementation of certain civil or military kinds of work which are hazardous to human it is advisable to use a group of robots or a group of unmanned objects. However, this application requires solving a number of control tasks [1]. These tasks include the traffic control of the group of mobile objects. The works [2, 3, 5] suggest to set out a group of moving objects consisting of a leader and several agents.

The task of controlling the traffic of the group is to maintain a command position of the agent according to the leader moving along a complex path. To solve this problem is known for a number of different approaches. The article [2] used a modified nonlinear leader tracking method. The work [3] shows the control law, which is built on the basis of synergetic principles, for the group of unmanned objects. The analysis of the control laws presented in the articles [2, 3] disclosed a range of limitations for their application. Firstly, they can only be used if the traffic programme of the leader has no noise. Secondly, the position of an agent relatively to the leader is measured most accurately and without phase shifts. Thirdly, the assumed hypothesis is that there are no disturbance signals.

At the same time most objects that move in the air, overland and underwater are affected by disturbance signals. As a rule the disturbance signals have random character and their dynamics is well studied.

Measurement of any physical values is accompanied by regular as well as random miscalculations.

Therefore, the research of the development of the method of designing on-board controllers of the leader and the agents which control the command position of the agent relatively to the leader with minimal error in the conditions of disturbance signals is important and up to date.

II. PURPOSE AND TASK OF RESEARCH

Assuming that the group of mobile robots (Fig.1) consists of $\eta+1^{\text{st}}$ element. The leader has

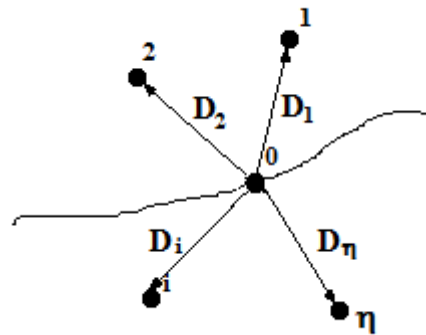


Figure 1 – Group of mobile objects with the leader

number 0 and the agents have numbers from 1 to η . The dynamics of movement of each robot is characterised by the system of linear ordinary differential equations

$$P_i x_i = M_i u_i + \psi_i, \quad (1)$$

where P_i is the matrix of size $n \times n$, the elements of which are the polynomials of the differentiation operator; n is the number of outputs of the elements with number i ; x_i is the vector of output signals of the element; M_i is the polynomial matrix of size $n \times m$; u_i is m -dimensional vector of controlling signals of the element i ; ψ_i is the vector of disturbance effect. Let us suppose that the movement trajectory of the leader (the solid line in Fig. 1) is given with the help of n -dimensional vector r_0 , the components of which determine the set value of the vector x_0 . The task of the control system is to make the leader to move along the commanded trajectory and to make the agents to keep constant radius-vectors D_i .

$$D_i = x_0 - x_i. \quad (2)$$

To perform this task it is suggested to create the control system with the scheme showed in Fig. 2.

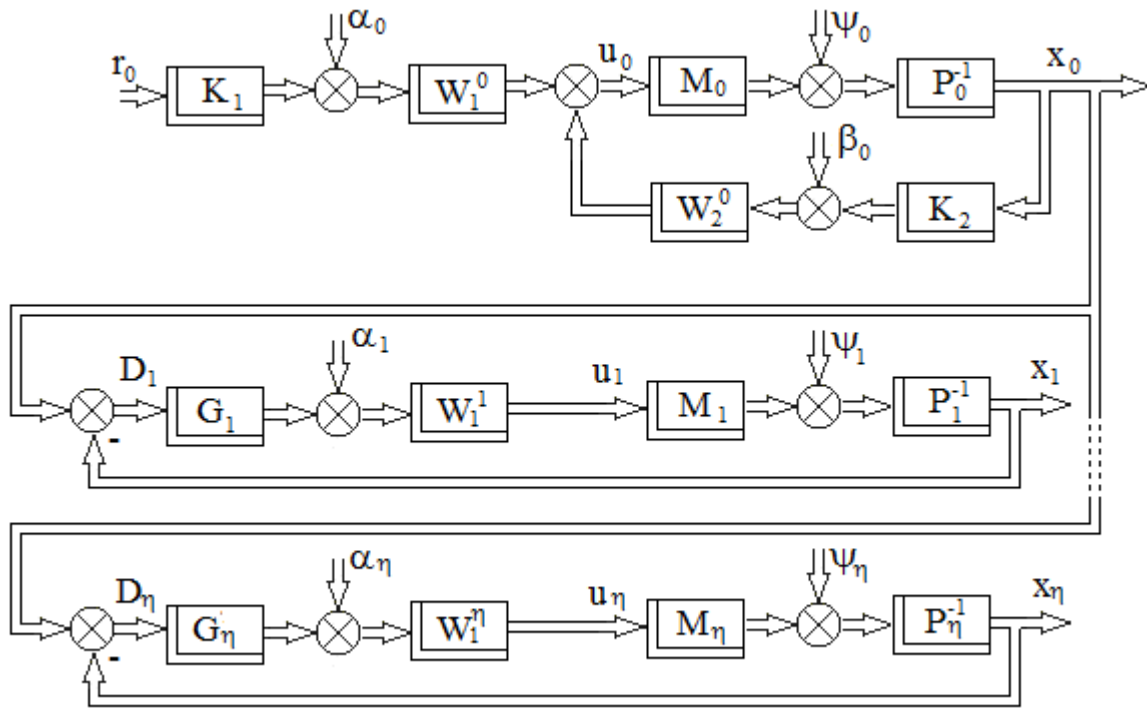


Figure 2 – Function scheme of the control system

The vector of programme signals r_0 is formed on the board of the leader with the help of programme generator with the transfer function matrix K_1 . The formation of the programme is accompanied by the distorting actions with the vector α_0 . The received vector of signals is transmitted to the first input of multidimensional regulator (transfer function matrix W_1^0) of the leader movement. The second input of the multidimensional regulator (W_2^0) receives the vector of distorted signals of the current position of the leader. Its measurement is carried out with the help of a sensor system with the transfer function matrix K_2 and the vector of additive noise β_0 . The vector of control signals u_0 is formed at the output of the multidimensional regulator of the leader. The vector of output signals of the leader x_0 is delivered to the input of a system which is designed to the measuring of the vector D_i parameters and is installed on the board of the agent with number i . The transfer functions matrix of this system and the vector of its measuring noises are denoted as G_i and α_i respectively.

The result of measuring the position of the agent relatively to the leader controls the operation of the multidimensional regulator with the matrix of transfer functions W_1^1 .

By analogy with the monograph [6], in order to assess the quality of task fulfilment of the control system the vector of the errors of performance of the programmed trajectory ε_0 and the vector of the errors

of keeping the distance ε_i were introduced for consideration. They are bound with the vectors at the input and output of the system (Fig. 2) by the equations

$$\varepsilon_0 = x_0 - \Phi_0 r_0, \quad (3)$$

$$\varepsilon_i = x_i - \Phi_i x_0, \quad (4)$$

where Φ_0 is the matrix of transfer functions of the desired transformation of vector r_0 by the leader; Φ_i is the transfer function matrix of the desired connection between the leader and a corresponding numbered agent i .

If the vectors of input signals r_0 , α_i , β_i , ψ_i ($i=0,1,\dots,\eta$) are multidimensional centred stationary random processes then the indicators of system quality [7] is the sum of specially weighted dispersions of the components of the vectors of control signals and error control. This criterion can be written in a frequency domain as

$$J = \frac{1}{j} \int_{-j\infty}^{j\infty} tr(RS'_{\varepsilon\varepsilon} + CS'_{uu}) ds, \quad (5)$$

where j is a complex unit; tr is a sign of the search of matrix spur; R is a positive definite weight matrix; « / » is a transposing sign; $S'_{\varepsilon\varepsilon}$ is a transposing matrix of spectral density of the vector ε , which unites vectors (3), (4) according to the rule

$$\varepsilon = [\varepsilon'_0 \quad \dots \quad \varepsilon'_i \quad \dots \quad \varepsilon'_\eta]';$$

C is a nonnegative defined weight matrix; S'_{uu} is a transposed matrix of spectral densities of the vector of signal control u which is composed from the vectors u_i

$$u = \begin{bmatrix} u'_0 & \cdots & u'_i & \cdots & u'_\eta \end{bmatrix};$$

$s=j\omega$.

Taking into account the task of the control system and the selected criteria of control quality (5) the research task is formulated in the following way. According to the known polynomial matrixes M_i , P_i , that characterize the dynamics of all the objects of the group, transfer functions matrixes of the measurers K_1 , K_2 , G_i , transposed matrixes of spectral and cross-spectral densities of the useful signals, disturbances and

noises to find the formula for the calculation of the physically implemented matrixes of transfer functions of the leader's regulator W_{01} , W_{02} and matrixes of transfer functions of the agents' regulators of W_{i1} . Their insertion into the system provides its stability and minimizes the quality criterion (5).

III. METHODOLOGY OF SOLVING THE PROBLEM

In order to solve the problem we introduced a fictitious variable

$$z_0 = r_0; \quad (6)$$

and the function scheme (pic. 2) was changed in the way presented in Figure 3.

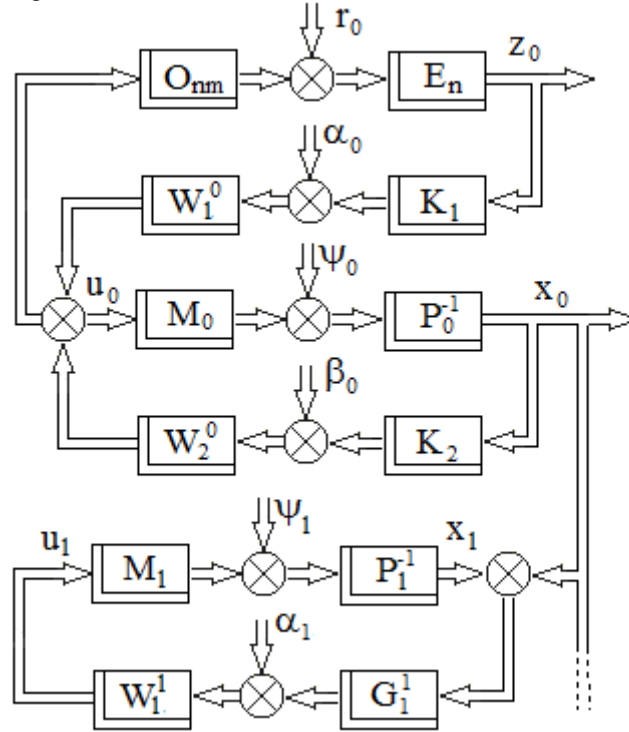


Figure 3 – Changed function scheme of the system

Consideration of the scheme (Fig. 3) enables forming new vectors of signals at the output of the system x , disturbance signals ψ and measuring noises φ in the following way

$$x = \begin{bmatrix} z'_0 & x'_0 & x'_1 & \cdots & x'_\eta \end{bmatrix}, \quad (7)$$

$$\psi = \begin{bmatrix} r'_0 & \psi'_0 & \psi'_1 & \cdots & \psi'_\eta \end{bmatrix}, \quad (8)$$

$$\varphi = \begin{bmatrix} \alpha'_0 & \beta'_0 & \alpha'_1 & \cdots & \alpha'_\eta \end{bmatrix}. \quad (9)$$

Taking into consideration the type of the vector of control signals u , the equations (7)-(9) and the structure of connection between the leader and the agents, we have built the functional scheme (Fig. 4) which is equivalent to the system of stabilization (Fig. 3). The extended object of control on the scheme is presented by the following block polynomial matrixes P_c and M_c

$$P_c = \begin{bmatrix} E_n & O_n & O_n & \cdots & O_n \\ O_n & P_0 & O_n & \cdots & O_n \\ O_n & O_n & P_1 & \cdots & O_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_n & O_n & O_n & \cdots & P_\eta \end{bmatrix}, \quad (10)$$

$$M_c = \begin{bmatrix} O_{nm} & O_{nm} & O_{nm} & \cdots & O_{nm} \\ M_0 & O_{nm} & O_{nm} & \cdots & O_{nm} \\ O_{nm} & M_1 & O_{nm} & \cdots & O_{nm} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_{nm} & O_{nm} & O_{nm} & \cdots & M_\eta \end{bmatrix}, \quad (11)$$

where E_n is a unity matrix of size $n \times n$; O_{nm} is a zero matrix of size $n \times m$.

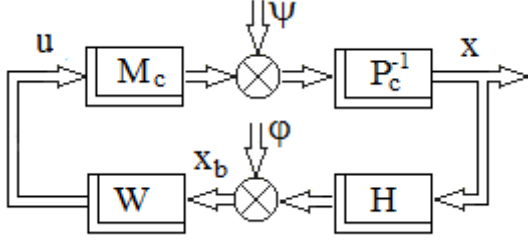


Figure 4 – Extended equivalent system of stabilization

The system of measurers is described by the block matrix of transfer functions H of

$$H = \begin{bmatrix} K_1 & O_n & O_n & \cdots & O_n \\ O_n & K_2 & O_n & \cdots & O_n \\ O_n & G_1^1 & -G_1^1 & \cdots & O_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_n & G_1^n & O_n & \cdots & -G_1^n \end{bmatrix}. \quad (12)$$

The matrix of transfer functions of compensators W has a block structure as well

$$W = \begin{bmatrix} W_1^0 & W_2^0 & O_{mn} & \cdots & O_{mn} \\ O_{mn} & O_{mn} & W_1^1 & \cdots & O_{mn} \\ O_{mn} & O_{mn} & O_{mn} & \cdots & O_{nm} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_{mn} & O_{mn} & O_{mn} & \cdots & O_{mn} \end{bmatrix}. \quad (13)$$

So, the set research objective is traced to the problem of synthesis of the optimal multidimensional system of stabilization [6, 7]. In order to solve it the combined method of synthesis in the frequency domain was presented in the article [8].

In accordance with this method the search of the transfer function matrix (13) which guarantees stability of the close-loop control system (Fig. 4) and minimizes the quality functional (5) is carried out on a phased basis.

At the first phase as a result of multivariable fraction decomposition (MFD) [9] of the fraction-rational matrix (12) it is necessary to seek the factors H_2 and H_{20} consistent with equation

$$H_1 = H_{20}^{-1} H_2, \quad (14)$$

where H_1 is the matrix resulting from the right-sided removal of the poles [10] from the product

$$HP_c^{-1} = P_d^{-1} H_1. \quad (15)$$

The second phase requires the formation of intermediary matrixes in the form of

$$P = H_{20} P_d, \quad M = H_2 M_c, \quad (16)$$

$$R_0 = H_*^{-1} \Phi_* R \Phi H^{-1}, \quad (17)$$

where Φ is the matrix of desired transformations which is equal to

$$\Phi = \begin{bmatrix} -\Phi_0 & E_n & O_n & \cdots & O_n \\ O_n & -\Phi_1 & E_n & \cdots & O_n \\ O_n & -\Phi_2 & O_n & \cdots & O_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_n & -\Phi_\eta & O_n & \cdots & E_n \end{bmatrix}, \quad (18)$$

if one takes into account the equations (3), (4). The index «*» in the Equation (18) means Hermitean conjugate matrix.

At the third phase it is necessary to carry out Naumenko factorisation [11] of the following block polynomial matrix

$$\aleph = \begin{bmatrix} O_q & P & -M \\ P_* & -R_0 & O_{qf} \\ -M_* & O_{fq} & -C \end{bmatrix}, \quad (19)$$

where

$$q = \eta n, \quad f = \eta m.$$

As a result of the factorisation of the matrix (19) we can determine the factors F and Σ , consistent with the equation

$$\aleph = F_* \Sigma F,$$

where Σ is a numerical matrix of the following structure

$$\Sigma = \begin{bmatrix} O_q & E_q & O_{qf} \\ E_q & O_q & O_{qf} \\ O_{fq} & O_{fq} & E_f \end{bmatrix},$$

F is a block polynomial matrix with its determinant that contains only stable zeroes:

$$F = \begin{bmatrix} E_q & -S & N \\ O_q & P & -M \\ O_{fq} & A & B \end{bmatrix}. \quad (20)$$

The fourth phase is the factorisation of an extended impact matrix by Davis [10]:

$$DD_* = (H_2 \quad P) \begin{pmatrix} S'_{\psi\psi} & O_q \\ O_q & S'_{\varphi\varphi} \end{pmatrix} \begin{pmatrix} H_{2*} \\ P_* \end{pmatrix}, \quad (21)$$

where D is a fraction-rational matrix with its determinant that contains zeroes and poles with a negative real part; $S'_{\psi\psi}$ is a transposed matrix of spectral densities of the vector (8); $S'_{\varphi\varphi}$ is a transposed matrix of spectral densities of the extended vector of noises (9).

The fifth phase presupposes the search for the optimal block matrix of regulators' transfer functions

$$W = (V_{10} B + V_1 M)^{-1} (-V_{10} A + V_1 P). \quad (22)$$

In this case V_{10} and V_1 are the polynomial matrixes resulting from the MFD of the optimal matrix of the varied transfer functions V

$$V_{10}^{-1} V_1 = V = -(T_0 + T_+ + Q_0 + Q_+) D_*, \quad (23)$$

T_0+T_+ is a fraction-rational matrix resulting from separation [6] of the following product

$$T_0 + T_+ + T_- = z_{22}^* (M_* P_*^{-1} S_* - N_*) D; \quad (24)$$

Q_0+Q_+ is the result of separation of this product

$$Q_0 + Q_+ + Q_- = -z_{22} M_* P_*^{-1} R_0 S'_{\varphi\varphi} P_* D_*^{-1}; \quad (25)$$

$$z_{22} = (B + A P^{-1} M)^{-1}. \quad (26)$$

In order to assess the control quality of the group of objects it is necessary to calculate the value of the quality functional (5) as a result of substitution of the optimum matrix of transfer functions (22) in the functional (5).

IV. THE RESULTS OF THE RESEARCH

To demonstrate the efficiency of the method of the synthesis of the control system we considered a group of robots which consists of two unmanned planes analogous to the work [2]. In this case the basic data for the synthesis of the optimal control system will be the following

$$\begin{aligned} \eta = 2; \quad x_i &= [V \quad \alpha \quad \theta \quad \dot{\theta}]^T; \quad u = [\delta_t \quad \delta_e]^T; \\ P_i &= \begin{bmatrix} s & -25 & 9.8 & 0 \\ 0.41 & s+12 & -0.94 & -2.2 \\ 0 & 0 & s & -1 \\ 0.12 & 24 & 0 & s+0.66 \end{bmatrix}; \\ M_i &= \begin{bmatrix} 8 \cdot 10^{-3} & 3 \cdot 10^{-4} & 0 & -3.9 \cdot 10^{-3} \\ 10 & 3.21 & 0 & -15.3 \end{bmatrix}^T; \\ \psi_i &= [u_x \quad \alpha_T]^T; \end{aligned}$$

where V is a linear velocity along the longitudinal axes; α is the angle of attack; θ is the pitch angle of an aircraft; δ_t is a throttle opening; δ_e is the elevator deflection angle; u_x is the velocity of turbulence along the longitudinal axes; α_T is the turbulence angle of attack.

In order to receive the results compared to the work [2] we presume that the disturbance signals u_x and α_T are centered stationary independent random processes of the type "white noise" with the intensities σ_u and σ_α correspondently. In this case [7] the transposed matrix of the spectral densities of the disturbances $S'_{\psi\psi}$ is as follows

$$S'_{\psi\psi} = \begin{bmatrix} S'_{rr} & O & O \\ O & S'_{\psi_0\psi_0} & O \\ O & O & S'_{\psi_0\psi_0} \end{bmatrix},$$

here S'_{rr} is the transposed spectral density matrix of the vector r which is equal

$$S'_{rr} = \frac{1}{\pi} \begin{bmatrix} \sigma_{v_0}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\alpha_0}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\theta}^2 & 0 \\ 0 & 0 & 0 & \sigma_q^2 \end{bmatrix} \frac{1}{|100s+1|^2};$$

$S'_{\psi_0\psi_0}$ is the following transposed spectral density matrix of the vector ψ_0

$$S'_{\psi\psi_i} = \frac{\sigma_u^2}{\pi} \Gamma_i \begin{bmatrix} 1 & 0 \\ 0 & \gamma^2 \end{bmatrix} \Gamma_i^*;$$

Γ_i is a matrix which characterises the impact of disturbance signals on the motion of the airplane

$$\Gamma_i = \begin{bmatrix} 0 & 0.41 & 0 & 0.12 \\ -25 & 12 & 0 & 23.5 \end{bmatrix};$$

γ is the ratio of intensities of disturbance effects ($\gamma=0.1$); σ_{v_0} , σ_{α_0} , σ_{θ} , σ_q are the intensities of random processes V , α , θ , $d\theta/dt$ correspondly.

As a result of fulfilment of all the above-mentioned phases of the research we obtain the following matrixes of the regulators' transfer functions

$$\begin{aligned} W_1^0 &= 10^{-3} \begin{bmatrix} 2.4 & -0.066 & 0.47 & 4.7 \cdot 10^{-3} \\ 890 & -25 & 33 & -0.325 \end{bmatrix}; \\ W_2^0 &= 10^{-3} \begin{bmatrix} -2.3 & -1.4 & 0.5 & -1.2 \\ -770 & -1120 & 3580 & -560 \end{bmatrix}; \\ W_1^1 &= 10^{-3} \begin{bmatrix} 2.3 & 1.4 & -0.5 & 1.2 \\ 770 & 1120 & -3580 & -560 \end{bmatrix}. \end{aligned}$$

CONCLUSIONS

The application of the above-grounded structural transformations and the obtained on their basis instrumental matrixes and vectors enabled proving a new approach to the synthesis of control systems of the groups of robots in frequency domain.

The received ratios make possible to find the structure and parameters of the optimal matrixes of regulators' transfer functions which should be installed on the board of the leader and the agents.

A joint coordinated work of the regulators in the conditions of disturbances and noises provides the system stability on the whole and minimizes the chosen functional of quality.

REFERENCES

- [1] Group control of movable objects in nondefinite environment: Monograph/Beloglazov D.A., Gaiduk A.R., Kosenko E.Yu., Medvedev M.Yu., Pshikhopov V.Kh., Solovyov V.V., Titov A.E., Finaev V.I., Shpvalov I.O. Under editorship of V. Kh. Pshikhopov. – PHYZMATLIT, 2015. 305 p. (in Russian)
- [2] Chulvoo P. Real-Time Leader-Follower UAV Formation Fight Based on Modified Nonlinear Guidance / P. Chulvoo, H.J. Kim, Y. Kim // 29th Congress of the International Council of the Aeronautical Sciences, St. Petersburg, Russia, September 7-12, 2014. pp. 1-8

- [3] Kucherov D. Group Behavior of UAVs in Obstacles Presence / D. Kucherov, A. Kozub, O. Kostyna // 4th International Conference on Methods and Systems of Navigation and Motion Control, Kiev. Pp. 51-54
- [4] Y. P. Kondratenko. "Robotics, Automation and Information Systems: Future Perspectives and Correlation with Culture, Sport and Life Science, in Decision Making and Knowledge Decision Support Systems," *Lecture Notes in Economics and Mathematical Systems*, vol. 675, A. M. Gil-Lafuente, C. Zopounidis, Eds. Springer International Publishing Switzerland, 2015, pp. 43–56.
- [5] Abraham A. Swarm Intelligence in Data Mining: Studies in Computational Intelligence, Volume 34. / A. Abraham, C. Grosan, V. Ramos - Springer Berlin Heidelberg New York, 2006. 275 p.
- [6] Azarskov V.N. Methodology of Constructing Optimal Systems of Stochastic stabilization: Monograph / V.N. Azarskov, L.N. Blokhin, L.S. Zhitetskiy / Edited by Blokhin L.N. – Kyiv: Published by the National Academy of Sciences, 2006. – 437p. (in Russian)
- [7] Statistic dynamics of control systems: textbook. L.B. Blokhin, M.Y. Burichenko, N.V. Bilan [at al.] – K.:NAU, 20014.-300p. (in Ukrainian)
- [8] Osadchiy S.I. Combined method for the synthesis of optimal stabilization systems of multidimensional moving objects under stationary random impacts / S.I. Osadchiy, V.A. Zozulya // *Journal of Automation and Information Sciences*. Vol. 45, 2013, Issue 6. pp. 25-35
- [9] Boegra H. O.Design Methods for Control Systems: Notes of course / Okko H. Boegra, Hubert Kwakernaak, Gjerrit Meinsma. – Dutch Institute of Systems and Control, 2006. – 325 p.
- [10] Davis M.C. Factoring the spectral matrix/ M.C.Davis // *IEEE Trans. Automat. Contr.* – 1963.- AC-8, N 4. – p. 296-305.
- [11] Naumenko K.I. Observation and Control of Motion of Dynamic Systems/K.I. Naumenko – K.: Naukova Dumka, 1984. – 208 p. (in Russian)